Development of a Mathematical Model for Heat Pump Fruit Drying

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ABSTRACT

A mathematical model for papaya glace drying using heat pump has been developed. It comprises of drying rate equation, mass and energy balance equations, thermo-physical property of papaya glace equations and heat exchanger equations. Successive substitution method was used for finding the solution. It was found that the model was fairly accurate for predicting final moisture content, air temperature at various parts in the system and temperature of refrigerant especially at low moisture level of papaya glace.

The mathematical model was then used to find out strategy for drying papaya glace with a specific airflow rate of 29.8 kg/h-kg dry papaya glace. Simulated results showed that drying time and energy consumption decreased when the drying air temperature increased. In addition, the appropriate by-pass air ratio was in the range of 86%-90%.

1. INTRODUCTION

Britnell, et al. [1] studied drying of ginger and potato by using heat pump. They found that quality of agricultural product, dried by heat pump, in terms of smell, taste and color was better than those dried by rotary drying. Clement, et al. [2] studied continuous drying of rubber using heat pump. They found that COP increased with air relative humidity and suitable by-pass air ratio was 60%-70%. Meyer and Grevenstein[3] studied economic grain drying using heat pump. They found that heat pump drying which operated along with other purpose such as heating water was more economical than drying using electric hot wire and diesel fuel. Soponronnarit, et al. [4] studied suitable strategy for papaya glace drying in thermal tunnel and developed a mathematical model. Main assumption of the model was heat equilibrium between moist air and papaya glace. Simulation result showed that the simulated drying rate was slightly higher than the experimental one.

The objective of this research is to develop a mathematical model for papaya glace drying using heat pump. Successive substitution method was used for finding solution. Strategy of drying was studied by adjusting the suitable by pass air ratio with consideration of drying time and energy consumption.
2. MATERIALS AND METHODS

2.1 Development of Mathematical Model

Saponronnarit, et al. [4] developed a mathematical model for predicting papaya glaçé drying in tunnel. Assumption of the model was thermal equilibrium between moist air and product. In this research, the model was further developed for predicting papaya glaçé drying using heat pump. The additional assumption was that temperature at the surface of evaporator tube equaled to refrigerant temperature in the evaporator and by-pass factor depended on type of evaporator. Refrigerant temperature at condenser and evaporator was the average value between inlet and outlet temperatures of both equipment. Temperature difference at inlet and outlet of recuperator was 4°C.

Calculation of Papaya Glaçé Moisture Content

Diffusion equation of cubic material was used to calculate papaya glaçé moisture content. The equation is as follows:

\[
MR(t) = \left(\frac{8}{\pi^2}\right)^3 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \left[ \frac{1}{(2i+1)^2} \right] \left[ \frac{1}{(2j+1)^2} \right] \left[ \frac{1}{(2k+1)^2} \right] \exp \left[-\left(\frac{(2i+1)^2}{L_x^2} + \frac{(2j+1)^2}{L_y^2} + \frac{(2k+1)^2}{L_z^2}\right) \pi^2 Dt \right]
\]

where:

\[
MR(t) = \frac{(M(t) - M_{eq})}{(M_{in} - M_{eq})}
\]

Diffusion coefficient of Achariyaviriya and Saponronnarit [5] was used for predicting drying rate. It is written as follows:

\[
D = 0.000917 \exp \left[-2877.49/(T_{max} + 273) \right]
\]

Equation (3) can be used for temperature of 40°C to 80°C and initial moisture content not higher than 50% dry basis. Achariyaviriya and Saponronnarit [5] found that equilibrium moisture content of papaya glaçé and air relative humidity can be calculated as follows:

\[
M_{eq} = CM_{in} RH/[100^* (1.01 + CRH + 2RH - CRIF + RIP)]
\]

\[
C = 163.15 \exp (-0.0647 T_{max})
\]

\[
M_{in} = 3.1987 + 0.14077 T_{max}
\]

where:

- \(MR(t)\) = moisture ratio, fraction
- \(M(t)\) = average moisture content of papaya glaçé at any time, decimal dry basis
- \(M_{in}\) = average initial moisture content of papaya glaçé, decimal dry basis
- \(M_{eq}\) = equilibrium moisture content of papaya glaçé, decimal dry basis
- \(D\) = diffusion coefficient, m²/h
- \(RH\) = relative humidity of drying air, fraction
- \(T_{max}\) = drying air temperature, °C
- \(t\) = drying time, h
- \(L\) = dimension of papaya glaçé, m
From Eq. 1, average moisture content changing at any time can be calculated from differential of Eq. 1 to time, using Runge-Kutta method.

