Enhanced augmented Lagrange Hopfield network for economic dispatch with piecewise quadratic cost functions

Vo Ngoc Dieu and Weerakorn Ongsakul

Abstract—This paper proposes a simple enhanced augmented Hopfield Lagrange neural network (EALH) for solving economic dispatch (ED) problem with piecewise quadratic cost functions. The EALH is an augmented Lagrange Hopfield neural network (ALH), which is a combination of continuous Hopfield neural network and augmented Lagrangian relaxation function as its energy function, enhanced by a heuristic search algorithm for determination of fuel type. The proposed EALH solve the problem in two phases. In the first phase, a heuristic algorithm based on average production cost of generating units is used to determine the most suitable fuel type of units satisfying load demand. In the last phase, the ALH is applied to solve economic dispatch to find optimal solution with the fuel types selected. The proposed method is tested on two test systems with various load demands and compared to hierarchical approach based on the numerical method (HNUM), Hopfield neural network (HNN), adaptive Hopfield neural network (AHNN), enhanced Lagrangian artificial neural network (ELANN), improved evolutionary programming (IEP), modified particle swarm optimization (MPSO), and hybrid real coded genetic algorithm (HRCGA). The results have shown that the proposed method is efficient and fast for the ED problems with multiple fuel types represented by quadratic cost functions.

Keywords—Augmented Lagrange Hopfield neural network, economic dispatch, piecewise quadratic cost function.

NOMENCLATURE

\[ a_{ik}, b_{ik}, c_{ik} \] cost coefficients for unit \( i \) with fuel type \( k \);
\[ B_{ij}, B_{i0}, B_{00} \] transmission loss formula coefficients;
\[ E_{max}\ ] \] maximum error of neural network;
\( i \) \] index of generating units;
\( k \) \] index of fuel types;
\( M_{ik} \) \] priority index for the unit \( i \) with fuel \( k \), in \$/MWh;
\( N \) \] number of online generating units;
\( N_{\text{max}} \) \] maximum number of iterations;
\( n \) \] iterative index of neural network;
\( P_{D} \) \] total load demand of the system, in MW;
\( P_{L} \) \] total network loss of the system, in MW;
\( P_{i} \) \] output power of unit \( i \), in MW;
\( P_{i}^{(0)} \) \] output power of unit \( i \) neglecting power loss, in MW;
\( P_{ik}^{\text{avg}} \) \] average output power of unit \( i \) with fuel \( k \), in MW;
\( P_{i}^{\text{min}}, P_{i}^{\text{max}} \) \] lower and upper generation limits of unit \( i \), in MW;
\( PF_{i} \) \] participation factor of unit \( i \);
\( R_{i} \) \] second derivative of cost function of unit \( i \);
\( U_{i} \) \] input of continuous neuron \( i \) corresponding to the output \( V_{i} \);
\( U_{\lambda} \) \] input of multiplier neuron corresponding to the output \( V_{\lambda} \);
\( V_{i} \) \] output of continuous neuron \( i \) representing for output power \( P_{i} \);
\( V_{\lambda} \) \] output of multiplier neuron representing Lagrangian multiplier \( \lambda \);
\( V_{i}^{(0)} \) \] initial value of \( V_{i} \);
\( V_{\lambda}^{(0)} \) \] initial value of \( V_{\lambda} \);
\( w_{1} \) \] weighting of \( P_{i}^{\text{max}} \) in priority index \( M_{ik} \);
\( w_{2} \) \] weighting of \( M_{ik} \) in the combined average priority index \( x_{ik} \);
\( x_{ik} \) \] combined priority index based on priority index \( M_{ik} \) and average incremental cost \( \lambda_{ik}^{\text{avg}} \);
\( \sigma \) \] slope of sigmoid function of continuous neurons;
\( \varepsilon \) \] maximum tolerance for neural network;
\( \alpha_{i} \) \] continuous neuron updating step sizes;
\( \alpha_{\lambda} \) \] multiplier neuron updating step size;
\( \beta \) \] penalty factor associated with power balance;
\( \lambda \) \] Lagrangian multiplier associated with power balance;
\( \lambda_{ik}^{\text{avg}} \) \] average incremental cost of fuel \( k \) of unit \( i \);

