Transmission Constrained Unit Commitment in Competitive Environment by Enhanced Adaptive Lagrangian Relaxation

Weerakorn Ongsakul, Member, IEEE and Nit Petcharaks

Abstract—This paper proposes an enhanced adaptive Lagrangian relaxation (ELR) for centralized dispatch of transmission constrained unit commitment (TCUC) problem in competitive environment. ELR minimizes the consumer payment rather than the total supply cost subject to the power balance, spinning reserve, transmission lines, and generator operating constraints. ELR algorithm is enhanced by new initialization of Lagrangian multipliers and adaptive adjustment of Lagrangian multipliers. The transmission constrained economic dispatch is solved by quadratic programming. If congestion exists, the alleviating congestion index is proposed for congestion management. Moreover, the side payment to each generator is proposed to ensure payment adequacy without resulting in excessive system marginal price, leading to a lower consumer payment. The ELR is tested on the IEEE 24 bus reliability test system.

Index Terms—Congestion management, DC flow, Lagrangian relaxation, Payment adequacy, Unit commitment.

I. NOMENCLATURE

\( ACI^t_i \) 
the congestion index of unit \( i \) at hour \( t \);

\( C \)
the total number of congested lines at hour \( t \);

\( CP \)
the total consumer payment ($);

\( CP_k \)
the least total consumer payment reached ($);

\( CP(U_{i,k}^{1/k}) \)
the total consumer payment at iteration \( k \) ($);

\( F_1(P_{i,ed}^t) \)
the generator supply cost function in a quadratic form,

\[ F_1(P_{i,ed}^t) = a_i + b_i P_{i,ed}^t + c_i P_{i,ed}^t \ ($/hr) \]

\( F_{avg}(P_{i,ed}^t) \)
the average supply cost of unit \( i \) ($/MWh);

\( f_l \)
the high operating limit of line \( l \) (MW);

\( G^{(k)} \)
the relative duality gap at iteration \( k \);

\( gp_{i,l}^t \)
the unit price of unit \( i \) at hour \( t \) ($/MWh);

\( k \)
the ALR iteration counter;

\( K_{max} \)
the maximum allowable number of iterations;

\( MC_{i,l}^t \)
the incremental cost of unit \( i \) at hour \( t \), \( b_l + 2c_i P_{i,l}^t \ ($/MWh);

\( NL \)
the total number of lines;

\( N \)
the total number of generator units;

\( NB \)
the total number of buses;

\( P_{i,avg} \)
the average output power of unit \( i \) (MW);

\( P_{i,l}^t \)
the generation output power of unit \( i \) at hour \( t \) (MW);

\( P_{i,ed}^t \)
the economic dispatch generation output of unit \( i \) at hour \( t \) (MW);

\( P_{i,ed,j}^t \)
the economic dispatch generation output at bus \( j \) at hour \( t \) (MW);

\( P_{l}^t \)
the total real power flow in line \( l \) (j-k) at hour \( t \) (MW);

\( P_{load}^t \)
the total load demand at hour \( t \) (MW);

\( P_{load,j}^t \)
the load demand at hour \( t \) at bus \( j \) (MW);

\( P_{max} \)
the maximum real power generation of unit \( i \) (MW);

\( P_{min} \)
the minimum real power generation of unit \( i \) (MW);

\( PC \)
the pool cost ($);

\( R_i \)
the spinning reserve at hour \( t \) (MW);

\( S(ACI^t_i) \)
the set of uncommitted units at hour \( t \) sorted in the descending order of the value of \( ACI^t_i \);

\( SEP_i \)
the system electricity price at hour \( t \) ($/MWh);

\( SMC_t \)
the system marginal cost at hour \( t \) ($/MWh);

\( SMP_i \)
the system marginal price at hour \( t \) ($/MWh);

\( SP_i \)
the side payment of unit \( i \) ($);

\( ST_i \)
the startup cost of unit \( i \) ($);

\( ST_{on} \)
the startup cost of unit \( i \) at hour \( t \) ($);

