The effect of transformer maintenance parameters on reliability and cost: a probabilistic model

Panida Jirutitijaroen* , Chanan Singh

Department of Electrical Engineering, Texas A& M University, College Station, TX 77840, USA

Received 19 February 2004; accepted 30 April 2004
Available online 23 July 2004

Abstract

Transformer is an equipment common to most power systems. Preventive maintenance is performed to extend the equipment lifetime. Models relating probability of failure to maintenance activity are proposed in [Panida Jirutitijaroen, Chanan Singh, Oil-immersed transformer inspection and maintenance: probabilistic models, in: Proceedings of the 2003 North American Power Symposium Conference, pp. 204–208]. The model parameters which are mean time in each stage, inspection rate of each stage, and probabilities of transition from one stage to others, have an effect on reliability and cost of maintenance. In order to establish a cost-effective maintenance process, analysis of model parameters should be conducted thoroughly.

This paper develops detailed models relating maintenance parameters to reliability and cost and then investigates the effect of varying model parameters. Simulation results from the proposed model are shown and corroborated by mathematical analysis of a simpler equivalent model. The analysis covers mean time to the first failure, maintenance cost, inspection cost, and failure cost.

© 2004 Elsevier B.V. All rights reserved.

Keywords: Inspection model; Maintenance; Maintenance model; Probabilistic model; Reliability; Transformer model

1. Introduction

There is relatively little literature on quantifying the effect of maintenance on reliability in power systems. A probabilistic model of the effect of maintenance on reliability was first introduced in [2,3]. Based on this general model, oil-immersed transformer inspection and maintenance model is proposed in [1] utilizing the concept of device of stage [6].

Model parameters in the proposed model [1] are assumed to be known from data collected. The parameters include inspection rate of each stage, mean time in each stage, failure cost, maintenance cost, and inspection cost. This paper investigates the effect of model parameters on reliability and maintenance cost. An equivalent model is introduced for simpler analysis. The analysis covers mean time to the first failure, maintenance and failure cost, and inspection cost. Simulation results of the proposed model are corroborated by mathematical equations of the equivalent model using first passage time and steady state probability calculations [6].

The objective of this paper is to give an insight into the effect of the model parameters on reliability and all associated cost.

2. Transformer maintenance model

A general probabilistic model of the effect of maintenance on reliability is proposed in [2,3]. The model represents the deterioration process of a device by discrete stages [6]. The transformer probabilistic model in [1] utilizes this general model. The model is based on the proposed model in [2,3] and concept of device of stages [6].

In Fig. 1, deterioration process of a transformer is approximated by three discrete stages: D1, D2, and D3. At each stage, oil is inspected to determine its condition. After the inspection, oil condition is determined by the criteria indicated in [4,5]. The criteria categorize oil condition into four groups ranging from normal (C1) to adverse condition (C4). To simplify the model, C3 and C4 are grouped together.
These oil conditions are categorized as:

1. Good working condition (C1).
2. Required reconditioning before use (C2).
3. Poor condition (C3).
4. Adverse condition (C4).

Maintenance action is assigned corresponding to the oil condition. If oil condition is C1, nothing is done. If oil condition is C2, C3, or C4, two options are available and are assigned with different probabilities: oil filtering or oil replacement. For example, if the present stage is D2 with oil condition C2, the probability of oil filtering will be higher than oil replacement. On the other hand, if the present stage is D2 with oil condition C3 or C4, the probability of oil replacement will be higher. After maintenance, the device will have three options, going to stage D1, D2, or D3. The probability of transferring to other stages depends on the present stage and maintenance practice.

3. Model parameters

Parameters in transformer maintenance model are listed below.

1. **Mean time in each stage**: These parameters determine the transition rate of each stage in the deterioration process.
2. **Inspection rate of each stage**: This parameter can be treated as maintenance rate of each stage under the assumption that inspection, test and maintenance actions are implemented sequentially.
3. **Probabilities of transition from one stage to others**: These parameters are the probabilities of oil condition after the inspection process, probabilities of transferring from any oil condition to a given stage, probabilities of filtering or replacing the oil, and probabilities of transferring to each stage after maintenance. These probabilities can be treated as equivalent transition rates from one stage to others. The equivalent model is introduced to clarify this point later.

