FINITE-WAVE VECTOR EFFECT OF PLASMON DISPERSION ON CRITICAL TEMPERATURE IN La$_{1.85}$Sr$_{0.15}$CuO$_4$

Samnao Phatisena$^1$* and Nouphy Hompanya$^2$

Received: Jul 6, 2005 Revised: Sept 9, 2005 ; Accepted: Sept 21, 2005

Abstract

Using the plasmon exchange model in the framework of the Eliashberg theory for strong-coupling superconductors, the critical temperature of La$_{1.85}$Sr$_{0.15}$CuO$_4$ has been calculated. The finite-wave vector effect on acoustic plasmon dispersion has been included in the expression for the effective interaction between charge carriers. This effect is shown to enhance the critical temperature significantly as compared with the result without this term. The critical temperature is sensitively dependent on the spacer dielectric constant $\varepsilon_M$ which is not known precisely. The Coulomb repulsion strength $\mu$ has been tested around the commonly used value of 0.1. It is found that their proper values of them are $\varepsilon_M \approx 8.0$ and $\mu \approx 0.1$.

Keywords: Plasmon exchange model, finite-wave vector effect, acoustic plasmons, critical temperature

Introduction

After the discovery of high-temperature superconductivity in cuprates (Bednorz and Müller, 1986), various kinds of theoretical models for the mechanism of high-temperature superconductivity were proposed. Even today there is no consensus among theoretical physicists as to how to develop a more detailed theoretical description of the cuprates. It is known that all cuprate high-temperature superconductors (HTS) have a layered structure. The layers are composed of Cu-O planes (or sheets) separated from each other by planes of various other oxides and rare earths. The Cu-O layer is assumed to form a two-dimensional electron gas (2DEG) and the electrons in a given layer can interact with each other within the same layer as well as from layer to layer via an effective interaction. An isolated layer has only one plasmon mode with a dispersion relation $\omega_p \propto q^{1/2}$. Interlayer interaction leads to a noticeable modification of the pure two-dimensional (2D) dispersion relation, namely, to the formation of plasmon bands. Indeed, it is a well-known fact that the spectrum of a layered electron gas contains low-energy electronic collective modes, often called acoustic plasmons, with a dispersion relation $\omega_p \propto q$. That such modes could not be observed experimentally at finite q so far is related to the fact that the only technique known to date to determine the plasmon energy as a function of its wave-vector (i.e., electron energy loss spectroscopy), has a resolution of 0.2 - 0.5 eV at best (Nücker et al., 1989; Stöckli et al., 2000). It remains thus an experimental challenge to measure collective charge excitations down to very low energies at finite q. It is also worth noting that the largest contribution of acoustic plasmons to physical quantities such as

---

$^1$ School of Physics, Institute of Science, Suranaree University of Technology, Nakornratchasima, 30000 Thailand
$^2$ Department of Physics, Faculty of Science, National University of Lao, Vientiane, Laos
* Corresponding author

condensation energy is expected to come from finite but rather small values of q with respect to the Fermi wave-vector. To study the effect of acoustic plasmons on superconductivity requires thus to probe finite q's. Recently, the effect of temperature dependence and the inclusion of the leading higher order in wave-vector q on the plasmon dispersion relation in layered superconductors has been reported (Hompanya and Phatisena, 2005). At a low temperature limit the slope of the layered plasmon dispersion was shown to increase significantly due to the inclusion of the higher order in q while the thermal enhancement is nearly negligible.

The influence of acoustic modes on superconducting properties has been studied within the strong-coupling phonon-plasmon scheme (Bill et al., 2000). It was shown that the density of states is peaked at \( q_x = \pi \) and \( q_z = 0 \), where \( q_z \) is the wave-vector perpendicular to the planes. The optical branch \( (q_z = 0) \) has a smaller attractive interaction than the acoustic branch. Therefore, the largest collective-mode contribution to \( T_c \) is provided by the lowest acoustic branch. Screening of the Coulomb interaction in a layered conductor is incomplete due to the nature of layering (Visscher and Falicov, 1971; Fetter, 1974). The response to a charge fluctuation is time dependent and the frequency dependence of the screened Coulomb interaction becomes important. The additional impact of dynamic screening on pairing in layered superconductors has been evaluated (Bill et al., 2003). The plasmon contribution in conjunction with the phonon mechanism was used. The presence of only phonons is assumed to be sufficient to overcome the static Coulomb repulsive interaction and the dynamic screening acts as an additional factor. The full temperature, frequency and wave-vector dependence of the dielectric function was used to calculate \( T_c \) of three classes of layered superconductors. In metal-intercalated halide nitrides the contribution arising from acoustic plasmons is dominant while the contribution of phonons and acoustic plasmons is of the same order in layered organic superconductors and the contribution of acoustic plasmons is significant but not dominant in high-temperature oxides.