\( T \)
the total number of hours;

\( T_{up} \)
the minimum up time of unit \( i \);

\( T_{down} \)
the minimum down time of unit \( i \);

\( T_{on} \)
the continuously on time of unit \( i \);

\( T_{off} \)
the continuously off time of unit \( i \);

\( TST_i \)
the total startup cost for unit \( i \) ($);

\( U_{i,l}^t \)
status of unit \( i \) at hour \( t \) (on = 1, off = 0);

\( \Omega^t \)
the set of committed units at hour \( t \);

\( \tau_{i,j} \)
the element in the matrix relating line flow \( l \) to a net injection at bus \( j \);

\( AP_i^t \)
the excess line flow over \( f_i \) at hour \( t \) (MW);

\( \epsilon \)
the duality gap tolerance;

\( k^{(0)}(\lambda^{(0)}) \)
the initial Lagrange multipliers at hour \( t \) ($/MWh, $/MWh);

\( k^{(k)}(\lambda^{(k)}) \)
the Lagrange multipliers at hour \( t \) at iteration \( k \) ($/MWh, $/MWh).
THE electricity industry in many countries is undergoing major regulatory changes. The market clearing price is one of the key issues of the design of a wholesale electricity market. In many countries, simple bid is adopted where generator operating constraints and costs such as no load and sunk capital costs are internalized in the supply bid curves. The corresponding schedule may be financially unattractive or technically infeasible to implement resulting in economic losses for generators and unrealistic prices [1]. Attempts to deal with these difficulties ranging from being able to state minimum income and indivisible bid in Spain, to a proposal of iterative bidding scheme for the California PX which was proven to be impractical for implementation [1]. On the other hand, some competitive power pools such as PJM (Pennsylvania, New Jersey and Maryland) and New York power pool have used unit commitment as centralized scheduling in a deregulated environment [1].

Electricity supply industry (ESI) structure in Thailand is a singer buyer model. The Energy Policy and Planning Office (EPPO) of Thailand has sought to transform the ESI into a full wholesale and retail competition model [2]. However, during the transition period, complex bids which reflect the actual cost structure of generators and their technical constraints, could be implemented in Thailand Power Pool [2].

In the framework of centralized optimization, many issues such as market clearing algorithm, payment adequacy constraints, equity, and efficiency, were addressed [3]-[11]. It was indicated that a payment minimization objective lead to a different unit commitment and dispatch solution, and lower market clearing prices [4],[9]. The objective of generation scheduling in a centralized market implemented in [3]-[11] is either total supply cost minimization or consumer payment minimization. In [4], payment adequacy constraint was introduced to ensure that all units winning the auction recover their fixed costs (startup costs and no load cost) and variable cost. The allocation of the generators’ fixed cost plays a crucial role in the pricing mechanisms of pool-based electricity markets with a complex bidding structure. In [8], four alternative fixed cost allocation schemes to determine the spot prices of electricity in a centralized market were presented. Different fixed cost allocation schemes resulted in different electricity prices, consumer payment and generators’ profit. It was found that the Table A/B scheme used in the former England and Wale market [4] could result the electricity prices at peak time period lower than the prices during periods of low demand [8]. This agreed with an empirical analysis of the prices in E&W market for the period April-June 1992 [8] which revealed that the system marginal price was an unpredictable function of demand. On the other hand, the fixed cost allocation scheme, amortized in proportion to the total output of continuous running periods, leads to a higher consumer payment than Table A/B scheme.

In [9], a forward dynamic programming algorithm was used to search for the most economic schedule. However, this method was complex and computationally demanding.

As the transmission network constraints are not taken into account, the obtained day ahead generation scheduling may not be feasible. Hence, redi dispatching and/or rescheduling must be performed economically, equitably, and efficiently. In [11], when transmission line congestion existed, the criterion to reschedule unit was checked whether to decommit the committed unit(s) or to commit more unit(s). Then, the corresponding indices were calculated and rescheduling was performed based on these indices. Nevertheless, the procedure was computationally time consuming.

