Abstract—This paper presents three alternative maximum peak power tracking-based control algorithms with stall regulation for optimal wind energy capture: torque reference-based MPPT algorithm, searching-based MPPT algorithm and fuzzy-based MPPT algorithm. The first algorithm uses a torque reference as a guideline for tracking. The second method searches an optimal operating point from the slope of the power-rotational speed curve. The last one achieves the control objective using a developed fuzzy logic rule base. These three algorithms were implemented on a low cost digital signal controller board and tested with a real-time wind turbine simulator. It is found from the experiment that the three algorithms can manage to obtain the maximum power for any input wind speeds but with different control performance in terms of power fluctuation, rotational speed fluctuation and tracking time.

Index Terms—maximum peak power tracking, wind turbine control, searching algorithm, fuzzy logic control

I. INTRODUCTION

As the wind energy has become one of the fastest growing renewable energy sources, the key issue of wind energy conversion systems is how to efficiently operate the wind turbines in a wide range of wind speeds. In general, a wind turbine is mechanically designed to produce its rated power at a certain wind speed which is referred as rated wind speed. Accordingly, there are two main operating regions: below-rated wind speed and above-rated wind speed. In the below rated wind speed region, a wind turbine controller starts to operate at its cut-in speed to extract energy as much as possible. For the above-rated wind speed, it is necessary to limit the mechanical power generated by the rotor to the rated value of the plant to prevent damage on the turbine. This can be done by either pitch control or stall regulation control but the former could be impossible for most of the small- and medium-sized fixed pitch wind turbines as their pitch angles are fixed. If wind speed increases, more energy will flow into the turbine and generator until the input energy reaches a specified power limit. At that point, the controller will operate to limit the power by reducing or increasing wind turbine speed to achieve the desired power.

A great deal of research has been done on optimum wind energy capture. For example, in [1], the authors used a torque-speed characteristic to locate the optimum operating point. In other words, the tracking process would have been impossible without the turbine characteristic, which may, in practice, not always available. This drawback is not involved in [2], where a fixed-step searching algorithm without knowledge of turbine characteristics was proposed for battery charging application. However, the disadvantage of that paper is that the algorithm is slow to respond a change of wind speeds. Therefore, [3] proposed a variable-step searching algorithm to eliminate such a disadvantage but power limitation in the above rated wind speed was not included. Fuzzy logic control was introduced in [4] for maximum energy tracking; nevertheless, the torque reference was still required.

This paper presents three alternative maximum peak power tracking (MPPT)-based algorithms with stall regulation for optimal energy capture from fixed pitch wind turbines. The first algorithm is guided by a torque reference. The second method searches an optimal operating point from the slope of the power-rotational speed curve. The last one achieves the control objective using a fuzzy logic rule base. For the above wind speed, stall regulation is employed to keep the power within a specified power limit. All the control algorithms were implemented on a digital signal controller (DSC) board and tested on a developed 7.5 kW wind turbine simulator. A comparative performance among the three methods is given in detail. The paper is organized as follows. Section II describes the wind turbine characteristic, Section III describes the three MPPT-based algorithms. The control block diagrams are described in Section IV. Case studies are given in Section V. Section VI concludes the paper.
II. WIND TURBINE CHARACTERISTIC

The power captured by the wind turbine, $P_{\text{turb}}$, is described (1). The output power is a function of wind speed cube and power coefficient which depend on the wind characteristic and its operating point.

$$P_{\text{turb}} = \frac{1}{2} \rho \pi R^2 v_t^3 c_p(\lambda, \beta)$$  

(1)

where

- $\rho$ = air density
- $R$ = turbine radius
- $v_t$ = wind speed
- $c_p$ = power coefficient
- $\beta$ = pitch angle
- $\lambda = \text{the tip speed ratio}$

The tip speed ratio is mathematically expressed by

$$\lambda = \frac{\omega_t R}{v_t}$$  

(2)

where $\omega_t$ = turbine rotational speed

The aerodynamic torque acting on the blades, $T_{\text{turb}}$, is obtained from

$$T_{\text{turb}} = \frac{1}{2} \rho \pi R^3 v_t^2 c_p(\lambda, \beta)$$  

(3)

where $c_T = \text{torque coefficient}$

If $c_p$ is known, the aerodynamic torque can also be calculated from

$$T_{\text{turb}} = \frac{1}{2} \rho \pi R^3 v_t^2 c_p(\lambda, \beta) / \omega_t$$  

