

## Optimization Using a Central Composite Rotatable Design for Jewelry-Bodied Casting

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### Abstract

Jewelry-bodied casting is the most important upstream process causing defects in jewelry industries. Ingate Angle, Mold Temperature, and Pouring Temperature influence on the yield of products were studied by using response surface methodology. A 2<sup>3</sup> full factorial central composite rotatable design (CCRD) was employed. The relationship of Ingate Angle, Mold Temperature, Pouring Temperature, and yield of products was determined by using regression analysis. The models of expected function with quadratic model were used for analysis of the experimental data. The results indicated that the maximum predictive yield of products in alloy 1 by using optimal setting conditions of three controllable factors was 100% and estimated optimum factors as follows : Ingate Angle=37°, Mold Temperature=482°C, and Pouring Temperature=934°C. The maximum predictive yield of products in alloy 2 by using optimal setting conditions of three controllable factors was 100% and estimated optimum factors as follows : Ingate Angle=35°, Mold Temperature=535°C, and Pouring Temperature=974°C. The maximum predictive yield of products in alloy 3 by using optimal setting conditions of three controllable factors was 99.72% and estimated optimum factors as follows : Ingate Angle=35°, Mold Temperature=514°C, and Pouring Temperature=1034°C.

**Keywords :** optimization, response surface methodology, central composite design, regression analysis, jewelry-bodied casting

### Introduction

Jewelry-bodied casting is the most important upstream process causing defects in jewelry industries. The casting conditions have affected the occurrence of defects and quality requirement. There are many controllable factors that affect the quality such as Types of Alloy, Ingate Angle, Mold Temperature, and Pouring Temperature.[1] These controllable factors have to be varied regarding of product appearance. The jewelry casting operators sometimes work by using their skills with trial and error in controllable factors setting and

cause problems in product quality. Therefore, if they know the exact controllable factor level setting, they can control the process quality better.

The application of statistical experimental design[2] in jewelry-bodied casting can result in improving product quality; reducing process variability, i.e., closer confirmation of the output response to nominal and target requirements; and reducing development time and overall costs. Conventional practice of classical method of maintaining other factors involved at an unspecified constant level does not depict the combined

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effect of all the factors involved. This method is also a time consuming process and requires a number of experiments to determine optimum levels, which is unreliable. This limitation of a classical method process can be eliminated by optimizing all the affecting parameters collectively by statistical experimental design using response surface methodology (RSM). Response surface methodology [3] is the statistical and mathematical technique which is useful for developing, improving, and optimizing processes. It also has important applications for the design, development, formulation of new products, as well as the improvement of existing product designs. This approach can help the jewelry casting operators in the Quality Control area work better, and it can control the consistency of product quality with less effort. Moreover, it can help them with new product development when they do not know the exact appropriate casting conditions used in the process. There are three controllable factors involved in jewelry-bodied casting process affecting the yield of products that are Ingate Angle, Mold Temperature, and Pouring Temperature. Three types of silver alloys that were used in product casting called Alloy 1, Alloy 2 and Alloy 3 were investigated whether they resulted in the casting quality. The products were pendants, square shape and 5 grams weight. We measured the quality characteristics of casting product in the form of yield. When there were defects[4], especially the porosity that most normally occurred inside the product, the yield of products would decrease. The target of quality requirement was the maximum yield of 100%.

In this study, the central composite rotatable design (CCRD) was employed as RSM tools for optimizing jewelry-bodied casting process. The regression analyses were used as the tools for building the relationship between controllable factors and response. The estimated functions were in the forms of quadratic functions. The performance measures were the coefficient of Determination ( $R^2$ ) and Mean Square Error (MSE). This paper illustrated the optimization procedure with two stages. In the first stage, the RSM was introduced

as powerful method to build the statistical approximation to provide the description of the relationship between the controllable factors and response. In the second stage, the predictive model would be defined as the objective function of optimization to accomplish the optimization procedure by using Excel Solver.

## Materials and Methods

### Materials

There were three types of silver alloy used in jewelry-bodied casting as follows:

1. Alloy 1 consisted of silver 94.22 % by weight and copper 5.78 % by weight.
2. Alloy 2 consisted of silver 94.22 % by weight, copper, zinc, and silicon 5.78 % by weight.
3. Alloy 3 consisted of silver 94.22 % by weight, copper, zinc, silicon and indium 5.78 % by weight.