In this paper the Eliashberg theory for strong-coupling superconductors as modified by McMillan (McMillan, 1968) and Kresin (Kresin, 1987) will be used to calculate the superconducting transition temperature, \( T_c \). The plasmon exchange model will be reconsidered and the effective interaction between electrons is described within the random phase approximation (RPA). This model was previously used to calculate the \( T_c \) in HTS (Longe and Bose, 1992). Here, the finite-wave vector effect on the plasmon dispersion relation in a layered conductor will be included in our calculation. The appropriated values of the dielectric constant \( \varepsilon_M \) and the effective Coulomb repulsion \( \mu^* \) for the cuprate superconductor La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) will be determined. Numerical results and discussions will be presented.

**Acoustic Plasmon Exchange Model**

The simple layered electron gas system consists of two conducting sheets along the z-axis separated by a dielectric spacer with the dielectric constant \( \varepsilon_M \) and with the interlayer distance \( L \). The description of layered conductors can be made by neglecting the small interlayer hopping in a first approximation. The electrons in a Cu-O plane interact via the Coulomb interaction with charge carriers both within and between the planes. The effective interaction between the electrons are described within the RPA and can be written in the standard form (Bose and Longe, 1992)

\[
\nu(q,\omega) = V_0(q) \left[ 1 + \int_{-\pi}^{\pi} dq_z \frac{2\omega_0(q,q_z)M(q,q_z)}{\omega^2 - \omega_0^2(q,q_z)} \right]
\]

where \( V_0(q) = 2\pi e^2/\varepsilon_0 q \) is the bare 2D Coulomb interaction, \( \omega_0(q, q_z) \) is the plasmon frequency and \( |M(q, q_z)|^2 \) is the square of the electron-plasmon matrix element. The plasmon frequency in the present model is shown to be (Hompanya and Phatisena, 2005)

\[
\omega_0(q, q_z) = \sigma q \left( \frac{3 \hbar^2 q}{4 m^* c^2} \right) R(q, q_z)
\]

\[
2\omega_0(q, q_z) |M(q, q_z)|^2 = \sigma q \left( \frac{3 \hbar^2 q}{4 m^* c^2} \right) R(q, q_z)
\]
where
\[ \sigma = 2ne^2/\left(\varepsilon_Fm^*\right) \]
and
\[ R(q,q_z) = \frac{\sinh(q_zL)}{\cosh(q_zL) - \cos(q_zL)} \]

n is 2D electron density, \( m^* \) is the electron effective mass, and \( R(q,q_z) \) is the layer form factor which reflects the layered nature of the system. Note that for \( L \to \infty \), the effective interaction \( V(q,\omega) \) becomes
\[ \frac{1}{\pi} \int dq_z \frac{\sinh(q_zL)}{\cosh(q_zL) - \cos(q_zL)} \]
which is the effective electron-electron interaction in a single 2DEG.

Also to be noted is the appearance of the second term in the parenthesis of Eqn. (2). This term reflects the finite-wave vector effect on plasmon dispersion relation, which is always missing in the other's calculation. This paper shows the significance of this term to \( T_c \) of the layered superconductors.

Indeed, it has been shown (Allen and Dynes, 1975) that if the effective interaction between electrons in a superconductor can be written as given by Eqn. (1), then the coupling strength \( \lambda \) due to the attractive part of the effective interaction and the average value \( \langle \omega^2 \rangle \) of the plasmon frequency can be obtained from
\[ \lambda = N(0) \left\{ \frac{2\pi^2}{\varepsilon_M} \int dq_2 \frac{V_2(q,q_2)}{\varepsilon_2(q,q_2)} \right\} \]
\[ \langle \omega^2 \rangle = N(0) \left\{ \frac{2\pi^2}{\varepsilon_M} \int dq_2 \frac{V_2(q,q_2)}{\varepsilon_2(q,q_2)} \right\} \]

where \( N(0) \) is the density of states of the electrons at the Fermi surface and \( \langle ... \rangle_{FS} \) indicates that an average of the expression is taken over the Fermi surface. Eqns. (6) and (7) can be shown (Longe and Bose, 1992) to be
\[ \lambda = N(0) \left\{ \frac{2\pi^2}{\varepsilon_M} \int dq_2 \frac{V_2(q,q_2)}{\varepsilon_2(q,q_2)} \right\} \]
\[ \langle \omega^2 \rangle = N(0) \left\{ \frac{2\pi^2}{\varepsilon_M} \int dq_2 \frac{V_2(q,q_2)}{\varepsilon_2(q,q_2)} \right\} \]