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1. INTRODUCTION

The operation cost in power systems needs to be minimized at each time via economic dispatch (ED). In practical power system operation conditions, many thermal generating units, especially those units are supplied with multiple fuel sources like...
coal, natural gas and oil, require that their fuel cost functions may be segmented as piecewise quadratic cost functions for different fuel types. The ED problem with piecewise fuel cost functions is to minimize fuel cost among the available fuels of each unit satisfying load demand and generation limits. This is a non-convex optimization problem. Since this representation contains discontinuous values at each boundary and many local optima, thus classical solution methods are difficult to deal with this problem. One approach of solving the problem with such units with multiple fuel options is to linearize the segments and solve them by traditional methods [1]. A better approach is to retain the assumption of piecewise quadratic cost functions and proceed to solve them. A hierarchical approach based on the numerical method (HNUM) was proposed in [2] as a one way to approach the problem. However, the major problem with the numerical methods is their exponentially growing time complexities for the larger systems, especially with non-convex constraints. The application of Hopfield neural network (HNN) [3] with merit of simplicity created difficulties in handling some kinds of inequality constraints, and dealing with large scale problems with many constraints. Moreover, the convergence of the HNN is also sensitive to the choice of penalty factors for constraints. In an enhanced Lagrangian neural network (ELANN) [4], the dynamics of Lagrange multipliers including equality and inequality constraints were improved to guarantee the convergences to the optimal solutions, and momentum was also employed to achieve fast convergence. However, both HNN and EALNN were involved a large number of iterations for convergence and often shown oscillation during transients. Adaptive Hopfield neural network (AHHN) [5], [6] is an improvement of the HNN by adjusting slope and bias of neurons during convergence process to obtain faster performance.

Recently, heuristic optimization techniques have been applied to solve the problem such as genetic algorithm (GA) [7], [8], evolutionary programming (EP) [9], [10], and particle swarm optimization (PSO) [11]. The GA is critically dependent on the fitness function and sensitive to the mutation and crossover rates, the encoding scheme of its bits, and the gradient of the search space curve leading toward solutions. With parallel searching mechanism, EP method has a high probability to find optimal solutions. For large scale complex problems, the solution may get trapped in a suboptimal state, where the variation operators cannot produce any offspring outperforming its parents. These applications involve a large number of iterations and were susceptible to the related control parameters [11]. More recently, particle swarm optimization (PSO) has been widely applied in power system optimization problems. Although this technique can generate high-quality solutions within shorter calculation time and stable convergence characteristic than some other methods [12], it seems to be sensitive to the tuning of some weights or parameters. This technique is still in research for proving its potential in solving complex power system problems. A hybrid system of EP, tabu search and quadratic programming (ETQ) was proposed in [13] to overcome the difficulties of single techniques.

In this paper, an enhanced augmented Lagrangian Hopfield neural network (EALH) is proposed for solving economic dispatch problem with piecewise quadratic cost functions for units having multiple fuels. The EALH is an augmented Lagrange Hopfield neural network (ALH) enhanced by a heuristic search based algorithm for fuel type determination. The proposed method solves the problem in two phases. In the first phase, a heuristic search algorithm is used to find out most suitable fuel types for units based on combined priority list index for each unit satisfying load demand. In the last phase, ALH is applied to solve final ED problem based on the fuel types determined from the heuristic search. To illustrate the effectiveness of the proposed EALH, it is tested on two test systems with various load demands and compared to hierarchical approach based on the numerical method (HNUM) [2], Hopfield neural network (HNN) [3], adaptive Hopfield neural network (AHHN) [5], [6], enhanced Lagrangian artificial neural network (ELANN) [4], improved evolutionary programming (IEP) [10], modified particle swarm optimization (MPSO) [11], and hybrid real coded genetic algorithm (HRCGA) [8].

The organization of this paper is follows. Section 2 addresses the formulation of economic dispatch problem with quadratic fuel cost functions. Heuristic search based algorithm for fuel type determination is presented in Section 3. Augmented Lagrange Hopfield neural network for the problem is described in Section 4. Numerical results are followed in Section 5. Finally, the conclusion is given.

2. PROBLEM FORMULATION

The objective of conventional ED problem is to minimize total cost of thermal generating units while satisfying various constraints including power balance and generator power limits. In the ED problem with piecewise quadratic cost functions, the piecewise quadratic function is used to represent multiple fuels which are available for each generating units [3].

The problem is formulated as follows.