Notice that model parameters 1 and 3 can be approximated from historical data of oil condition of a physical transformer; thus, these parameters are given. However, inspection rate of each stage can be varied to achieve high reliability with minimum cost. Therefore, this parameter is of the most concern in the analysis.

In the following section, sensitivity analysis of inspection rate of each stage is implemented on the model in Fig. 1. Other model parameters are listed in Appendix A. The analysis covers two aspects: mean time to the first failure, and all associated costs (failure, maintenance and inspection costs, respectively). The simulation results from MATLAB are presented and examined in each section.
4. Sensitivity analysis of inspection rate on mean time to the first failure (MTTFF)

Mean time to the first failure is the expected operating time before failure of the transformer starting from initial stage. This analysis will provide information of how the transformer operating time changes when the inspection rate of each stage changes.

Let \( i_1 \) = inspection rate of D1 (per year), \( i_2 \) = inspection rate of D2 (per year), \( i_3 \) = inspection rate of D3 (per year).

The simulation results of the relationship of each inspection rate and MTTFF are shown in Fig. 2a and b.

The following observations can be drawn from these simulation results.

1. In Fig. 2a, MTTFF decreases with \( i_1 \). This is caused by the assumption of exponential distribution of time spent in each stage. The exponential distribution implies constant failure rate. This is of particular significance in stage D1. This means that the inspections, which lead back to D1, will not improve the time to failure in D1; however, those leading to D2 and D3 will result in degradation. Thus, the effect of inspection will always be degradation. In other words, if we assume an exponential distribution for stage 1, then maintenance can not be useful.

2. In Fig. 2b, MTTFF increases at a decreasing rate with \( i_2 \) and stays at some value.

3. In Fig. 2c, MTTFF has a positive and linear relationship with \( i_3 \).

Next, the model in Fig. 1 is modified by representing stage D1 by three sub-stages in order to relax the assumption of exponential distribution. Although each sub-stage is exponentially distributed, the overall D1 will have a deterioration. The simulation results of relationship of each inspection rate and MTTFF are shown in Fig. 3a–c.

In Fig. 3a, MTTFF increases rapidly when increasing \( i_1 \) and then slightly decreases at high \( i_1 \). The simulation results in Fig. 3b and c give the same observations as in Fig. 2b and c.

In conclusion, the simulation results suggest that inspection rate of D1 helps extending MTTFF; however, too high inspection rate of D1 might reduce MTTFF. In addition, inspection rate of D2 beyond a certain value has a minimal impact on reliability. Fig. 3c indicates that transformer lifetime will be longer with improved inspection rate of D3.

5. Sensitivity analysis of inspection rate on all associated cost

Costs from maintenance practice in model in Fig. 1 are inspection cost, oil filtering cost, oil replacement cost, and failure cost. This analysis will provide information about the effect of inspection rate on all associated cost. We assume cost parameters in Appendix A.

The simulation result of relationship between each inspection rate and all associated costs are shown in Figs. 4–6. The
Fig. 3. The relationship between MTTFF and inspection rate when stage 1 is represented by three sub-stages.

Fig. 4. The relationship between all associated cost and $i_1$. 

---

Fig. 5. The relationship between all associated cost and $i_2$.

Fig. 6. The relationship between all associated cost and $i_3$. 
following observations can be made from the simulation results.

1. In Figs. 4a, 5a and 6a, failure cost decreases exponentially as inspection rate of D1, D2 and D3 increases.

2. In Fig. 4b, maintenance cost first decreases as inspection rate of D1 increases and then increase with inspection rate of D1. The optimal region of inspection rate of D1 that will minimize maintenance cost is 0.5–1 per year.

3. In Figs. 5b and 6b, maintenance cost increases with inspection rate of D2 and D3 and stays at constant value at higher inspection rate of D2 and D3.