(4)

It can be seen from the above two equations that $c_T$ and $c_p$ are a function of $\lambda$ and $\beta$. In this paper, the fixed pitch wind turbine is of interest and therefore $\beta$ is kept constant. This is generally true for small- and medium-sized fixed-pitch wind turbines. Therefore, $c_T$ and $c_p$ depend only on $\lambda$. Figure 1 shows the $c_T$ and $c_p$ curves as a function of $\lambda$ of a three blade horizontal axis wind turbine. These curves represent an important characteristic that determines the starting torque and the value of maximum power coefficient, $c_p^{\text{max}}$. The $c_p$ curve, indicating the efficiency of power conversion of the rotor blades, can be obtained by the multiplication of $c_T$ and $\lambda$. Note that the power and torque coefficient of a wind turbine depends on aerodynamic design of the blades.

III. MPPT ALGORITHMS

The main purpose of MPPT algorithms is to maintain operating points on $c_p^{\text{max}}$ for any wind speeds (see Fig. 1). The operating points can be on the positive slope (the left side of $c_p^{\text{max}}$), zero slope (at $c_p^{\text{max}}$), and negative slope (the right side of $c_p^{\text{max}}$). If an operating point is in the positive slope region, the controller will move it to the right to get closer to the optimum tip speed ratio, $\lambda$. This can be achieved by decreasing load current which results in an increase in the rotational speed. Conversely, if the operating point lies on the right hand side of the peak, the load current has to be increased, resulting in a decrease in the rotational speed. With this principle, the operating point can be maintained at $c_p^{\text{max}}$ for various wind speeds.

![Fig. 1. Power and torque with MPPT tracking process](image)

A. Torque Reference-Based MPPT Algorithm

This method requires the wind turbine characteristics (e.g., $R$, $c_p^{\text{max}}$ and $\lambda$). The below rated wind speed reference torque, $T_{\text{ref}}$, can be calculated by substituting (2) into (4).

$$T_{\text{ref}} = k_i \omega_t^2, \quad P_i < P_{\text{rated}}$$  

(5)

where

- $k_i = \frac{1}{2} \rho \pi R^3 c_p^{\text{max}} / \lambda$  
- $\lambda_0 = \text{tip speed ratio at } c_p^{\text{max}}$

The reference torque for the above rated wind speed, $T_{\text{sh}}$, is calculated from the rating of the output power, $P_{\text{rated}}$.

$$T_{\text{sh}} = P_{\text{rated}} / \omega_t, \quad P_i \geq P_{\text{rated}}$$  

(6)

The controller uses (5) and (6) as the reference torque to control the plant.

The torque reference-based MPPT algorithm can be alternatively achieved by testing the wind turbine to find the optimal torque-rotation speed curve with various wind speeds as shown in Fig. 2 a). The reference torque trajectory can be mathematically written as a function of torque and rotational speed or it can be stored as a look up table which is easy to be programmed in a microcontroller or a DSC board. Figure 2 a) and b) show the reference torque trajectory and corresponding output.
power in the below and above rated wind speed, respectively [5].

**B. Searching-Based MPPT Algorithm**

This algorithm brings the operating point toward $P_{\text{opt}}$ by increasing or decreasing the rotational speed step by step. This tracking methodology is called the perturbation and observation method. To limit the output power at a specified power limit by the stall regulation, a controller will reduce the rotational speed until the power coefficient reduces to the power limit while if the power is lower than the power limit, the controller will increase the rotational speed until the power equals the power limit. The control flowchart of the maximum power tracking system in Fig. 3 illustrates the details of decision processes based on the tracking procedure in Fig. 2. If the rotational speed is higher than the cut-in speed, $\omega_{\text{cut-in}}$, the MPPT controller will start the procedure. If a given perturbation leads to a positive or negative slope, the next perturbation increases or decreases the rotational speed until the slope becomes zero (i.e., maximum power point is reached). An updated load current reference, $i_{\text{ref}}$, for each sampling period, $T_s$, and an instantaneous power slope are calculated by (7) and (8), respectively [6, 7].