Min \( F = \sum_{i=1}^{N} F_i(P_i) \) \hspace{1cm} (1)

\( \begin{align*}
F_i(P_i) &= \begin{cases} 
\alpha_i + \beta_i P_i + \gamma_i P_i^2, & \text{fuel 1}, P_i^\text{min} \leq P_i \leq P_i^\text{max}, \\
\alpha_j + \beta_j P_i + \gamma_j P_i^2, & \text{fuel 2}, P_i^\text{min} \leq P_i \leq P_i^\text{max}, \\
\vdots & \\
\alpha_k + \beta_k P_i + \gamma_k P_i^2, & \text{fuel } k, P_i^\text{min} \leq P_i \leq P_i^\text{max} 
\end{cases}
\end{align*} \) \hspace{1cm} (2)

subject to

(a) power balance constraint

\( \sum_{i=1}^{N} P_i - P_L - P_D = 0 \hspace{1cm} (3) \)

(b) generator operating limits

\( P_i^\text{min} \leq P_i \leq P_i^\text{max} \hspace{1cm} (4) \)

3. HEURISTIC SEARCH FOR FUEL TYPE DETERMINATION

Each unit has multiple fuel types with each of them is represented by a quadratic cost function. To solve the ED problem with multiple fuel types, a heuristic search algorithm is first used to find out most appropriate fuel type for each unit satisfying load demand based on a combined priority index before the final ED problem is solved.

The average production cost for each fuel type is defined as follows.

\( M_i = \frac{F_i(P_i)}{P_i^\text{avg}} \) \hspace{1cm} (6)

where

\( P_i^\text{avg} = w_i P_i^\text{min} + (1 - w_i) P_i^\text{max} \hspace{1cm} 0 \leq w_i \leq 1 \) \hspace{1cm} (7)

Average incremental cost for each fuel type is defined as follows [14].
A combined priority index based on the priority index \( M_{ik} \) and average incremental cost \( \lambda_{ik}^{opt} \) is defined as follows.

\[
x_{ik} = w_1 M_{ik} + (1 - w_2) \lambda_{ik}^{opt}; \quad 0 \leq w_2 \leq 1
\]  

For each unit, which segment has lower \( x_{ik} \) will have priority to be selected for operation.

Procedures for fuel type determination for each unit satisfying load demand as follows.

1. Step 1: Determine the combined priority index \( x_{ik} \) for each fuel type of each unit as in (9).
2. Step 2: Create a priority list of fuels of all units.
3. Step 3: Starting from the first fuel type of each unit for checking total power supply to load demand. Delete these fuels from the list and check the total power supply to the load demand.
4. Step 4: Sort the list in ascending checking total power supply to load demand.
5. Step 5: Select the next fuel in the list. Delete this fuel from the list and check the total power supply to the load demand.
6. Step 6: Repeat Step 5 until the total power supply from the units is sufficient to the load demand.
7. Step 7: Stop the procedure.

4. AUGMENTED LAGRANGE HOPFIELD NEURAL NETWORK FOR ED

After obtaining fuel types for each unit by heuristic search, the ALH is applied to solve the final ED problem satisfying power balance and generator limits.

The augmented Lagrange function \( L \) of the problem is formulated as follows.

\[
L = \sum_{i=1}^{N} F_i(P_i)
\]

\[
+ \lambda \left( P_k + P_D - \sum_{i=1}^{N} P_i \right) + \frac{1}{2} \beta \left( P_k + P_D - \sum_{i=1}^{N} P_i \right)^2
\]

To represent in augmented Lagrange Hopfield neural network, \( N \) continuous neurons representing output powers of \( N \) generating units and one multiplier neuron representing the Lagrangian multiplier are required. The energy function \( E \) of the problem is formulated based on the augmented Lagrangian function in terms of neurons as follows.

\[
E = \sum_{i=1}^{N} F_i(V_i)
\]

\[
+ V_2 \left( P_k + P_D - \sum_{i=1}^{N} V_i \right) + \frac{1}{2} \beta \left( P_k + P_D - \sum_{i=1}^{N} V_i \right)^2
\]

\[
+ \sum_{i=1}^{N} \frac{1}{2} g^{-1}(V_i) dV + \frac{1}{2} g^{-1}(V_i) dV
\]

where the sums of integral terms are Hopfield terms where their global effect is a displacement of solutions toward the interior of the state space [15].