A day-ahead generation scheduling in centralized market were solved by Lagrangian relaxation (LR) [3],[5], linear programming [11], and dynamic programming [9]. LR has been successfully applied to the complex UC problem including various hard constraints (e.g. ramp rate constraints, minimum up and down time, etc.). LR has many advantages over other methods used for UC, e.g. the computational requirement of using LR varies linearly with the number of generation units (N) and stages (T) while the computational requirement of dynamic programming (DP) varies exponentially with N and T, $(2^N - 1)^T$.

In this paper, the market model used is a centralized market with complex bidding structure without demand bid. The system operator is responsible for coordinating the auction to produce a feasible and efficient day-ahead schedule and dispatch the scheduled units by evaluating the day-ahead bids. All winning bidders are paid two parts: energy payment and side payment. The energy payment for each bidder is paid with a uniform price. Whereas, the proposed side payment is given to each supplier when the energy payment could not recover their fixed cost and operating cost. Under a fully competitive environment, it is assumed that the generator bids are close to their incremental costs. Generation scheduling problem can be formulated as a unit commitment problem by evaluating the day-ahead bids instead of the production costs of the generating units.

This paper proposes an enhanced adaptive Lagrangian relaxation (ELR) for centralized dispatch of transmission constrained unit commitment (TCUC) problem during the transition period. ELR minimizes the consumer payment subject to the power balance, spinning reserve, transmission lines, and generator operating constraints. ELR algorithm is enhanced by new initialization to obtain a high quality initial solution and adaptive Lagrangian multipliers to speedup the convergence. For transmission constrained economic dispatch, quadratic programming is used to minimize the total modified consumer payment subject to power balance, generator operating, and DC modelled line flow constraints. In addition, the alleviating congestion index is proposed for congestion management. The proposed ELR algorithm is tested on the IEEE 24 bus reliability test system [12].

The organization of this paper is as follows. Section III addresses the UC problem formulation. The ELR algorithm
III. PROBLEM FORMULATION

A. Pricing Mechanisms

In a centralized market with a complex bidding structure, the generators’ fixed cost allocation and payment adequacy constraints are included. Both influence greatly on hourly electricity prices, daily consumer payment, and daily generators’ profit.

1) Uniformly Fixed Cost Amortized Allocation: A unit’s fixed cost amortized over every committed period in proportion to its total power output had many advantages [8]. Hence, Mendes adopted this allocation type to compute unit prices, \( g_{p_i} \) [9], expressed by

\[
TST_i + \sum_{t=1}^{T} a_t U_{i,t} + \frac{\sum_{t=1}^{T} P_{i,t} U_{i,t}}{P_{load}}.
\]

where \( a_t \) is no load cost of unit \( i \) and

\[
TST_i = \sum_{t=1}^{T} ST_i (1 - U_{i,t-1}) U_{i,t}.
\]

The incremental cost \( MC_i \) of quadratic supply cost function is computed by

\[
MC_i = b_i + 2c_i P_{i,t}.
\]

The consumer payment is calculated based on the market clearing prices or system marginal prices \( SMP' \) which is set at the price of the most expensive generating unit.

\[
SMP' = \max (g_{p_i}), \quad i \in \Omega.
\]

The consumer payment minimization problem is modified as,

\[
\text{Minimize } \sum_{i=1}^{N} \sum_{t=1}^{T} MC_{i} \cdot P_{i,t},
\]

subject to:

a) power balance constraints

\[
P_{load} - \sum_{i=1}^{N} P_{i,t} U_{i,t} = 0,
\]

b) committed unit constraints

\[
\sum_{i=1}^{N} P_{i,max} U_{i,t} \leq P_{load},
\]

c) spinning reserve constraint

\[
f_l \leq \sum_{j=1}^{NL} T_{i,j}(P_{j,t} - P'_{load,j,\Omega}) \leq f_l, \quad i = 1, \ldots, N,
\]

d) transmission line constraints

\[
P_{load} + R_{j} - \sum_{i=1}^{N} P_{i,max} U_{i,t} \leq P_{load},
\]

e) generation limit constraints

\[
P_{i,\text{max}} U_{i,t} \leq P_{i,t}, \quad i = 1, \ldots, N,
\]

f) minimum up and down time constraints.