It is interesting to note that \( \lambda \), as given by Eqn. (8), does not depend on the interlayer distance \( L \). This is due to the analytic properties of the RPA potential given by Eqn. (1). Another important point is that the integrals (8) and (9) diverge for small \( q_m \), but their ratio, i.e. \( \langle \omega^2 \rangle \), however does not. For small \( q_m \), \( \langle \omega^2 \rangle \) tends rapidly to the lower limit of \( \sigma / L \) which is the 3D electron density. On the other hand, for \( q_m \) large or \( q \approx 2k_F \), the average \( \langle \omega^2 \rangle \) tends rather slowly to the upper limit \( 2k_F \varepsilon \coth(2k_F L) \approx 2k_F \varepsilon \).

It is simpler to scale the parameter,
\[ y = q / k_F \]
Eqn. (8) and (9) then become
\[ \lambda = \frac{N(0)2\pi^2}{\varepsilon_M} \int dq_2 \frac{V_2(q,q_2)}{\varepsilon_2(q,q_2)} \]
\[ \langle \omega^2 \rangle = \frac{N(0)2\pi^2}{\varepsilon_M} \int dq_2 \frac{V_2(q,q_2)}{\varepsilon_2(q,q_2)} \]

where \( N(0) = \frac{m^*}{2\pi} \) and \( k_F^2 = 2\pi \sigma \). The average plasmon frequency, \( \langle \omega^2 \rangle \), is given by the square root of the ratio of (10) and (11).

It is seen from Eqns. (10) and (11) that the two parameters obviously depend on the dielectric constant \( \varepsilon_M \), the effective mass \( m^* \), the surface density \( \sigma \) of the electron gas (or equivalently the Fermi wave-vector \( k_F \) and hence the Fermi energy \( \varepsilon_F \)), and the coherence length \( \xi \).

**Critical Temperature of La1.85 Sr0.15 CuO4**

In this section we will focus on the La1.85 Sr0.15 CuO4 for which most parameters have been determined and it deserves special attention because of the simplicity of its structure. This system plays a role similar to the hydrogen atom in atomic physics. It is the best test system for understanding the basic principles of high-temperature superconductivity.

Following are the normal state parameters (Bill et al., 2003):
- the interlayer distance \( L = 6.5 \AA \)
- the Fermi wave-vector \( k_F = 3.5 \times 10^7 \mathrm{cm}^{-1} \)
the dielectric constant \( \varepsilon_M = 5 - 10 \)
the effective mass \( m^* = 1.7 m_e \)
the coherence length \( \xi = 35 \text{Å} \)
and the Coulomb pseudopotential is taken to be \( \mu^* = 0.1 \) (here, \( m_e \) being the mass of the bare electron).

Two equations, McMillan’s equation and Kresin’s equation, both of which were modified from the Eliashberg theory for strong-coupling superconductors will be used for the calculation of \( T_c \) of this material. The McMillan’s equation for the plasmon exchange model has the form

\[
T_c^{pl} = \frac{\langle \omega \rangle}{1.45} \exp \left[ -\frac{1.04(1 + \lambda)}{\lambda - \mu^* (1 + 0.62\lambda)} \right] \quad (12)
\]

where the Debye frequency \( \theta_D \) is replaced by the average frequency of plasmon, \( \langle \omega \rangle \), the exchange of which is responsible for superconductivity.

Kresin’s equation for the plasmon exchange model to calculate the value of \( T_c \) is given by

\[
T_c^{pl} = \frac{0.25 \langle \omega \rangle}{\left( \frac{\langle \omega \rangle}{\sqrt{2\lambda}} - 1 \right)^{3/2}} \quad (13)
\]

where the effective interaction strength \( \lambda_{eff} \) is given by

\[
\lambda_{eff} = \frac{\lambda - \mu^*}{1 + 2\mu^* + \lambda \mu^* t(\lambda)} \quad (14)
\]

and the analytical expression for the function \( t(\lambda) \) is given (Longe and Bose, 1992) by

\[
t(\lambda) = 0.75 + 0.8/(1 + \lambda) - 0.12(\lambda - 0.5) \quad (15)
\]