The dynamics of the ALH for updating neuron inputs are defined as follows.

\[
\frac{dU_i}{dt} = -\frac{\partial E}{\partial V_i}
\]

\[
+ \left[ V_2 + \beta \left( P_k + P_D - \sum_{i=1}^{N} V_i \right) \right] \left[ \frac{\partial P_i}{\partial V_i} - 1 \right] U_i
\]

\[
\frac{dU_i}{dt} = \frac{\partial E}{\partial V_i}
\]

where,

\[
\frac{\partial P_i}{\partial V_i} = 2 \sum_{j=1}^{N} B_{ij} V_j + B_{i0}
\]

The algorithm for updating inputs of neurons is as follows.

\[
U_i^{(n+1)} = U_i^{(n-1)} - \alpha \frac{\partial E}{\partial U_i}
\]

\[
U_i^{(n+1)} = U_i^{(n-1)} + \alpha \frac{\partial E}{\partial U_i}
\]

The outputs of continuous neurons representing for output power of generating units are limited by maximum and minimum values and calculated by a sigmoid function [16]:

\[
V_i = g(U_i) = \left\{ \begin{array}{ll}
\frac{P_{max} - P_{min}}{2} [1 + \tanh(\sigma U_i)] + P_{min} & \text{if } P_{min} < V_i < P_{max} \\
0 & \text{otherwise}
\end{array} \right.
\]

For each unit, which segment has lower \( x_{ik} \) will have priority to be selected for operation.

Procedure for fuel type determination for each unit satisfying load demand as follows.

Step 1: Determine the combined priority index \( x_{ik} \) for each fuel type of each unit as in (9).
Step 2: Create a priority list of fuels of all units.
Step 3: Starting from the first fuel type of each unit for checking total power supply to load demand. Delete these fuels from the list.
Step 4: If the total power supply from the units is sufficient to the load demand, go to Step 7.
Step 5: Sort the list in ascending \( x_{ik} \) associated with fuel types.
Step 6: Select the next fuel in the list. Delete this fuel from the list and check the total power supply to the load demand.
Step 7: Repeat Step 5 until the total power supply from the units is sufficient to the load demand.
Step 8: Stop the procedure.

Fig. 1. Sigmoid function with different values of the slope.
\[
\frac{dE}{dt} = \frac{\partial E}{\partial V_i} \frac{dV_i}{dt}
\]

Substituting (17) into (19):
\[
\frac{dE}{dt} = \frac{\partial E}{\partial V_i} \frac{dg(U_i)}{dU_i} \frac{dU_i}{dt}
\]

Substituting (12) into (20):
\[
\frac{dE}{dt} = \frac{1}{2} \frac{dU_i}{dt} \left( \frac{dU_i}{dt} \right)^2
\]

Since \(g(U_i)\) is a monotonically increasing function, the value of \(dE/dt\) is always negative. Therefore, the energy function of Hopfield network is minimized with respect to continuous neurons when these neurons change their status.

On the other hand, for example, the effect of the multiplier neuron associated with power balance constraint on energy function is considered as follows.
\[
\frac{dE}{dt} = \frac{\partial E}{\partial V_i} \frac{dg(U_i)}{dU_i} \frac{dU_i}{dt}
\]

Substituting (18) into (22):
\[
\frac{dE}{dt} = \frac{\partial E}{\partial V_i} \frac{dg(U_i)}{dU_i} \frac{dU_i}{dt}
\]

Substituting (13) into (23):
\[
\frac{dE}{dt} = \left( \frac{dU_i}{dt} \right)^2
\]

The right hand side of the equation (24) shows that \(dE/dt\) is always positive, that means the energy function (11) is always maximized with respect to the change of multiplier neurons.

**Selection of parameters**

The proper parameter selection will guarantee fast convergence of the network. By experiments, the values of \(\sigma\) and \(\beta\) are fixed at 1x10^6 and 1x10^3, respectively. The other parameters will vary depending on the data being processed. Too large values of \(\alpha_i\) and \(\alpha_c\) will cause the network to behave like a discrete system producing values at the upper and lower limits of each neuron. In contrast, too small values of \(\alpha_i\) and \(\alpha_c\) will cause the network to converge very slowly.