**Calculation of Air Condition after Drying**

Considering CV. 1 in Fig.1, from conservation of mass principle, it is found that air moisture change is equal to evaporated water from papaya glācé. From this equation, \( W_f \) can be determined.

\[
W_f = (M_i - M_f) R + W_{\text{mix}}
\]

(7)

where: \( R = \frac{M_p}{m_{\text{mix}} \Delta t} \)

From conservation of energy principle, it shows that enthalpy change of air flow including change of internal energy in drying material is equal to total heat transfer between system and environment. From this equation, \( T_f \) can be determined.

\[
T_f = (C_a T_{\text{mix}} + W_{\text{mix}} (h_f + C_v T_{\text{mix}}) + RC_p \theta - W_f h_f - Q_1) / (C_a + W_f C_v + RC_p)
\]

(8)

![Diagram of Heat Pump Drying System](image)

**Fig. 1.** Control volumes of the heat pump drying system.
where:  
\[ m \quad = \quad \text{dry air mass flow rate, kg/h} \]
\[ h_g \quad = \quad \text{latent of vaporization, kJ/kg} \]
\[ C \quad = \quad \text{specific heat, kJ/kg °C} \]
\[ \theta \quad = \quad \text{papaya glace' temperature before drying, °C} \]
\[ Q_p \quad = \quad \text{heat loss, kJ/kg dry air} \]
\[ M_p \quad = \quad \text{dry mass of product, kg} \]
\[ M \quad = \quad \text{average moisture content of papaya glace, decimal dry basis} \]
\[ T \quad = \quad \text{temperature, °C} \]
\[ W \quad = \quad \text{humidity ratio, kg water/kg dry air} \]

Subscripts:
\[ a \quad = \quad \text{dry air} \]
\[ f \quad = \quad \text{after drying} \]
\[ I \quad = \quad \text{before drying} \]
\[ p_w \quad = \quad \text{papaya glace} \]
\[ v \quad = \quad \text{water vapor} \]
\[ \text{mix} \quad = \quad \text{mixed air before entering to drying chamber} \]

**Calculation of Mixed Air Temperature and Recycled Air**

Considering CV 4 in Fig. 1, from conservation of mass principle, it is found that mass of water in hot air before drying is equal to total mass of water in reduced moisture air from evaporator and mass of water in recycled hot air. From this equation, \[ W_{\text{mix}} \] can be determined.

\[ W_{\text{mix}} = RCW_f + (1 - RC)W_{\text{evo}} \]  \hspace{1cm} (9)

where:  
\[ RC = \frac{m_p}{m_{\text{mix}}} \]
\[ W_{\text{evo}} = \text{humidity ratio of air leaving from evaporator, kg water/kg dry air} \]
\[ m_p = \text{mass flow rate of bypass air, kg/h} \]

From conservation of energy principle, enthalpy of air flow entering to CV 4 is equal to enthalpy of air flow leaving from CV 4 when heat loss between system and environment is neglected. From this equation, \[ T_{\text{cai}} \] can be determined.

\[ T_{\text{cai}} = \{RC \left[ C_a T_f + W_f (h_f + C_v T_f) \right] + (1 - RC) \left[ T_{\text{reco}} (C_a + W_{\text{evo}} C_v) + W_{\text{evo}} h_f \right] - W_{\text{mix}} h_f \}/(C_a + W_{\text{mix}} C_v) \]  \hspace{1cm} (10)

where:  
\[ T_{\text{reco}} = \text{outlet air temperature of recuperator, °C} \]
\[ T_{\text{cai}} = \text{inlet air temperature of condenser, °C} \]

**Calculation of Humidity Ratio and Outlet Air Temperature of Evaporator**

Considering CV 6 in Fig. 1, it is assumed that humidity ratio and temperature of the outlet air of evaporator can be calculated by the following equations :  

\[ W_{\text{evo}} = BFW_f + (1 - BF) W_{\text{er}} \]  \hspace{1cm} (11)

\[ T_{\text{eo}} = BFT_{a} + (1 - BF) T_{e} \]  \hspace{1cm} (12)
where: \( BF \) = fraction of air which does not touch refrigerant coil tube with value of 30%
\( T_{ei} \) = inlet air temperature of evaporator, °C
\( T_{eo} \) = outlet air temperature of evaporator, °C
\( T_{f} \) = refrigerant temperature at evaporator, °C
\( W_{er} \) = saturated humidity ratio at evaporator surface, kg water/kg dry air

**Electricity Consumption**

Considering CV. 2 in Fig. 1, from conservation of energy principle, it is found that enthalpy change rate of air flow is equal to shaft power of main blower which can be written as follows:

\[
W_s = m_{max}(C_a + W_{max}C_v)(T_{mix} - T_{cao})
\]

where: \( T_{cao} \) = outlet temperature of condenser, °C
\( W_s \) = shaft power of blower, kJ/h