Pure silver[1] is a precious metal. It is white, ductile, and convenient for the process. It is harder than gold but less ductile than copper. It has the most inductility. Its melting point is 961 degree celsius. Copper mixing in sterling silver [1] will increase hardness and strength of alloy. It will increase wariness ability but decrease elongation. When the alloy is melted, the copper will interact with oxygen in the atmosphere and result in the copper oxide film coating over the surface of the product. This leads to the problems that the surface of the product looks dull, and it is not beautiful. Zinc mixing in sterling silver[1] will be used as deoxidizer. Zinc will interact with oxygen and result in zinc oxide that will prevent copper oxide film from coating over the surface of the product. Silicon mixing in sterling silver [1] will increase the smoothness of the surface, but the excessive silicon makes the product crack easily. It also creates hard spots inside the product. Indium mixing in sterling silver[1] will increase tarnish resistance but decrease hardness and strength. As a result, such elements mixing in sterling silver will especially increase some good properties of each alloy.

## Methods

Jewelry-bodied casting [1] is the process of investment casting process. It helps smoothing the surface of products and making more precise appearance. The casting process is called Lost wax casting uses vulcanized rubber mould producing wax products and lost wax by heating plaster mould of wax trees. After lost wax, it makes the product appearance have a hole inside like wax product appearance. This plaster mould is heated in the oven in order to strengthen the mould itself. Then products will be casted in the vacuum induction furnace. Jewelry-bodied casting in vacuum induction furnace can control the oxidation effect better than in centrifugal furnace because there is the inert gas such as nitrogen gas or argon gas that cover the atmosphere to prevent oxidation effect, which causes gas porosity inside the products.

There are three controllable factors affecting to the yield of products, i.e. Ingate Angle, Mold Temperature, and Pouring Temperature. Ingate Angle is a melting metal angle used as ingate in gating system. It varies corresponding to the product's appearance. Mold Temperature is the final temperature of heated plaster mold used for pouring melted metal. It affects the microstructure of products. It should appropriately differ from pouring temperature in order to make complete solidification. Pouring Temperature is the final temperature of melted metal used for pouring melted metal into the heated plaster mould. Melting Temperature of pure silver is 961 degree celcius. Pouring temperature should be over melting temperature of metal 50-100 degree celcius so that it can ensure that the solid metal becomes the whole liquid metal. When the liquid metal with high temperature is cool down, it can ensure that the liquid metal has complete solidification. Low pouring temperature makes microstructure equiaxial grain. High pouring temperature makes microstructure columnar grain. Equiaxial grain is preferred because it has more strength.

Casting products used as specimens in this work were pendant, square shape, and approximate 5 grams weight. Casting specimens were polished and

inspected by vision. The defects were detected and recorded. After that, the microstructure of specimens was analyzed. Central Composite Rotatable Design (CCRD) that is the experimental design used in this research. The Central Composite Design was proposed by Box and Wilson [5]. It consists of  $2^k$  full factorial points or  $2^{k-q}$  resolution V fraction factorial points called cubic points,  $2k$  axial or star points and  $n_0 \geq 2$  runs in the design center [6] (where  $k$  is the number of controllable factors,  $q$  is the number of fraction, and  $n_0$  is the number of design center runs). CCD with the rotating property is conducted by choosing an appropriate axial distance [6]. Rotating property is important for a second-order design to possess a reasonably stable distribution of scaled prediction variance throughout the experimental design region. The reasonably stable scaled prediction variance ensures that the quality of the predicted response values is roughly the same throughout the region of interest.

A  $2^3$  full factorial central composite design with five coded levels leading to 19 runs of experiments was performed. There were 8 cubic points of  $2^3$  full factorial points, 6 axial points (star points) and 5 center points in design. The design was rotatable CCD, using an axial distance  $\alpha = 1.682$ . Response was measured as yield of products. There were three controllable factors influencing to response that were Ingate Angle, Mold Temperature, and Pouring Temperature.