4. In Fig. 4c, inspection cost increases linearly with inspection rate of D1.

5. In Figs. 5c and 6c, inspection cost increases as inspection rate of D2 and D3 increases and remains constant at high inspection rate of D2 and D3.

6. In Fig. 4d, the optimum region of inspection rate of D1 that will minimize total cost depends on inspection rate of D2 and D3. If the inspection rate of D2 and D3 are higher, the optimal value of inspection rate of D1 will be smaller. Failure cost dominates total cost at small inspection rate of D1 while maintenance cost dominates total cost at high inspection rate of D1.

7. In Figs. 5d and 6d, the minimum total cost will occur at very high inspection rate of D2 and D3. Failure cost dominates total cost at small inspection rate of D2 and D3 while maintenance cost dominates total cost at high inspection rate of D2 and D3.

In conclusion, simulation result suggests that cost effective maintenance occurs at small inspection rate of D1 and high inspection rate of D2 and D3.

The sensitivity analysis of inspection rate on MTTFF and all associated costs are discussed in the previous section based on simulation results of model in Fig. 1. In the next section, equivalent mathematical models are presented for simpler analysis. Equations derived from mathematical analysis will provide an explicit relationship of each inspection rate with MTTFF and costs.

6. Equivalent models for mathematical analysis

Two equivalent models are introduced to simplify the transformer maintenance model shown in Fig. 1. The equivalent models have three discrete stages representing the deterioration processes. Assume that maintenance is implemented at every inspection, maintenance and inspection rate of each stage is considered to be an equivalent repair rate.

Let \( y_1 \) = mean time in stage 1 (year), \( y_2 \) = mean time in stage 2 (year), \( y_3 \) = mean time in stage 3 (year), \( \mu_{21} \) = repair rate from stage 2 to 1 (/year), \( \mu_{32} \) = repair rate from stage 3 to 2 (/year), \( \mu_{31} \) = repair rate from stage 3 to 1 (/year).

6.1. Perfect maintenance model

It is assumed that in the initial stage the transformer is in good working condition that needs no maintenance. Moreover, it is assumed that maintenance will always improve the device to the previous stage; therefore, repair rate of stage 2 will improve the device to stage 1 and repair rate of stage 3 will improve the device to stage 2. The model is shown in Fig. 7.

6.2. Imperfect maintenance model

This model is slightly different from the model in Fig. 7. Transition rate from stage 1 to 3 is introduced (\( \lambda_{13} \)) to describe an imperfect inspection of stage 1. This model accounts for the probability that inspection of stage 1 might cause the system to transit to stage 3. Note that this model is an equivalent model for transformer maintenance model in Fig. 1 since it accounts for a transition of stages 1–3 (Fig. 8).

The equivalent models will be employed in analyses in the next section, MTTFF and cost analysis. The first passage time and steady state probability calculation will be used. The equations obtained from the analyses will be used to verify the simulation results from the previous analyses.

7. Mean time to the first failure analysis

MTTFF equations are derived using the methodology of first passage time calculation [6]. These equations will
explain the simulation results in Fig. 3. The analysis is based on equivalent math models, perfect maintenance model and imperfect maintenance model.

7.1. Perfect maintenance model

MTTFF is calculated in Appendix B.1.

Let \( T_0 = \) life time without maintenance, \( T_E = \) extended life time with maintenance, \( \lambda_{12} = 1/\gamma_1 \) transition rate from D1 to D2, \( \lambda_{23} = 1/\gamma_2 \) transition rate from D2 to D3, \( \lambda_{3i} = 1/\gamma_3 \) transition rates from D3 to F. Then,

\[
T_0 = \gamma_1 + \gamma_2 + \gamma_3
\]

(1)

\[
T_E = \frac{\mu_{21}}{\lambda_{12} \lambda_{23}} + \frac{\mu_{32}}{\lambda_{23} \lambda_{3} f} + \frac{\mu_{21} \mu_{32}}{\lambda_{12} \lambda_{23} \lambda_{3} f}
\]

(2)

\[
MTTFF = T_0 + T_E
\]

(3)

The extended time of perfect maintenance model is a summation of all possible combinations of ratios between maintenance rate of the current stage and failure rate of the current and previous stage. Since \( T_E \) can only be positive in this model, inspection and maintenance will always extend the equipment life time.