\[
U_{i,t} = \begin{cases} 1, & \text{if } T_{i,on} < T_{i,up} \\ 0, & \text{if } T_{i,off} < T_{i,down} \\ 0 \text{ or } 1, & \text{otherwise} \end{cases}
\]

g) Startup cost is assumed to be constant and independent on the number of ‘off’ hours,

\[
ST_{i,t} = ST_i (1-U_{i,t-1}) U_{i,t}.
\]

IV. AN ELR FOR TCUC

The LR procedure solves the UC problem by relaxing or temporarily ignoring the coupling constraints and solving the problem as if they did not exist. This is done through the dual optimization procedure attempting to reach the constrained optimum by maximizing the Lagrangian,
\[ L(P, U, \lambda, \mu) = \sum_{i=1}^{T} \sum_{t=1}^{N} MC_i^t P_i^t U_{i,t} + \sum_{i=1}^{T} \lambda^i (P_{\text{load}}^i - \sum_{i=1}^{T} P_i^t U_{i,t}) + \sum_{i=1}^{T} \mu^i (P_{\text{load}}^i + R^i - \sum_{i=1}^{T} P_{\text{max}} U_{i,t}). \]  

Consequently, the Lagrangian function is rewritten as
\[ L = \sum_{i=1}^{N} \sum_{t=1}^{T} \{MC_i^t P_i^t U_{i,t} - \lambda^i P_i^t U_{i,t} - \mu^i P_{\text{max}} U_{i,t}\} + \sum_{i=1}^{T} (\lambda^i P_{\text{load}}^i + \mu^i (P_{\text{load}}^i + R^i)). \]

The term \[ \sum_{i=1}^{T} \{MC_i^t P_i^t U_{i,t} - \lambda^i P_i^t U_{i,t} - \mu^i P_{\text{max}} U_{i,t}\} \] can be minimized separately for each generating unit, when the coupling constraints are temporarily ignored.

### A. Dynamic Programming

In the conventional Lagrangian relaxation method, the dual solution is obtained by using dynamic programming for each unit separately. This can be visualized in Fig. 1 showing the only two possible states for unit \( i \) (i.e., \( U_{i,t} = 0 \) or 1):

\[ \begin{align*}
U_i &= 1 \\
U_i &= 0 \\
& \quad i = 0, 1, 2, \ldots, T-1 \\
\text{Fig. 1. Search paths of each unit in dynamic programming} \\
\end{align*} \]

At the \( U_{i,t} = 0 \) state, the value of the function to be minimized is trivial (i.e., it equals zero), at the state where \( U_{i,t} = 1 \), the function to be minimized is
\[ \min[(b_i + 2c_i P_i^t) P_i^t - \lambda^i P_i^t - \mu^i P_{\text{max}}]. \]

The dual power at each hour is obtained from
\[ P_i^{\text{opt}} = \frac{\lambda^i - b_i}{4c_i}. \]

There are three cases to check \( P_i^{\text{opt}} \) against its limits:
1. If \( P_i^{\text{opt}} < P_{\text{min}}, P_i^t = P_{\text{min}}. \)
2. If \( P_{\text{min}} \leq P_i^{\text{opt}} \leq P_{\text{max}}, P_i^t = P_i^{\text{opt}}. \)
3. If \( P_i^{\text{opt}} > P_{\text{max}}, P_i^t = P_{\text{max}}. \)

Dynamic programming is used to determine the optimal schedule of each unit over the scheduled time period by comparing the unit accumulated payments from two historical routes. At hour \( t \), the dual power calculated by (21) and within the limit, will be substituted in
\[ (b_i + 2c_i P_i^t) P_i^t - \lambda^i P_i^t - \mu^i P_{\text{max}}. \]