**Initialization**

The algorithm requires initial conditions for all neurons. For the continuous neurons representing for output power of units, the inputs are initiated by “mean distribution” [2] as follows.
\[
V_i^{(0)} = P_D \frac{p_{i,max}}{\sum_{i=1}^{N} p_{i,max}}
\]

For the multiplier neuron associated with power balance constraint, the output is initialized by solving \(\partial E/\partial V_i = 0\) from (12), in which \(\beta\) and \(U_i\) are neglected.
\[
V_i^{(0)} = \frac{1}{N} \sum_{i=1}^{N} b_i + 2c_i \frac{V_i^{(0)}}{1 - \frac{\partial P_i}{\partial V_i}}
\]

**Termination criteria**

The algorithm of the ALH will be terminated when either the maximum error \(Err_{max}\) including both constraint and iterative errors of continuous and multiplier neurons is lower than a predefined tolerance \(\varepsilon\) or the maximum number of iterations \(N_{max}\) is reached.

Procedure of the ALH for the ED problem is as follows.

Step 1: Select sigmoid function for continuous neurons and updating step sizes for all neurons.

Step 2: Initialize all neurons as (25)-(26).

Step 3: Calculate dynamics of all neurons using (12)-(13).

Step 4: Calculate new inputs of all neurons using (15)-(16).

Step 5: Calculate outputs of all neurons using (17)-(18).

Step 6: Determine maximum error \(Err_{max}\) of the ALH.

Step 7: If \(Err_{max} < \varepsilon\), go to Step 3. Otherwise, stop.

**Participation factor method**

Transmission loss can be calculated by power load flow instead of the typical B-coefficient method as in (4). In this case, the transmission power loss is computed from load flow analysis and distributed optimally to generating units by using participation factor is used [17]. The participation factor of the \(i\)th generating unit is calculated as follows
\[
PF_i = \frac{1}{R_i} \sum_{j=1}^{N} \frac{1}{R_j}
\]

The total generation of each generating unit is updated by
\[
P_i = P_i^{(0)} + PF_i \cdot P_t
\]

**Overall procedure**

Overall procedure for the EALH for the ED problem with quadratic fuel cost functions and transmission loss computed by B-coefficient method consist of only phases. Firstly, the heuristic search for fuel determination for each unit in Section 3 is executed satisfying load demand. Finally, the ALH in Section 4 is applied to solve the final ED problem with the determined fuel types. Procedure of the EALH for the problem with transmission loss computed from power load flow as is follows

Step 1: Use heuristic search to determine feasible fuel for each generating unit so that total power supply from generating units satisfying load demand as in Section 3.

Step 2: Apply ALH in Section 4 to solve economic dispatch problem with results from Step 1 neglecting transmission power loss.

Step 3: Run power flow with results from Step 2 to find transmission power loss.

Step 4: Distribute the losses among all generators using participation factor as (28).

Step 5: Compute total cost of all generations.

5. **NUMERICAL RESULTS**

The proposed EALH is tested on two test systems with various load demand. The algorithm for solving the test system is implemented on Matlab 6.5 platform and run on 1.1 GHz Celeron, 128 MB of RAM PC. For stopping criteria, the tolerance of the ALH \(\varepsilon\) is set to 10^-4.

**Case 1: 10 units system**

The test system consists of 10 generating units [2], each with two or three piecewise quadratic cost functions representing different fuel types. Total demands are gradually changed from
2,400 MW to 2,700 MW with power loss neglected. The values of \( w_1 \) and \( w_2 \) are equally set to 0.5 for load demand of 2,400 MW, 0.75 for 2,500 MW and 2,600 MW, and 0.8 for 2,700 MW.

The results of the proposed EALH are compared to those from HNUM [2], HNN [3], AHNN [5], ELANN [4], IEP [10], MPSO [11], real coded genetic algorithm (RCGA) [8], and HRCGA [8] for various load demands of 2,400 MW, 2,500 MW, 2,600 MW, and 2,700 MW shown in Tables 1, 2, 3 and 4, respectively.

The proposed method obtains better solutions than HNUM [2], ELANN [4] and IEP [10] in all cases except the HNUM at 2,600 MW case. The proposed method also obtains better solution than HNN [3] for 2,400 MW and 2,700 MW cases, and AHNN [5] for 2,700 MW case. Total costs of the proposed method are close to those from the others for the rest cases. Note the power balance constraint in HNUM [2] and HNN [3] are not satisfied.

**Case 2: IEEE 30 bus system**

The proposed EALH is also tested on the IEEE 30 bus system with piecewise quadratic cost functions for generating units given in [6] and line data given in [19]. Power load flow in this study for transmission loss computation is from a power system toolbox included in [18]. Results from the proposed method are compared to those from conventional Hopfield network (HNN) and AHNN in [6] for various load demands of 220 MW, 283.4 MW, and 380 MW as in Table 5.