If the repair rate of each stage is high relative to the transition rate of that stage and the previous stage (\( \mu_{31} \gg \lambda_{12} \lambda_{23}, \mu_{32} \gg \lambda_{23} \lambda_{3} f \)), the lifetime before failure of the device will be high.

7.2. Imperfect maintenance model

MTTFF is calculated in Appendix B.2 using first passage time technique. Then,

\[
MTTFF = \frac{T_0 + T_E}{1 + \lambda_{13}/\lambda_{12} + \lambda_{13} \mu_{21} / \lambda_{12} \lambda_{23}}
\]

(4)

\[
T_E = \frac{\mu_{21} \mu_{31}}{\lambda_{12} \lambda_{23} \lambda_{3} f} + \frac{\mu_{21} \mu_{32} + \mu_{21} \lambda_{13} + \mu_{32} \lambda_{13}}{\lambda_{12} \lambda_{23} \lambda_{3} f} + \frac{\mu_{21}}{\lambda_{12} \lambda_{23}}
\]

(5)

The relationships of inspection rate of each stage and MTTFF are listed in the following.

7.2.1. Inspection rate of stage 1

It is possible that inspection and maintenance will reduce MTTFF at very high inspection rate of stage 1 (recall that high inspection in stage 1 will increase \( \lambda_{13} \); thus, denominator may be large). This will increase the failure rate from stage 1 to 3; therefore, MTTFF may decrease. This conclusion is verified by the simulation result in Fig. 3a.

7.2.2. Inspection rate of stage 2

High inspection rate of stage 2 will increase the repair rate from stage 2 to 1 (\( \mu_{21} \)).

Assuming that this repair rate is very high,

\[
MTTFF \approx \frac{1 + \gamma_3 (\mu_{31} + \mu_{32} + \lambda_{13})}{\lambda_{13}}
\]

(6)

Then, MTTFF will increase to a constant value. This is verified by the simulation result in Fig. 3b.

7.2.3. Inspection rate of stage 3

High inspection rate of D3 will increase the repair rate from stage 3 to 2 (\( \mu_{32} \)) and also repair rate of stage 3 to 1 (\( \mu_{31} \)). These rates are linearly related to MTTFF; therefore, the lifetime will increase linearly with inspection rate of stage 3. This is verified by the simulation result in Fig. 3c.

8. Cost analysis

Cost equations are derived using steady state probability calculation. The cost analyses include failure cost, maintenance cost, and total cost. Maintenance cost in this analysis includes inspection cost based on the assumption of the equivalent model that maintenance is implemented at every inspection. These equations will explain the simulation results in Figs. 4–6.

8.1. Perfect maintenance model

The transitional probability matrix and resulting steady state probability are derived in Appendix C.1.

Let \( FC = \) repair cost after failure (dollar/time), \( MC = \) maintenance cost (dollar/time), \( P(i) = \) steady state probability of stage \( i \); \( i = 1, 2, \) or \( 3, \) \( C_F = \) expected annual failure cost (dollar/year), \( C_M = \) expected annual maintenance cost (dollar/year), \( C_T = \) expected annual total cost (dollar/year), \( T_R = \) repair time (year).

8.1.1. Failure cost analysis

The expected failure cost per year is

\[
C_F = FC \times \text{frequency of failure}
\]

(7)

\[
C_F = FC \times P(3) \times \frac{1}{\gamma_3} = \frac{FC}{T_R + MTTFF}
\]

(8)

The failure cost is an average cost over lifetime in one cycle of the device. This indicates that as MTTFF increases, the annual failure cost will reduce and it can also reduce to zero.