Then, dynamic programming searches for the optimal scheduling for each unit to obtain the lowest value of the term
\[ \sum_{i=1}^{T} \{b_i + 2c_i P_i^t) P_i^t - \lambda^i P_i^t - \mu^i P_{\text{max}}\}. \]

### B. Initialization

Our initialization procedure intends to create the feasible schedule in the first iteration. For each hour, the group of identical units with the least incremental cost at \( P_{\text{max}}, b_i + 2c_i P_{\text{avg}} \) will be committed one group by one group until the power balance constraint is satisfied. Subsequently, unconstrained economic dispatch is carried out to obtain the hourly equal lambda which is initially set to Lagrangian multipliers \( \lambda^{(0)} \), in each hour. For the hours with insufficient spinning reserves, more unit(s) are needed to be committed to give the initial feasible solution by committing a group of identical units with the least incremental cost, \( b_i + 2c_i P_{\text{avg}} \) one group by one group until the spinning reserve is satisfied. Since dynamic programming will commit the unit with negative value of the term in (22) at each hour to minimize the term in (23), \( \mu^{(t)} \) is determined by setting the term in (22) equal to zero as
\[ (b_i + 2c_i P_i^t) P_i^t - \lambda^i P_i^t - \mu^i P_{\text{max}} = 0. \]

For each hour, each nonnegative \( \mu^{(t)} \) is determined by
\[ \mu^{(t)} = \max[\frac{1}{P_{\text{max}}} - \{b_i + 2c_i P_{\text{avg}}\} P_{\text{avg}} - \lambda^{(t)} P_{\text{avg}}], 0], \]

where \( P_{\text{avg}} = \frac{P_{\text{max}} - P_{\text{min}}}{2} \).

Note the value of \( \lambda^{(0)} \) may be too low because the committed capacity is just sufficient for load supply at each hour and may be insufficient for spinning reserve. Thus, the dual power calculated by (21) may be inappropriate and the average output power \( P_{\text{avg}} \) by (26) is used instead. The initial \( \mu^{(t)} \) is determined by the highest \( \mu^{(t)} \) among the committed units as,
\[ \mu^{(t)} = \max[\mu^{(0)}_1, \ldots, \mu^{(0)}_m], \]

where \( m \) is the marginal unit with the highest \( b_i + 2c_i P_{\text{avg}} \) giving the sufficient spinning reserve at hour \( t \).

### C. Adaptive Updating of the Lagrangian multiplier

In this paper, the Lagrangian multiplier update rule is designed the step size to be large at the beginning of iterations and smaller as the iteration grows as proposed in [5]. The value of \( \alpha \) and \( \beta \) are determined heuristically [13]. Each nonnegative \( \lambda^i \) and \( \mu^i \) are adaptively updated by considering load and spinning reserve balance and Euclidean norm.
\[ \lambda^{(k)} = \max[\lambda^{(k-1)} + \frac{\text{pdif}^i}{\sqrt{(\alpha + \beta \times k) \times \text{norm(pdif)}}}], \]

where \( \text{pdif}^i \) is the difference between the current and previous values of the dual power.
implies that the committing unit will increase the power flow in line and

\[ \text{norm}(\text{pdif}) = \sqrt{(\text{pdif}_1^t)^2 + (\text{pdif}_2^t)^2 + \ldots + (\text{pdif}_T^t)^2}, \]  

where, 

\[ \mu^{tk} = \max[\mu^{(t-k)} + \frac{\text{rdif}^t}{(\alpha + \beta \times k) \times \text{norm}(\text{rdif})}, 0], \]  

where, \( \text{rdif}^t = P_{l,\text{load}}^t + R^t - \sum_{i=1}^{N} P_{i,\text{max}} U_{i,l}, \)  

\[ \text{norm}(\text{rdif}) = \sqrt{(\text{rdif}_1^t)^2 + (\text{rdif}_2^t)^2 + \ldots + (\text{rdif}_T^t)^2} \]

\[ \alpha \text{ and } \beta \text{ are divided into three cases depending on the signs of } \text{pdif}^t \text{ and } \text{rdif}^t \text{ as follows:} \]