The range and levels of variables investigated in this research are given in Table 1. The initial setting of the levels of Ingate Angle was selected corresponding to the normally performing uses in practice. The initial setting of the levels of Mold Temperature was selected corresponding to the cooling curve of metal. The initial setting of the levels of variables was selected corresponding to the phase diagrams of casting alloy mixture. Such phase diagrams could indicate the approximate range of Pouring Temperature used in casting process.

**Table 1** Coded variable levels and natural variable levels used in Alloy 1 , Alloy 2 , and Alloy 3

Factors	Coded variable levels and natural variable levels for								
	Alloy type 1			Alloy type 2			Alloy type 3		
	-1	0	1	-1	0	1	-1	0	1
Ingate Angle ( ° )	45	60	75	45	60	75	45	60	75
Mold Temperature ( °C )	470	480	490	520	530	540	520	530	540
Pouring Temperature ( °C )	940	950	960	980	990	1000	1040	1050	1060

In this research, we used quadratic model for predicting the relationship between the response and three controllable factors. The quadratic model for predicting the relationship between the response and three controllable factors was expressed according to equation (1)

$$y = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \beta_{ii} x_i^2 + \sum_{i=1}^2 \sum_{j=i+1}^3 \beta_{ij} x_i x_j \tag{1}$$

where  $\beta_0$  is intercept on y-axis,  $\beta_i$  is linear coefficients,  $\beta_{ii}$  is quadratic coefficients,  $\beta_{ij}$  is cross-product coefficients, and  $x_i, x_j$  are coded independent variables.

The Minitab (version 13.0) software was used for regression analyses of the obtained data. The statistical significance of the regression coefficients was determined by Student’s t test. The model equations were determined by F test. The appropriate adequate

models were determined by lack of fit test, and the proportion of variance explained by the models obtained were given by the coefficients of determination ( $R^2$ ) and  $R^2$  (adjusted). The response was transformed into log(yield) for variance stabilizing[7], and the Excel Solver was used to determine the optimized process parameters .

## Results and Discussions

### Results

The model development was in the forms of quadratic models by using regression analyses. The student’s t statistics indicated that the regression coefficients were significant. The F test indicated that the regression models were significant, and the Lack of Fit test indicated that the regression models were appropriate.

**Table 2** The optimal setting of three controllable factors and maximum predictive yield of product in each type of alloy

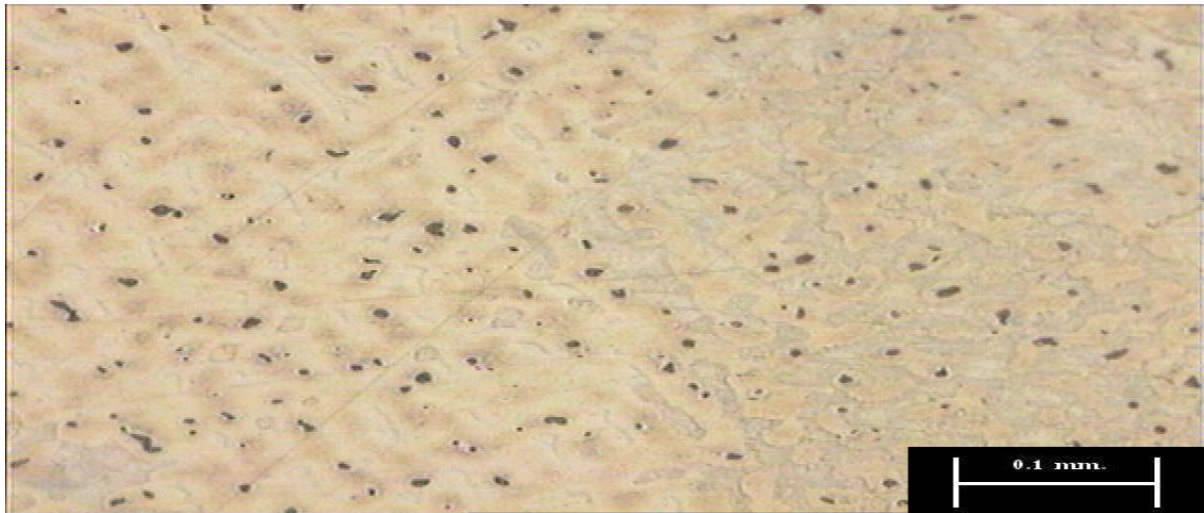
Alloy Type	Factors			Optimal Yield ( $\hat{y}$ ) (%)
	Ingate Angle ( $x_1$ ) (°)	Mold Temperature ( $x_2$ ) (°C)	Pouring Temperature ( $x_3$ ) (°C)	
1	37	482	934	100
2	35	535	974	100
3	40	514	1034	99.72