Consider the case of very frequent maintenance, this cost will approach zero. On the other hand, without maintenance; this cost will be an average cost over a total life time (life time without maintenance plus repair time). This indicates that failure cost will be the highest without maintenance; therefore, maintenance helps reducing failure cost.

8.1.2. Maintenance cost analysis

The expected maintenance cost per year is
8.1.3. Total cost analysis

The expected total cost is a summation of failure and maintenance cost. Clearly, without maintenance the total cost will be only a failure cost which is an average cost over a total lifetime. Consider very frequent maintenance, failure cost will be zero while maintenance cost will be the highest. Thus, total cost is dominated by failure cost at small inspection rate and is dominated by maintenance cost at high inspection rate.

8.1.3.1. Should we do the maintenance at all? Since maintenance is introduced in order to reduce the total cost, it should be implemented only if the highest possible total cost without maintenance is less than the highest possible total cost with maintenance, i.e.,

\[ C_T(\mu_{21} = 0, \mu_{32} = 0) < C_T(\mu_{21} \neq 0| \mu_{32} \neq 0) \] (11)

\[ C_T(\mu_{21} = 0, \mu_{32} = 0) = \frac{FC}{T_R + T_0} \] (12)

\[ C_T(\mu_{21} \neq 0| \mu_{32} \neq 0) = \begin{cases} C_T(\mu_{21} = 0, \mu_{32} \to \infty) = \frac{MC}{y_2} \\ C_T(\mu_{21} \to \infty, \mu_{32} = 0) = \frac{MC}{y_1} \\ C_T(\mu_{21} \to \infty, \mu_{32} \to \infty) = \frac{MC}{y_1} \end{cases} \] (13)

Thus, the following inequality should be considered.

\[ \frac{FC}{T_R + T_0} > \frac{MC}{y_1} \text{ or } \frac{MC}{y_2} \] (14)

Similarly,

\[ \frac{FC}{MC} > \frac{T_R + T_0}{y_1} \text{ or } \frac{T_R + T_0}{y_2} \] (15)

The inequality tells that if the ratio of failure cost and maintenance cost is higher than a constant value, then the maintenance should be implemented. Intuitively, if the failure cost is not expensive, we would rather replace the device than maintain it.

8.2. Imperfect maintenance model

The transitional probability matrix and the resulting steady state probabilities are derived in Appendix C.2.

8.2.1. Failure cost analysis

The expected failure cost per year is

\[ C_F = FC \times \text{frequency of failure} \] (16)

\[ C_F = FC \times P(3) \times \frac{1}{y_3} = \frac{FC}{T_R + \text{MTTFF}} \] (17)

Failure cost equation of this model is the same as that of perfect maintenance model; however, MTTFF equation is different. From MTTFF analysis, MTTFF will be greater than the lifetime without maintenance as long as the probability of transferring from stage 1 to 3 is not high which is usually true. Therefore, failure cost will reduce to a constant value as inspection rate of any stage increases. This conclusion is verified by simulation results in Figs. 4a, 5a and 6a.

8.2.2. Maintenance cost analysis

The expected maintenance cost per year is

\[ C_M = MC \times \text{frequency of maintenance} \] (18)

\[ C_M = MC \times (P(1)\lambda_{13} + P(2)\mu_{21} + P(3)(\mu_{31} + \mu_{32})) \] (19)

If the probability of transferring from stages 1 to 3 is very small then the analysis is the same as in perfect maintenance model. Maintenance cost will increase from zero to some constant value when inspection rates of D2 and D3 increase. This is verified by simulation results in Figs. 5b and 6b. However, when inspection rate of D1 increases (probability of transferring from stages 1 to 3 is higher), maintenance cost could increase to infinity. This is verified by the simulation result in Fig. 4b. It might be the case that the device condition gets worse and worse with every inspection and maintenance.

8.2.3. Total cost analysis

Failure cost dominates total cost at small inspection rate while maintenance cost dominates total cost at high inspection rate. Total cost will be smallest at optimum region of inspection rate of stage 1 and high inspection rate of stages 2 and 3. This conclusion is verified by simulation results in Figs. 4d, 5d and 6d.