$$i_{\text{ref}}[(k+1)T_s] = i_{\text{ref}}[(k)T_s] + M \frac{\Delta p[(k)T_s]}{\Delta \omega[(k)T_s]}$$  \hspace{1cm} (7)

$$\text{slope} = \frac{\Delta p[(k)T_s]}{\Delta \omega[(k)T_s]}$$  \hspace{1cm} (8)

where $M = \text{update factor}$ \hspace{1cm} $i_{\text{ref}}[(k)T_s] = \text{current at } k$

**C. Fuzzy-Based MPPT Algorithm**

A fuzzy logic control (FLC) algorithm is characterized by “IF-THEN” rules. The algorithm is suitable for wind turbine control with complex nonlinear models and parameters variation. Like the second algorithm, the fuzzy-based MPPT uses the perturbation and observation to track the maximum output power in the below rated wind speed without knowledge of wind turbine characteristic. The input variables of fuzzy-based MPPT are a rotational speed and an aerodynamic torque observer $\hat{T}_{a}$. In the above rated wind speed, the FLC uses a torque reference calculated from (6) to limit the output power at the power limit. Another two input parameters, $\Delta \hat{T}_{a}$ and $\Delta \omega$, are used to limit torque and speed fluctuation. The MPPT with FLC uses $\hat{T}_{a}$ to classify the operating regions.

**Membership Functions of Input and Output Variables**

In the fuzzification process, the relevant numerical parameters are linguistically converted into equal-base symmetric triangles (for the output) and trapezoidal membership functions [8, 9]. The membership functions for six input parameters consisting of $E_{\text{be}}$ and $E_{\text{ab}}$, defined in (9), $T_{a}$, $\Delta T_{a}$, $\omega_{l}$ and $\Delta \omega_{l}$ are shown in Fig. 4. The membership function for one output parameter,
\( \Delta T_g \), is shown in Fig. 5. All the membership functions are normalized in a range of \([-1,1]\).

\[
E_{he} = T_{he}^{ref} - T_a \\
E_{ab} = T_{ab}^{ref} - T_a
\]  

(9)

**Fuzzy Rule Base**

Fuzzy rule base that associates the fuzzy output to the fuzzy inputs is derived from the system behavior. It basically contains the knowledge acquired by designers as fuzzy rules and is expressed in forms of IF-THEN rules. The sole objective of the fuzzy rules designed here is to keep the wind turbine operating at the optimal point by using torque control for the two regions [10].

**Below-Rated Wind Speed Region**

The FLC knows how to operate in this region by observing whether the aerodynamic torque is lower than \( T_a \) (9). In this region, only three input parameters are needed to determine the output: \( E_{he} \), \( T_a \), and \( \omega_l \). The FLC tracks the torque reference given in (4) to obtain the maximum peak power. The relationship between \( T_a \) and \( \omega_l \) generates \( E_{he} \) with 35 rule base when \( T_a \) is s (see Fig. 4 c).

**Above-Rated Wind Speed Region**

When the aerodynamic torque is greater than \( T_a \) (i.e., \( T_a \) is ns), the FLC tracks the torque reference by (7) and uses five input parameters: \( E_{ab} \), \( \Delta \omega_l \), \( \Delta T_a \), \( T_a \) and \( \omega_l \) to generate \( \Delta T_g \). The first three parameters are used to create the 105 rule base whereas \( T_a \) decides which region the controller will operate. The final parameter is for overspeed protection; to be specific if \( \omega_l \) is b, then \( \Delta T_g \) is pbb [11].

**Defuzzification**

Defuzzification is a process that converts the fuzzy set representing the overall conclusion into a real number after the evaluation of the rule base module. There are various types of defuzzification but a widely used one is the center of gravity (or centroid) defuzzification method, which determines the center of gravity of output membership function. If the discrete fuzzy set is applied in Fig. 6, the center of gravity of output membership function, \( U_0 \), can be obtained from

\[
U_0 = \frac{1}{n} \sum_{i=1}^{n} \mu(U_i)
\]  

(10)

where \( \mu(U_i) \) = fuzzy output value at \( i \)

\[
\mu(U_i) = \text{area of final fuzzy value at } i
\]

\[
\text{Center of gravity}
\]