- Case 1: \( \text{pdif}^t \geq 0 \) and \( \text{rdif}^t \geq 0 \) : updating both \( \lambda^t \) and \( \mu^t \) by using \( \alpha = 0.02 \) and \( \beta = 0.05 \).
- Case 2: \( \text{pdif}^t < 0 \) and \( \text{rdif}^t < 0 \) : updating both \( \lambda^t \) and \( \mu^t \) by using \( \alpha = 0.3, \beta = 0.1 \).
- Case 3: \( \text{pdif}^t < 0 \) and \( \text{rdif}^t > 0 \) : updating only \( \mu^t \) by using \( \alpha = 0.02, \beta = 0.05 \).

**D. Stopping Criteria**

The relative duality gap is used to measure the solution quality, by checking against the stopping criteria. The iteration process stops when either the relative duality gap is less than the specified tolerance or the iteration counter exceeds the maximum allowable number of iterations.

\[ G^{(tk)} = \frac{CP(U_{i,t}^{(tk)}) - L(P^{(tk)}, U^{(tk)}, \lambda^{(tk)}, \mu^{(tk)})}{L(P^{(tk)}, U^{(tk)}, \lambda^{(tk)}, \mu^{(tk)})} \]  

where \( CP(U_{i,t}^{(tk)}) = \sum_{i=1}^{T} \sum_{t=1}^{N} MC_i(P_{i,t}^{(tk)} \cdot U_{i,t}) \) and \( L(P^{(tk)}, U^{(tk)}, \lambda^{(tk)}, \mu^{(tk)}) \) is calculated from (20).

**E. Congestion Management**

For each hour, transmission constrained economic dispatch by quadratic programming is used to minimize the modified consumer payment subject to power balance equation and DC modelled line flow constraints.

Minimize \( \sum_{i=1}^{T} (b_i + 2c_i P_{l,\text{load}}^t) P_{i,\text{ed}}^t \), subject to:

a) power balance constraints

\[ \sum_{i=1}^{T} P_{i,\text{ed}}^t = P_{l,\text{load}}^t, \]  

b) generation limit constraints

\[ P_{i,\text{max}} \leq P_{i,\text{ed}}^t \leq P_{i,\text{min}} \]  

c) transmission line constraints

\[ -f_i + \sum_{j=1}^{NB} r_{ij} P_{l,\text{load}}^t \leq \sum_{j=1}^{NB} r_{ij} P_{j,\text{ed}}^t - f_i + \sum_{j=1}^{NB} r_{ij} P_{i,\text{load}}^t. \]

If the solution of transmission constrained ED is not feasible, rescheduling will be needed. The uncommitted units at this congested hour are considered to be committed by evaluating their capability to relieve all congested lines in the least cost manner. If there are \( C^l \) congested lines at hour \( t \), the congestion index \( (ACI_i^t) \) of uncommitted units is calculated as:

\[ ACI_i^t = \left\{ \sum_{j=1}^{C^l} \tau_{i,j} P_{i,\text{ed}}^t \right\} \cdot MC_i(P_{i,\text{opt}}^t) \]  

where \( \Delta P_{i}^t = f_i - P_{i}^t \),

\[ P_{i}^t = \sum_{j=1}^{NB} (\tau_{i,j} P_{j,\text{ed}}^t - \tau_{i,j} P_{j,\text{load}}^t). \]