Note that in this cost analysis, the inspection cost is accounted in the maintenance cost. However, if the inspection is used only to determine the stage of the device then the inspection cost need to be addressed in the model separately.

9. Inspection model and inspection cost analysis

An inspection stage is added to the perfect maintenance model (Fig. 9). Note that the inspection stage has no
transition rate to other stage under an assumption of perfect inspection that the device after inspection will stay in the same stage. Transitional probability matrix and resulting steady state probability are derived in Appendix D. The MTTFF equation is the same as that of the model without inspection. Moreover, the steady state probability equations are the same as those of perfect inspection model.

Intuitively, inspection by itself should not improve operating lifetime of the device since it is introduced only to determine the stage of the device. Clearly, the inspection does not affect the failure and maintenance cost.

9.1. Inspection cost analysis

Let IC = inspection cost (dollar/time), CI = expected inspection cost (dollar/year).

The expected annual inspection cost is

\[ C_I = IC \times P(1) \times \mu_1 \]

\[ C_I = IC \times \mu_2 (y_1 + y_1 y_2 \mu_{21} + y_1 y_2 y_3 \mu_{21} \mu_{32}) / (T_R + \text{MTTFF}) \]

Inspection cost is a linear function of inspection rate and probability of being in stage 1; therefore, higher inspection rate and repair rate of going from any stage to stage 1 will increase the inspection cost.

9.2. What is the advantage of inspection?

Obviously, inspection increases the total cost. However, inspection is intended to determine the stage of the device which is a crucial issue. Inspection is neither introduced to extend the device lifetime nor to reduce the cost. As long as the inspection does not cause the system to transit to higher stages, it should be implemented.

10. Conclusion

Analysis of inspection rate of each stage on MTTFF, failure cost, maintenance cost and inspection cost has been recognized in the paper. Simulation results from MATLAB are shown and verified by mathematical equations of the equivalent model. The paper suggests the criteria of implementing maintenance by comparing the failure and maintenance cost. In addition, inspection model has been constructed for inspection cost analysis. The analysis suggests the inspection is introduced only to determine the stage of device.

The proposed model in [1] will be selected for different load type of transformer in the system. The implementation using Monte Carlo simulation is in progress.

Acknowledgements

The work of this paper was supported by the Power System Engineering Research Center.

Appendix A. Model Parameters used in simulation

Cost parameters
- Inspection cost = US$100
- Oil filtering cost = US$1000
- Oil replacement cost = US$10,000
- Failure cost = US$100,000
- Mean time in D1 = 10 years
- Mean time in D2 = 7 years
- Mean time in D3 = 3 years

Appendix B. Mean time to the first failure

B.1. Perfect maintenance model

Truncated transitional probability matrix \( Q \) is constructed by deleting row 4 and column 4 which associated with the absorbing stage [6].

\[
Q_n = \begin{bmatrix}
1 - \frac{1}{y_1} & \frac{1}{y_1} & 0 \\
\mu_{21} & 1 - \left( \mu_{21} + \frac{1}{y_2} \right) & \frac{1}{y_2} \\
0 & \mu_{32} & 1 - \left( \mu_{32} + \frac{1}{y_3} \right)
\end{bmatrix} 
\]

(B.1)
The expected number of time intervals matrix is calculated from \( N = [I - Q_n]^{-1} \) [6].

\[
\det(N) = \frac{\mu_{21}\mu_{32}}{y_1} + \frac{\mu_{21}}{y_1 y_3} + \frac{\mu_{32}}{y_1 y_2} + \frac{1}{y_1 y_2 y_3} - \left( \frac{\mu_{32}}{y_1 y_2} + \frac{\mu_{21}\mu_{32}}{y_1} + \frac{\mu_{21}}{y_1 y_3} \right) = \frac{1}{y_1 y_2 y_3} \quad (B.2)
\]

Entering from stage 1, MTTFF is the summation of matrix N(1).