Fig. 6. Discrete fuzzy defuzzification for center of gravity method

**D. Aerodynamic Torque Observer**

It is very important to know the aerodynamic torque to avoid the measurement of the wind speed. This paper uses the aerodynamic torque observer based on the standard state-space model of a plant expressed by (11). The system can be transformed from the continuous domain to the discrete domain by the z transformation with zero order hold (ZOH), as given in (12).

\[
\dot{x} = Ax + Bu, \quad y = Cx
\]  

(11)

\[
y[k + 1] = \Phi x[k] + \Gamma u[k], \quad y[k] = cz[k]
\]  

(12)

The closed-loop observer structure of (11) is

\[
\hat{x}[k + 1] = \Phi \hat{x}[k] + \Gamma u[k] + K e(x[k] - \hat{x}[k])
\]

where

- \( \text{nbb} = \text{negative big big} \)
- \( \text{nb} = \text{negative big} \)
- \( \text{nm} = \text{negative medium} \)
- \( \text{ns} = \text{negative small} \)
- \( \text{nss} = \text{negative small small} \)
- \( \text{z} = \text{zero} \)
- \( \text{ps} = \text{positive small} \)
- \( \text{pm} = \text{positive medium} \)
- \( \text{pb} = \text{positive big} \)
- \( \text{pbb} = \text{positive big big} \)
- \( \text{s} = \text{small} \)
\[ \dot{x}[k+1] = (\Phi - Kc)\dot{x}[k] + \Gamma u[k] + Ky[k] \]  

where \( K \) = observer gain

The observer connected to the plant calculates an error between the output of the plant and observer and drives the error toward zero. For the sake of simplicity, a wind power drive train is modeled as one mass with the assumptions that there is no interaction between the drive train and tower dynamics as well as no gravitational force that acts on the blade and causes periodic excitation. The mathematical equations to model the driving train consist of aerodynamic dynamic torque (14), rotational acceleration and auxiliary torque (15). To derive (14), it is assumed that the aerodynamic torque has much slower variation than sampling rate, \( T_s \), so giving (16).

The state equations of (15) and (16) are shown in (17).

With (17), an observe gain, \( K \), can be calculated.

\[
T_a = J_i \dot{\omega}_i + B_i \omega_i + T_g \tag{14}
\]

\[
\dot{\omega}_i = \frac{1}{J_i} (T_{aux} - T_g) \tag{15}
\]

where \( J_i \) = moment of inertia of turbine
\( B_i \) = viscous friction
\( T_g \) = generator torque
\( T_{aux} = T_a - B_i \omega_i \)
\( \dot{T}_a = 0 \)

\[
\begin{bmatrix}
\dot{\omega}_i \\
\dot{T}_{aux}
\end{bmatrix}
= \begin{bmatrix}
0 & 1/J_i \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\omega_i \\
T_{aux}
\end{bmatrix}
- \begin{bmatrix}
1/J_i \\
0
\end{bmatrix} T_g
\]

\[
y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_i \\ T_{aux} \end{bmatrix} \tag{17}
\]

IV. MPPT CONTROL BLOCK DIAGRAM

A. Control Block Diagram of Torque Reference-Based MPPT

The torque reference-based MPPT block diagram of the system is shown in Fig. 7. This system receives the rotational speed from the plant. The rotational speed is sent to the torque reference look up table to interpolate the aerodynamic torque reference, \( T^{ref} \). This reference will be sent to a load regulation loop (lower right corner of the figure), where a PI compensator is used to control the load current as desired and improve the system stability.

B. Control Block Diagram of Searching-Based MPPT

To implement the MPPT and stall regulation control in a DSC controller unit, total power \( P_t \) should be written in more detail as given in (18).

\[
P_t = J_i \dot{\omega}_i + P_e + T_f \omega_i + V_b i_g + i_g^2 R_g \tag{18}
\]

where \( P_e \) = electrical power
\( T_f \) = friction torque
\( V_b \) = brush voltage
\( i_g \) = generator current
\( R_g \) = generator resistance

The overall block diagram of control system is shown in Fig. 8. The dash line covers the MPPT controller built from (9) for below rated wind speed. The bottom block shows the stall control for power limit. It is connected to the plant consisting of the wind turbine coupled with a generator, a dc/dc converter, a load and sensors. The controller receives generator voltage and current signals from the sensors. The output power, the slope of power with respect to speed \( \dot{\omega}_i \), and current references are updated using (7). This reference will be sent to a load regulation loop (lower right corner of the figure), where a proportional-integral (PI) compensator is used to control the load current as desired and improve the system stability.