**Heat Exchangers**

Considering CV. 3 in Fig. 1., from conservation of energy principle, it is found that rate of heat rejected by condenser is equal to enthalpy change rate of air flow, when heat loss between environment and system is neglected.

\[
Q_{in,cond} = m_{max}(C_a + W_{max}C_v)(T_{cao} - T_{cai})
\]

where: \( Q_{in,cond} \) = heat received from internal condenser, kJ/h

From conservation of energy principle in CV. 5, rate of heat exchanged at recuperator or air to air heat exchanger is equal to enthalpy change rate of air flow.

\[
\text{Hot current } Q_{re} = m_p(C_a + W_{v}C_v)(T_f - T_{ei})
\]
\[
\text{Cold current } T_{re0} = \left[ Q_{re} / m_p(C_a + W_{v0}C_v) \right] + T_{so}
\]

where \( Q_{re} \) = heat exchange, kJ/h
\( m_p \) = air flow rate via evaporator, kg/h

Assumption of calculation is that the difference between inlet and outlet temperature is 4°C, i.e.

\[
T_{ei} = T_f - 4
\]

**Calculation of Refrigerant Temperature**

Heat transfer equation at heat exchanger with phase change of media substance can be expressed as follows:

At the condenser:

\[
T_c = \left[ T_{cai} - T_{cao} \exp \left( (T_{cao} - T_{cai})UA_e/Q_{in,cond} \right) \right] / \left[ 1 - \exp \left( (T_{cao} - T_{cai})UA_e/Q_{in,cond} \right) \right]
\]

\[
(18)
\]
At the evaporator:

\[ T_e = \frac{[T_{ei} - T_{eo}] \exp \left( (T_{ei} - T_{eo})UA_e/Q_{ev} \right)}{[1 - \exp \left( (T_{ei} - T_{eo})UA_e/Q_{ev} \right)]} \]  

(19)

**Calculation of Heat Pump Equipment**

Assumption of calculation is that the performance of compressor and evaporator are related to refrigerant temperature at evaporator and condenser. Mathematical function in terms of polynomial is as follows:

\[ P = A_1 + A_2 T_e + A_3 T_e^2 + A_4 T_c + A_5 T_c^2 + A_6 T_e T_c + A_7 T_e^2 T_c + A_8 T_c^2 + A_9 T_e^2 T_c \]  

(20)

\[ Q_{ev} = B_1 + B_2 T_e + B_3 T_e^2 + B_4 T_c + B_5 T_c^2 + B_6 T_e T_c + B_7 T_e^2 T_c + B_8 T_c^2 + B_9 T_e^2 T_c \]  

(21)

where \( A \) and \( B \) are constants from manufacturing company.

### 2.2 Calculation Procedure

To start, \( W_f \) is assumed and bypass factor is set according to the design of evaporator for calculating \( W_{env} \) value from Eq. 11. \( W_{env} \) is used to calculate \( W_{max} \) from Eq. 9, and then value of \( T_{max} \) is used to calculate \( RH \) value and \( M_{env} \) value is calculated from Eq. 4. Then \( M_c \) value is calculated from derivative of Eq. 1 to time. New value of \( W_f \) is calculated from Eq. 7 and \( T_f \) is calculated from Eq. 8. \( T_{max} \) and new \( W_f \) are used to calculate relative humidity of hot air leaving from dryer (\( RH_f \)) and then \( RH_f \) is checked. If \( RH_f \) is not acceptable (\( RH_f < 1 \)), subprogram for moisture condensation will be calculated. But if \( RH_f \) is acceptable (\( RH_f > 1 \)), assumed \( T_c \) and \( T_e \) are used to calculate \( P \) and \( Q_{ev} \) from Eq. 20 and Eq. 21, respectively. Then \( Q_e \) is calculated. Assumed \( W'_f \) value and \( W_{env}' \) value are used to find \( T_{ei}, T_{eo}, T_{cas}, T_{cas}, Q_{in,cond}, Q_{re} \) and \( T_{rep} \) from Eq. 17, Eq. 12, Eq. 10, Eq. 13, Eq. 14, Eq. 15 and Eq. 16, respectively. The new values of \( T_e \) and \( T_e \) are calculated from Eq. 18 and Eq. 19, respectively. \( W'_f, T_e \), and \( T_e \) values are checked. If different value between new \( W_f \) and assumed \( W_f \) is more than acceptable value (0.00001) and the different value between new \( T_e, T_e \) and assumed \( T_e, T_e \) are more than acceptable value (0.1), recalculation by using new value of \( W'_f, T_e \), and \( T_e \) in place of assumed value will be conducted. If those values are less than acceptable value, the following calculation will be done: energy consumption of main blower and compressor are calculated. Then final moisture content of papaya glace' is checked whether it is less than determined value or not. If it is less than or equal to determined value, program will be stopped. Figure 2 shows block diagram of calculation and Fig. 3 shows computer simulation flow chart.