Negative \( \tau_{i,j} \) implies that the committing unit \( i \) will decrease the power flow in line \( l \), whereas committing unit \( i \) with positive \( \tau_{i,j} \) will increase the power flow in line \( l \). To relieve congestion, negative \( \Delta P_{i}^t \) implies that the power flow in line \( l \) \((j-k)\) must be decreased whereas positive \( \Delta P_{i}^t \) requires the power flow in line \( l \) \((j-k)\) be increased or overload line \( l \) \((k-j)\) be decreased. Thus, positive value of \( \tau_{i,j} \Delta P_{i}^t \) means that committing unit \( i \) may help alleviating the congestion in line \( l \), and negative value will increase the overloading. However, unit \( i \) may help relieving one congested line but it may increase overloading in another congested line. Hence, the term \( \sum_{j=1}^{C^l} \tau_{i,j} P_{i,\text{max}} \Delta P_{i}^t \) represents the capability of unit \( i \) to relieve the total congested lines in arithmetic summation. The higher value of \( \sum_{j=1}^{C^l} \tau_{i,j} P_{i,\text{max}} \Delta P_{i}^t \), the better the effects on relieving the overload. Whereas the lower the incremental cost \( MC_i(P_{i,\text{opt}}^t) \), the less increase in cost. Therefore, the uncommitted unit with the highest \( ACI_i^t \) is the first unit required to alleviate the congestion. After committing one more unit, the transmission line constrained ED is carried out. If there is no feasible solution, more units based on the next highest \( ACI_i^t \), will be committed one at a time until all line constraints are satisfied while satisfying the minimum down time constraints.

**F. Overall ELR Procedure**

Step 1. Initialize \( \lambda^t \) and \( \mu^t \) described in Section IV.B.
Step 2. Initialize the ELR iteration counter, \( CP_B = 10^7 \), and \( k = 1 \).
Step 3. Solve the unit subproblems by using dynamic programming described in Section IV.A.
Step 4. If the dual solution is not feasible, go to Step 13.
Step 5. Carry out the transmission constrained ED by quadratic programming. If there is a feasible solution, go to Step 10.
Step 6. If there is no uncommitted unit, go to Step 13. Otherwise, calculate the congestion index \( ACI_i \) defined in Section IV.E and sort them in descending order of the value \( ACI_i \) to obtain \( S(ACI_i) \).

Step 7. Commit the first unit in \( S(ACI_i) \) and delete this unit from \( S(ACI_i) \).

Step 8. Carry out the transmission constrained economic dispatch by quadratic programming. If there is a feasible solution, go to Step 10.

Step 9. If \( S(ACI_i) \) is empty, go to Step 13. Otherwise, go to Step 7.

Step 10. Calculate the primal payment \( CP([U_{i,l}]) \), the dual payment \( L(P_{i,l}, U_{i,l}, \lambda_{i,l}, \mu_{i,l}) \) and the relative dual gap \( G^{(k)} \) as described in Section IV.D.

Step 11. If \( CP([U_{i,l}]) < CP,B, CP_B = CP([U_{i,l}]) \) and \( [U_{i,l}] = [U_{i,l}] \).

Step 12. If the relative dual gap \( G^{(k)} < \varepsilon \), go to Step 14.

Step 13. If \( k < K_{\text{max}}, k = k + 1 \), update Lagrangian multiplier adaptively as described in Section IV.C and return to Step 3.

Step 14. Terminate ELR.

### V. NUMERICAL RESULTS

The ELR algorithms is tested on the IEEE Reliability system consisting of 24 buses, 39 transmission lines and 32 generating units [12]. In the simulation, the spinning reserve is required to be 7% of the total load demand. As shown in Table 1, when the consumer payment is minimized, the consumer payment is 8.8% less than supply cost minimization even though the total supply cost is 3.8% higher.

If the consumer payment \( PC \) in the uniformly fixed cost amortized allocation in (5) is calculated by using ELR solution, \( PC \) will be $1,730,700 or 27.2% higher. The system marginal price calculated in (4) and the system electricity price computed in (9) are shown in Fig. 2. The fixed cost allocation marks up \( SMP^t \) to be much higher than \( SEP^t \) resulting in a much higher consumer payment. This is because \( SEP^t \) is obtained by the highest marginal cost and the uniformly distribution of side payment to each MW load, whereas, \( SMP^t \) is computed from the highest marginal cost plus the generator fixed cost amortized by its generating output power. The consumer payment and the base load generation units’ profit change significantly with small change in \( SMP^t \). Note the value of \( SEP^t \) and \( SMP^t \) are high at the beginning period because the expensive unit, unit no. 27, is committed due to minimum up time constraint. ELR for CPM requires less computation time than SCM.