C. Control Block Diagram of Fuzzy-Based MPPT

The FLC block diagram of the system is shown in Fig. 9. This system receives the current, voltage and rotational...
speed from the plant and then send them to the torque observer to estimate the aerodynamic torque, $T_a$. This controller converts the six input parameters to predefined membership functions in Fig. 4 and sends them to a FLC. The FLC calculates $\Delta T_g$ using the input membership functions, fuzzy rule base and defuzzification to update the output torque and output command.

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### V. Case Studies

#### A. Test System

The test system is composed of two main parts: 1) a developed wind turbine simulator on the left side of Fig. 10 and 2) a purposed wind turbine controller on the right side. The wind turbine simulator consists of a torque control inverter connected to a 7.5 kW induction motor and voltage and current sensors, data acquisition and a DSC controller board. The DSC controller board uses a high performance 16 bits dsPIC30f6010, which combines the advantage of a high performance microcontroller and high computation speed digital signal processors (DSP) [14]. The software used to control the simulator was implemented in this DSC linked with a personal computer via two RS232 ports: one for transferring wind speed data to the DSC board and the other for sending parameters (e.g., $P_L$, $i_L$, $v_L$, $\omega_L$, $\omega_g$) to the computer. The proposed wind turbine controller on which the feedforward and MPPT algorithms are implemented consists of a dc/dc converter connected between a generator and a load, voltage and current sensors, a data acquisition unit and a DSC controller with the same performance as the one used in the simulator [15].

#### B. Output Power and Control Trajectory

Figures 11-13 shows control trajectories of the three MPPT-based algorithms with five different wind speeds: 3, 3.5, 4, 4.5 and 5 m/s. As can be seen from the figures, in the below rated wind speed (less than 700W), the MPPT with torque reference controller succeed in tracking the maximum power for each wind speed with the lowest rotational speed variation. When the wind speed is stepped up from 4.5 to 5 m/s, the system starts to limit the output power at 700W. If the wind speed increases beyond 5 m/s, the operating point will move to the left hand side of the previous one with a constant output power of 700W but with higher aerodynamic torques. During the move, the controller decreases the rotational speed until the wind turbine has been stalled. It can be seen that there is a high fluctuation in the output power in the above rated wind speed.

The searching-based MPPT controller tracks the maximum peak power at each wind speed with highest rotational speed fluctuation in the below rated wind speed. Around the output power limit (700 W), it can be seen that this algorithm has a slightly lower power fluctuation than that of the first algorithm. Note that the second algorithm uses the lower part of the control diagram in Fig. 8 to calculate the reference torque in the above rated wind speed. Referring to Fig. 9, the fuzzy-based MPPT controller tracks the maximum peak power with the lowest power fluctuation as well as fairly low rotational speed fluctuation in the below rated wind speed. The tracking time for the three control algorithms is compared in Fig. 14. It is clearly seen that the torque reference-based MPPT and the fuzzy-based MPPT are fastest in the below rated wind speed and the above rated wind speed, respectively.

### VI. Conclusion

The three MPPT-based control algorithms for optimum energy capture from a wind turbine in its below- and above-rated wind speed were presented. The algorithms were implemented on a low cost DSC board and tested with a developed wind turbine simulator. The first algorithm tracks the maximum power using a torque reference obtained from the wind turbine characteristic. The second method is based on the observation that the power versus rotational speed curve has a single well-defined peak. Therefore, a necessary condition for the speed being at the maximum power point is that the first derivative of the power respect to the rotational speed is zero. The third algorithm employs a fuzzy logic as the key controller. It can be concluded from the experimental results that the MPPT with torque reference offers fastest tracking time in the below rated wind speed and the MPPT with fuzzy logic is favored in terms of power fluctuation and tracking time in the above rated wind speed. Although the second method has the slowest tracking time and the highest rotational speed fluctuation, it is attractive for a small amount of computational resource and therefore low cost for implementation.
REFERENCES