\[
\text{MTTFF} = y_1 + y_2 + y_3 + \frac{\mu_{21}y_1 y_2}{y_3} + \frac{\mu_{21}\mu_{32}y_1 y_2 y_3}{y_3} \quad (B.3)
\]

**B.2. Imperfect maintenance model**

Truncated transitional probability matrix \( Q \) is constructed by deleting row 4 and column 4 which associated with the absorbing stage [6].

\[
Q_n = \begin{bmatrix} 
1 - \left( \frac{\lambda_{13}}{y_1} + \frac{1}{y_1} \right) & \frac{1}{y_1} & \lambda_{13} \\
\mu_{21} & 1 - \left( \frac{\mu_{21}}{y_2} + \frac{1}{y_2} \right) & \frac{1}{y_2} \\
\mu_{31} & \mu_{32} & 1 - \left( \frac{\mu_{31} + \mu_{32} + \frac{1}{y_3}}{y_3} \right) 
\end{bmatrix} 
\]

The expected number of time intervals matrix is calculated from \( N = [I - Q_n]^{-1} \) [6].

\[
\det(N) = \frac{1}{y_1 y_2 y_3} + \frac{\lambda_{13}}{y_2 y_3} + \frac{\lambda_{13}\mu_{21}}{y_3} 
\]

Entering from stage 1, MTTFF is the summation of matrix N(1).

\[
\text{MTTFF} = \frac{1}{\det(N)} \left( \frac{\lambda_{13}}{y_2 y_3} + \frac{1}{y_1 y_3} + \frac{1}{y_1 y_2} + \mu_{21} \left( \mu_{31} + \mu_{32} \right) + \frac{\mu_{21}}{y_3} + \frac{\mu_{31} + \mu_{32} + \frac{1}{y_3}}{y_3} \right) 
\]

\[
+ \frac{\mu_{31}}{y_1} + \frac{\mu_{32}}{y_1} + \frac{\lambda_{13}}{y_2} + \mu_{21}\lambda_{13} \quad (B.6)
\]

**Appendix C. Steady state probability**

**C.1. Perfect maintenance model**

Using frequency balance approach, steady state probability is calculated from

\[
P = \begin{bmatrix} 
-\frac{1}{y_1} & \mu_{21} & 0 & \mu_F \\
1 & 1 & 1 & 1 \\
0 & \frac{1}{y_2} & -\left( \mu_{32} + \frac{1}{y_3} \right) & 0 \\
0 & 0 & \frac{1}{y_3} & -\mu_F 
\end{bmatrix}^{-1} 
\]

\[
\det(P) = -\frac{\mu_F}{y_1 y_2 y_3} (T_R + \text{MTTFF}) \quad (C.2)
\]

\[
P = \frac{1}{(T_R + \text{MTTFF})} \begin{bmatrix} 
\lambda_{13} & \frac{1}{y_1} & \frac{1}{y_2} & \frac{1}{y_3} \\
\frac{1}{y_1} & 1 - \left( \frac{\mu_{21}}{y_2} + \frac{1}{y_2} \right) & \mu_{32} & 0 \\
\lambda_{13} & \frac{1}{y_2} & -\left( \mu_{31} + \mu_{32} + \frac{1}{y_3} \right) & 0 \\
0 & 0 & \frac{1}{y_3} & -\mu_F 
\end{bmatrix} 
\]

\[
\text{Let } T_R = \frac{1}{\mu_F } \text{: the repair time (year) then,} 
\]

**C.2. Imperfect maintenance model**

Using frequency balance approach, steady state probability is calculated from

\[
P = \frac{1}{(T_R + \text{MTTFF})} \begin{bmatrix} 
\lambda_{13} & \frac{1}{y_1} & \frac{1}{y_2} & \frac{1}{y_3} \\
\frac{1}{y_1} & 1 - \left( \frac{\mu_{21}}{y_2} + \frac{1}{y_2} \right) & \mu_{32} & 0 \\
\lambda_{13} & \frac{1}{y_2} & -\left( \mu_{31} + \mu_{32} + \frac{1}{y_3} \right) & 0 \\
0 & 0 & \frac{1}{y_3} & -\mu_F 
\end{bmatrix} 
\]
\[ \text{det}(P) = -\frac{\mu_F}{\gamma_1\gamma_2\gamma_3} (T_R + \text{MTTFF}) \times (1 + \gamma_1\lambda_{13} + \gamma_1\gamma_2\mu_{21}\lambda_{13}) \] 