### Table 1 Supply cost and consumer payment

<table>
<thead>
<tr>
<th>Method</th>
<th>Problem</th>
<th>Time (s)</th>
<th>Total Cost ($)</th>
<th>Consumer Payment($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELR</td>
<td>CPM</td>
<td>100</td>
<td>816,000</td>
<td>1,259,619</td>
</tr>
<tr>
<td>ELR</td>
<td>SCM</td>
<td>320</td>
<td>784,872</td>
<td>1,370,068</td>
</tr>
</tbody>
</table>

### Fig. 2. The system marginal price and the system electricity price in CPM

Note that the supply cost includes the fuel cost and the operating and maintenance (O&M) cost. The fuel quadratic cost function \( \{ F_i(P_i) = \tilde{a}_i + \tilde{b}_i P_i + \tilde{c}_i P_i^2 \} \) is obtained from heat energy input (MBtu/h) – generation output (MW) curve multiplied by fuel cost ($/MBtu). Then, the O&M fixed cost ($/h) and variable cost ($/MWh) are added to \( \tilde{a}_i \) and \( \tilde{b}_i \) so that the total supply cost \( F_i(P_i) \) is obtained.

### VI. CONCLUSIONS

In this paper, the proposed ELR is efficiently and effectively minimize the consumer payment in a day ahead transmission constrained unit commitment. The proposed uniformly distribution side payment could ensure the revenue adequacy without resulting in excessive system marginal price, leading to a lower consumer payment. If the bidden cost functions are piecewise linear, using ELR to solve transmission constrained UC will be investigated in our future research work.

### VII. APPENDIX

The data were obtained from [12] and the unit data and load are summarized in Tables 2 and 3.

### Table 2 the daily load

<table>
<thead>
<tr>
<th>Hour</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L o a d (MW)</td>
<td>2223</td>
<td>2052</td>
<td>1938</td>
<td>1881</td>
<td>1824</td>
<td>1852.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hour</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>L o a d (MW)</td>
<td>1881</td>
<td>1995</td>
<td>2280</td>
<td>2508</td>
<td>2565</td>
<td>2593.5</td>
</tr>
</tbody>
</table>
Table 2 (continued)

<table>
<thead>
<tr>
<th>Load (MW)</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2565</td>
<td>2508</td>
<td>2479.5</td>
<td>2479.5</td>
<td>2593.5</td>
<td>2850</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load (MW)</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2821.5</td>
<td>2764.5</td>
<td>2679</td>
<td>2622</td>
<td>2479.5</td>
<td>2308.5</td>
</tr>
</tbody>
</table>

Table 3 Unit bidding costs and operational limits

<table>
<thead>
<tr>
<th>Unit</th>
<th>Bus</th>
<th>Pmin</th>
<th>Pmax</th>
<th>Init. Stat.</th>
<th>Tap</th>
<th>Tdown</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>ST cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
<td>140</td>
<td>350</td>
<td>11 8 48 567.9 9.62 0.0039</td>
<td>5362</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>54.2</td>
<td>155</td>
<td>5 8 8 345.1 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>3</td>
<td>68.9</td>
<td>197 12 12 10 548.5 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>3</td>
<td>68.9</td>
<td>197 12 12 10 548.5 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>51</td>
<td>54.2</td>
<td>155</td>
<td>5 8 8 345.1 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>61</td>
<td>54.2</td>
<td>155</td>
<td>5 8 8 345.1 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>72</td>
<td>54.2</td>
<td>155</td>
<td>5 8 8 345.1 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>82</td>
<td>54.2</td>
<td>155</td>
<td>5 8 8 345.1 18.31 0.0114</td>
<td>1783</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>97</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>18</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>19</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>22</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>23</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>24</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>26</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>27</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>28</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>29</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>31</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>32</td>
<td>2500</td>
<td>100</td>
<td>10 8 8 395.5 18.07 0.0274</td>
<td>1302</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VIII. REFERENCES