### Discussions

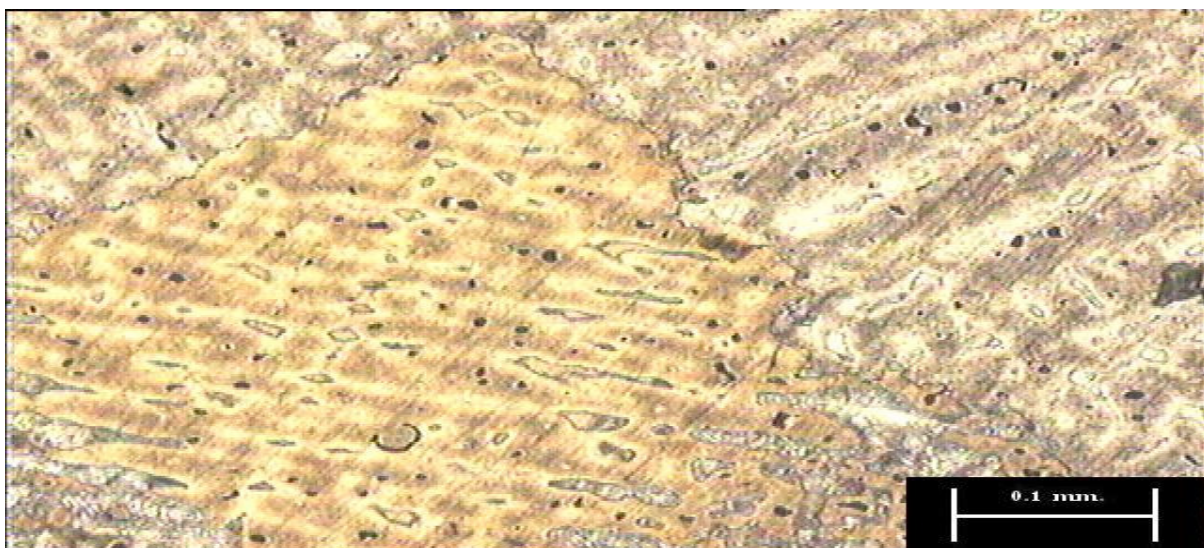
Three controllable factors that were Ingate Angle, Mold Temperature, and Pouring Temperature affected the yield of product. For the microstructure analysis, most defects were gas porosity, and a few defects were dendrite or shrinkage porosity. This was involved in the easy embrittlements of product. Therefore, if

we can reduce those defects, the quality of product will be better.

Gas porosity defects inside the specimens are illustrated in Figure 1, and dendrite defects inside the specimens are illustrated in Figure 2. If we can reduce those defects, the quality of product will be better.



**Figure 1** Gas porosity defects inside the specimens



**Figure 2** Dendrite defects inside the specimens

## Conclusions

This work gives the idea of quality engineering application based on the experimentation to jewelry industry. The limitations of application are the high costs of casting materials and complication of experimental procedure.

The results indicated that for casting process by using the CCRD, the maximum predictive yield of products in alloy 1 by using optimal setting conditions of three controllable factors was 100%, and the optimum conditions are as follows: Ingate Angle =  $37^{\circ}$ , Mold Temperature =  $482^{\circ}\text{C}$ , and Pouring Temperature =  $934^{\circ}\text{C}$ . The maximum predictive yield of products in alloy 2 by using optimal setting conditions of three controllable factors was 100% and estimated optimum factors as follows : Ingate Angle =  $35^{\circ}$ , Mold Temperature =  $535^{\circ}\text{C}$ , and Pouring Temperature =  $974^{\circ}\text{C}$ . The maximum predictive yield of products in alloy 3 by using optimal setting conditions of three controllable factors was 99.72% and estimated optimum factors as follows : Ingate Angle =  $35^{\circ}$ , Mold Temperature =  $514^{\circ}\text{C}$ , and Pouring Temperature =  $1034^{\circ}\text{C}$ .

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