The proposed method can obtain better solutions than both HNN and AHNN in [6] for all cases. Although the hardware is used in this study is about ten times faster than that was used in [6], the computational times from the proposed method are much more faster. Note the computer used in [6] was a Compaq 90 MHz Pentium PC.

**Table 1. Comparison of fuel cost and CPU time for load demand of 2,400 MW**

<table>
<thead>
<tr>
<th>Method</th>
<th>Total power (MW)</th>
<th>Cost ($/h)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HNUM [2]</td>
<td>2,401.2</td>
<td>488.50</td>
<td>1.08</td>
</tr>
<tr>
<td>HNN [2]</td>
<td>2,399.8</td>
<td>487.87</td>
<td>~60</td>
</tr>
<tr>
<td>AHNN [5]</td>
<td>2,400</td>
<td>481.72</td>
<td>~4</td>
</tr>
<tr>
<td>ELANN [4]</td>
<td>2,400</td>
<td>481.74</td>
<td>11.53</td>
</tr>
<tr>
<td>IEP [10]</td>
<td>2,400</td>
<td>481.779</td>
<td>-</td>
</tr>
<tr>
<td>MPSO [11]</td>
<td>2,400</td>
<td>481.723</td>
<td>-</td>
</tr>
<tr>
<td>RCGA [8]</td>
<td>2,400</td>
<td>481.723</td>
<td>49.92</td>
</tr>
<tr>
<td>HRCGA [8]</td>
<td>2,400</td>
<td>481.722</td>
<td>6.1</td>
</tr>
<tr>
<td>ETQ [13]</td>
<td>2,400</td>
<td>481.72</td>
<td>86.3</td>
</tr>
<tr>
<td>EALH</td>
<td>2,400</td>
<td>481.723</td>
<td>0.008</td>
</tr>
</tbody>
</table>

**Table 2. Comparison of fuel cost and CPU time for load demand of 2,500 MW**

<table>
<thead>
<tr>
<th>Method</th>
<th>Total power (MW)</th>
<th>Cost ($/h)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HNUM [2]</td>
<td>2,500.1</td>
<td>526.70</td>
<td>-</td>
</tr>
<tr>
<td>HNN [3]</td>
<td>2,499.8</td>
<td>526.13</td>
<td>~60</td>
</tr>
<tr>
<td>AHNN [5]</td>
<td>2,500</td>
<td>526.230</td>
<td>~4</td>
</tr>
<tr>
<td>ELANN [4]</td>
<td>2,500</td>
<td>526.27</td>
<td>12.25</td>
</tr>
<tr>
<td>IEP [10]</td>
<td>2,500</td>
<td>526.304</td>
<td>-</td>
</tr>
<tr>
<td>MPSO [11]</td>
<td>2,500</td>
<td>526.239</td>
<td>-</td>
</tr>
<tr>
<td>RCGA [8]</td>
<td>2,500</td>
<td>526.239</td>
<td>49.92</td>
</tr>
<tr>
<td>HRCGA [8]</td>
<td>2,500</td>
<td>526.238</td>
<td>6.1</td>
</tr>
<tr>
<td>CEP [9]</td>
<td>2,500</td>
<td>526.246</td>
<td>0.495</td>
</tr>
<tr>
<td>FEP [9]</td>
<td>2,500</td>
<td>526.262</td>
<td>0.394</td>
</tr>
<tr>
<td>IFEP [9]</td>
<td>2,500</td>
<td>526.246</td>
<td>0.558</td>
</tr>
<tr>
<td>EALH</td>
<td>2,500</td>
<td>526.239</td>
<td>0.006</td>
</tr>
</tbody>
</table>

For CPU time, it may not directly comparable between the methods due to various computers and programming languages used. However, a CPU time comparison to show the efficiency of performance among the compared methods. The CPU times from HNUM [2], HNN [3], AHNN [5], ELANN [4], and ETQ [13] were from VAX 11/780, IBM PC-386, Compaq 90 MHz, 133 MHz Pentium PC, and 600 MHz Pentium III PC with 128 MB RAM, respectively. The computer used for both RCGA and HRCGA in [8] was from a Pentium III PC 500 MHz. There is no CPU time reported from the MPSO [11] and computer used in EP [9]. The hardware used in the proposed method is faster than some of them. The CPU times from HNN [3] and AHNN [5] for all cases are about 60 seconds and 4 seconds, respectively. However, as shown in Tables 1, 2, 3, and 4 the CPU time from the proposed method is much faster than the others.
Table 5. Comparison of fuel cost and CPU time for IEEE 30 bus system