(C.5)

Then, the steady state probability is

\[ P = \frac{1}{(T_R + \text{MTTFF})} \begin{bmatrix} y_1 + y_1y_2\mu_{21} + y_1y_3\mu_{31} + y_1y_2y_3\mu_{21}\mu_{31} + y_1y_2y_3\mu_{21}\mu_{32} \\ y_2 + y_2y_3\mu_{31} + y_2y_3\mu_{32} + y_1y_2y_3\lambda_{13}\mu_{32} \\ y_3 \\ y_1 + y_2y_3\mu_{31} + y_1y_2y_3\mu_{21}\mu_{31} + y_1y_2y_3\mu_{21}\mu_{32} \\ (1 + \lambda_{13}\gamma_1 + \lambda_{13}\mu_{21}\gamma_1) \\ \end{bmatrix} \] 

Let \( T_1 \) = Time in inspection stage then,

\[ T_1 = \mu_1(y_1 + \mu_{21}y_1y_2 + \mu_{21}\mu_{32}y_1y_2y_3) \] 

(D.5)

\[ \text{det}(P) = \frac{\mu_F}{\gamma_1\gamma_2\gamma_3} (T_R + T_1 + \text{MTTFF}) \] 

(D.6)

Appendix D. Inspection model

D.1. Mean time to the first failure

Truncated transitional probability matrix \( Q \) is constructed by deleting row 4 and column 4 which associated with the absorbing stage [6].

\[ Q_\pi = \begin{bmatrix} 1 - \left( \mu_1 + \frac{1}{\gamma_1} \right) & \frac{1}{\gamma_1} & 0 & \mu_E \\ \mu_{21} & 1 - \left( \mu_{21} + \frac{1}{\gamma_2} \right) & \frac{1}{\gamma_2} & 0 \\ 0 & \mu_{32} & 1 - \left( \mu_{32} + \frac{1}{\gamma_3} \right) & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \] 

(D.1)

The expected number of time intervals matrix is calculated from \( N = [I - Q_\pi]^{-1} \) [6].

\[ \text{det}(N) = \frac{1}{\gamma_1\gamma_2\gamma_3} \] 

(D.2)

Entering from stage 1, mean time to the first failure is

\[ \text{MTTFF} = y_1 + y_2 + y_3 + \mu_{21}y_1y_2 + \mu_{32}y_2y_3 + \mu_{21}\mu_{32}y_1y_2y_3 \] 

(D.3)

D.2. Steady state probability

Using frequency state probability, steady state probability is calculated from

\[ P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ \frac{1}{\gamma_1} - \left( \mu_{21} + \frac{1}{\gamma_2} \right) & \mu_{32} & 0 & 0 \\ 0 & \frac{1}{\gamma_2} - \left( \mu_{32} + \frac{1}{\gamma_3} \right) & 0 & 0 \\ 0 & 0 & \frac{1}{\gamma_3} & -\mu_F & 0 \\ \frac{\mu_F}{T_R} & 0 & 0 & 0 & -1 \end{bmatrix}^{-1} \] 

(D.4)
The conditional probabilities of stage 1, 2 and 3 given that the stages are in working stages (excluding time spent in inspection stage) are

\[ P(1) = \frac{y_1 + y_1 y_2 \mu_{21} + y_1 y_2 y_3 \mu_{21} \mu_{32}}{T_R + \text{MTTFF}} \]  
\[ P(2) = \frac{y_2 + y_2 y_3 \mu_{32}}{T_R + \text{MTTFF}} \]  
\[ P(3) = \frac{y_3}{T_R + \text{MTTFF}} \]

These probabilities are the same as in Appendix C.1.

References