The results of \( T_c \) obtained by these two equations will be compared with the recent work by Bill et al. (2003). We will start with the calculation of \( \lambda \) and \( \langle \omega \rangle \) given by Eqns. (10) and (11) respectively. It can be seen from the given parameters that the value of dielectric constant \( \varepsilon_M \) is in the range 5 - 10, and \( \lambda \) and \( \langle \omega \rangle \) are obviously sensitive to this choice of \( \varepsilon_M \). We, therefore, calculate the value of \( \lambda \), \( \langle \omega \rangle \) and then \( T_c^{pl} \) by using different values of \( \varepsilon_M \). The result is shown in Table 1. The finite-wave vector (higher order in \( q \)) effect of the plasmon dispersion relation given by Eqn. (2), which is the term \((3/4) \hbar^2 q/m_e^2\), on \( T_c^{pl} \) is also shown in the Table. It is seen that the values of \( T_c^{pl} \) obtained by using Kresin’s equation are higher than those by McMillan’s equation and the finite-wave vector effect enhances the values of \( T_c^{pl} \) significantly.

As reported by Bill et al. (2003), the experimental value of \( T_c \) of \( \text{La}_1.85\text{Sr}_{0.15}\text{CuO}_4 \) is \( T_c^{exp} \approx 38\text{K} \). Their numerical result is \( T_c = 36.5\text{K} \) whereas in the absence of acoustic plasmons it is \( T_c^{ph} = 30\text{K} \). Therefore, the value of \( T_c \) due to acoustic plasmons is \( T_c^{pl} = 8\text{K} \). It is seen from Table 1 that the expected results correspond to the dielectric constant \( \varepsilon_M = 8 \) (8.38K and 8.97K by Kresin’s equation, 6.96K and 7.45K by McMillan’s equation). These results are quite different from the values that correspond to \( \varepsilon_M = 7 \) and \( \varepsilon_M = 9 \). It is, therefore, necessary to obtain \( T_c^{pl} \) that corresponds to the dielectric constant around \( \varepsilon_M = 8 \). The result is shown in Table 2. It is seen from Table 2 that the appropriate value of the dielectric constant is \( \varepsilon_M = 8.0 \) for \( \mu^* = 0.1 \).

<table>
<thead>
<tr>
<th>( \varepsilon_M )</th>
<th>By Kresin’s equation</th>
<th>By McMillan’s equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{c1} )</td>
<td>( T_{c2} )</td>
<td>( T_{c3} )</td>
</tr>
<tr>
<td>5</td>
<td>136.821</td>
<td>146.540</td>
</tr>
<tr>
<td>6</td>
<td>57.943</td>
<td>62.059</td>
</tr>
<tr>
<td>7</td>
<td>22.978</td>
<td>24.610</td>
</tr>
<tr>
<td>8</td>
<td>8.377</td>
<td>8.973</td>
</tr>
<tr>
<td>9</td>
<td>3.753</td>
<td>2.948</td>
</tr>
<tr>
<td>10</td>
<td>0.797</td>
<td>0.854</td>
</tr>
</tbody>
</table>

Table 1. The calculated values for \( T_c \) (in K) as obtained from Kresin’s equation \( (T_{c1} \text{ and } T_{c2}) \) and from McMillan’s equation \( (T_{c3} \text{ and } T_{c4}) \). \( T_{c1} \) and \( T_{c3} \) are the values without the finite-wave vector effect whereas \( T_{c2} \) and \( T_{c4} \) are the values including that effect.
Finally, to find the proper value of the effective repulsive strength \( \mu^* \) (rather than 0.1) that fit the expected result of \( T_{c1}^p \approx 8K \), we use it here as a second parameter varying from 0.07 to 0.13 in steps of 0.01. The critical temperature \( T_{c}^p \) as a function of the dielectric constant \( \varepsilon_M \) for 7 values of \( \mu^* \) is shown in Table 3 and Figure 1. It is seen from the Figure that the proper value of \( \varepsilon_M \) and \( \mu^* \) that fit the expected result of \( T_{c1}^p \approx 8K \) are \( \varepsilon_M \approx 8.0 \) and \( \mu^* = 0.1 \).