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>$\Sigma P_i$ (MW)</td>
<td>227.05</td>
<td>227.086</td>
<td>225.897</td>
</tr>
<tr>
<td></td>
<td>$P_L$ (MW)</td>
<td>7.053</td>
<td>7.0861</td>
<td>5.897</td>
</tr>
<tr>
<td></td>
<td>Cost ($/h)</td>
<td>589.63</td>
<td>589.647</td>
<td>584.140</td>
</tr>
<tr>
<td></td>
<td>CPU time (s)</td>
<td>-</td>
<td>25</td>
<td>0.107</td>
</tr>
<tr>
<td>283.4</td>
<td>$\Sigma P_i$ (MW)</td>
<td>294.99</td>
<td>295.252</td>
<td>294.133</td>
</tr>
<tr>
<td></td>
<td>$P_L$ (MW)</td>
<td>11.582</td>
<td>11.852</td>
<td>10.733</td>
</tr>
<tr>
<td></td>
<td>Cost ($/h)</td>
<td>810.06</td>
<td>810.278</td>
<td>806.118</td>
</tr>
<tr>
<td></td>
<td>CPU time (s)</td>
<td>-</td>
<td>30</td>
<td>0.111</td>
</tr>
<tr>
<td>380</td>
<td>$\Sigma P_i$ (MW)</td>
<td>396.39</td>
<td>396.324</td>
<td>395.265</td>
</tr>
<tr>
<td></td>
<td>$P_L$ (MW)</td>
<td>16.389</td>
<td>16.324</td>
<td>15.265</td>
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<tr>
<td></td>
<td>Cost ($/h)</td>
<td>1,391.00</td>
<td>1,390.997</td>
<td>1,355.670</td>
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<tr>
<td></td>
<td>CPU time (s)</td>
<td>-</td>
<td>25</td>
<td>0.103</td>
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6. CONCLUSION

In this paper, the enhanced augmented Lagrange Hopfield is simply and efficiently implemented for solving the economic dispatch problem with piecewise quadratic cost functions. The proposed method uses a heuristic search for fuel type determination first based on a combined priority index of average production cost and average incremental cost of units. Then an augmented Lagrange Hopfield is applied to solve the final economic dispatch. The proposed method has been tested and compared to many other methods and shown that it is very efficient to the problem compared other methods in terms of total costs and computational times. This shows the possibility of successfully being applied to complex nonlinear problems in power system operation of the proposed method.

APPENDIX

Table 6 shows the solutions of the proposed method for the test system for 10 units with various load demand from 2,400 MW to 2,700 MW.

Table 7 shows the solutions for IEEE 30 bus test system with three load levels of 220 MW, 283.4 MW, and 380 MW.

Table 6. Solutions for various load demands by EALH for 10-unit system

<table>
<thead>
<tr>
<th>Unit</th>
<th>2,400 MW</th>
<th>2,500 MW</th>
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<tbody>
<tr>
<td></td>
<td>$P_L$ (MW)</td>
<td>Fuel</td>
</tr>
<tr>
<td>1</td>
<td>189.7397</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>202.3427</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>253.8954</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>233.0456</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>241.8299</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>233.0456</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>253.2752</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>233.0456</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>320.3831</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>239.3973</td>
<td>1</td>
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</table>

Table 7. Solutions for various load demands by EALH for IEEE 30 bus system

<table>
<thead>
<tr>
<th>Unit</th>
<th>220 MW</th>
<th>283.4 MW</th>
<th>380 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_L$ (MW)</td>
<td>Fuel</td>
<td>$P_L$ (MW)</td>
</tr>
<tr>
<td>1</td>
<td>134.9538</td>
<td>2</td>
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<tr>
<td>2</td>
<td>35.6396</td>
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<td>3</td>
<td>16.1121</td>
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<td>19.2918</td>
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<td>4</td>
<td>11.3038</td>
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<td>12.3921</td>
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<td>5</td>
<td>15.4349</td>
<td>1</td>
<td>15.7965</td>
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</tbody>
</table>
REFERENCES