### 3. RESULTS AND DISCUSSIONS

#### 3.1 Comparison between Experimental Drying Rate and Drying Rate from Mathematical Model

From comparison between results of calculation from mathematical model and experimental results at temperature of 50°C with papaya glace' dimension of 1.1 x 1.2 x 1.3 cm³, initial moisture content of 38.3% dry basis, final moisture content of 23% dry basis and bypass air ratio of 63% as shown in Fig. 4, it was found that average moisture content of papaya glace' from calculation was
nearly the same as those from experiment. Air temperature at various points as shown in Fig. 5 show that inlet and outlet air temperature of any equipment such as condenser, evaporator and recuperator calculated by mathematical model was close to experimental data. Refrigerant temperature at evaporator and condenser as shown in Fig. 6 show that simulated value with mathematical model was in average value of experimental results.

3.2 Using a Mathematical Model for Finding Appropriate Drying Condition

Appropriate drying conditions were investigated by using the developed mathematical model. Suitable bypass air ratio was calculated by using criteria of minimum specific energy consumption and near minimum drying time as shown in Fig. 3. The quantity of papaya glacé was varied in three values as follows: 56.5 kg, 80.1 kg and 101.1 kg. Figures 7 to 9 show the results of simulation at various quantity of papaya glacé. From these figures, it was found that the suitable bypass air ratio was 90%, 88% and 86% with quantity of papaya glacé of 56.5 kg, 80.1 kg and 101.1 kg, respectively. At these conditions, specific energy consumption or drying time is at the minimum values.

Simulation result with varying drying air temperature showed that drying time and specific energy consumption decreased when drying air temperature increased. It was because high drying temperature caused low relative humidity of drying air. Therefore, moisture content of papaya glacé decreased quickly and resulted in low energy consumption. Suitable drying temperature in the range of study was 50°C as shown in Fig. 10.

![Fig. 2. Block diagram of calculation.](image-url)
Fig. 3. Computer simulation flow chart.
Fig. 4. Evolution of simulated and experimental moisture content (Test 3/2)
[By-pass air ratio = 63%, Temperature = 50°C,
Specific air flow rate = 29.83 kg/h-kg dry papaya glacé].

Fig. 5. Evolution of air temperature at various parts in system (Test 3/2)
[By-pass air ratio = 63%, Temperature = 50°C,
Specific air flow rate = 29.83 kg/h-kg dry papaya glace].
Fig. 6. Temperature of refrigerant at condenser and evaporator (Test 3/2)
[By-pass air ratio = 63%, Temperature = 50°C,
Specific air flow rate = 29.83 kg/h-kg dry papaya glace].

Fig. 7. Effect of by-pass air ratio on specific energy consumption, papaya glace 56.5 kg
[Temperature = 50°C, size = 1.1x1.2x1.3 cm³, initial moisture content = 38.3% dry basis,
final moisture content = 23.6% dry basis].
Fig. 8. Effect of by-pass air ratio on specific energy consumption, papaya glacé 80.1 kg [Temperature = 50°C, size = 1.1x1.2x1.3 cm³, initial moisture content = 38.3 % dry basis, final moisture content = 23.6 % dry basis].

Fig. 9. Effect of by-pass air ratio on specific energy consumption, papaya glacé 101.1 kg [Temperature = 50°C, size = 1.1x1.2x1.3 cm³, initial moisture content = 38.3 % dry basis, final moisture content = 23.6 % dry basis].
Fig. 10. Effect of drying air temperature on specific energy consumption [Temperature = 50°C, size = 1.1 x 1.2 x 1.3 cm³, initial moisture content = 38.3% dry basis, final moisture content = 23.6 % dry basis, by-pass air ratio = 30%].

4. CONCLUSION

Mathematical model of papaya glaceé drying using heat pump provided good prediction for drying at low initial moisture content. The model also provided good prediction for air temperature at various points in the system. Drying time and energy consumption should be considered in drying. From mathematical model, it was found that adjustment tendency of suitable by-pass air ratio which yielded minimum specific energy consumption depended on quantity of papaya glaceé to be dried. If quantity of papaya glaceé was large, by-pass air ratio decreased. The suitable by-pass air ratio in this study was in range of 86%-90%.

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6. REFERENCES