**Discussion and Conclusions**

Using the plasmon exchange model in the framework of the Eliashberg theory for strong coupling superconductors, the plasmon contribution to the critical temperature \( T_{c}^p \) could be obtained. In this model the plasmons are assumed to be attractive bosons in the pairing effect. The effective interactions between the electrons are described within the RPA. The electrons interact with each other within the same layer as well as from layer to layer via an effective interaction involving plasmon exchanges among all layers. Eliashberg’s equation for the calculation of \( T_c \) has been modified into McMillan’s equation and Kresin’s equation. These two equations contain two basic parameters to be evaluated, \( \lambda \) and \( \langle \omega \rangle \). The quantity \( \lambda \) represents the attractive strength between electrons, which in this model is essentially mediated by plasmons. The quantity \( \langle \omega \rangle \) is the average value of the frequency of the plasmons, the exchange of which is responsible for superconductivity. Both \( \lambda \) and \( \langle \omega \rangle \) obviously depend on the dielectric constant \( \varepsilon_M \), the effective mass \( m^* \), the Fermi wave-vector \( k_F \), the interlayer distance \( L \), and the coherence length \( \xi \) (to specify the lower limit of integration for \( \lambda \) and \( \langle \omega \rangle \)). The third parameter entered in the two equations for \( T_{c}^p \) is the Coulomb repulsion strength \( \mu^* \). This parameter is generally not well known, but one knows that it is limited by the condition \( 0 < \mu^* < 0.5 \). Many other investigators take its numerical value to be 0.1. In this work, \( \mu^* \) is kept as an undefined parameter around 0.1.

**Table 2.** The calculated values for \( T_{c1}^p \) (in K) as obtained from the same process as Table 1 with dielectric constant around \( \varepsilon_M = 8 \)

<table>
<thead>
<tr>
<th>( \varepsilon_M )</th>
<th>By Kresin’s equation</th>
<th>By McMillan’s equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T_{c1} )</td>
<td>( T_{c2} )</td>
</tr>
<tr>
<td>7.5</td>
<td>14.0379</td>
<td>15.0351</td>
</tr>
<tr>
<td>7.75</td>
<td>10.8781</td>
<td>11.6508</td>
</tr>
<tr>
<td>8</td>
<td>8.3779</td>
<td>8.9730</td>
</tr>
<tr>
<td>8.25</td>
<td>6.4107</td>
<td>6.8661</td>
</tr>
<tr>
<td>8.5</td>
<td>4.8721</td>
<td>5.2182</td>
</tr>
</tbody>
</table>

**Table 3.** The calculated values for \( T_{c1}^p \) (in K) as a function of \( \varepsilon_M \) by using Kresin’s equation for various values of effective repulsive strength around \( \mu^* = 0.1 \). Kresin’s equation without finite-wave vector effect has been used

<table>
<thead>
<tr>
<th>( \mu^* )</th>
<th>( \varepsilon_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.5</td>
</tr>
<tr>
<td>0.07</td>
<td>32.286</td>
</tr>
<tr>
<td>0.08</td>
<td>24.980</td>
</tr>
<tr>
<td>0.09</td>
<td>18.940</td>
</tr>
<tr>
<td>0.10</td>
<td>14.038</td>
</tr>
<tr>
<td>0.11</td>
<td>10.141</td>
</tr>
<tr>
<td>0.12</td>
<td>7.116</td>
</tr>
<tr>
<td>0.13</td>
<td>4.830</td>
</tr>
</tbody>
</table>
A specific cuprate superconductor, La$_{1.85}$Sr$_{0.15}$CuO$_4$, for which most parameters have been determined, is selected for the calculation of $T_c^{pl}$. Since the experimental value of $T_c^{exp}$ of this material is $T_c^{exp} \approx 38$ K and the phonon contribution to the $T_c$ is shown to be $T_c^{ph} = 30$ k, hence the plasmon contribution should be $T_c^{pl} \approx 8$ K. Indeed the critical temperature is sensitively dependent on parameters mentioned above. However, only $\varepsilon_M$ is not known precisely and the value of $\mu^*$ should be tested around the value of 0.1. Variation of $\varepsilon_M$ and $\mu^*$ shows that their proper values for $T_c^{pl} \approx 8$ K are $\varepsilon_M \approx 8$ and $\mu^* \approx 0.1$.

The plasmon exchange model is very simple since the microstructure of the superconductors is completely neglected. The model is characterized by four parameters only. For reasonable values of these parameters the calculated value of $T_c^{pl}$ is found to be in reasonable agreement with the experimental values of the materials. In the case of high-temperature oxides, the contribution of low-energy plasmons to the critical temperature is significant but not dominant. The phonon contribution is still largest in this model. In some classes of layered superconductors, the acoustic plasmon contribution is shown to be dominant or of the same order as the phonon contribution.

**References**


