The Development of Process Models for Consolidation of Titanium Alloy Coated Silicon Carbide Fibre Composites

J. Carmai* and F. P.E. Dunne**

Abstract
The paper addresses the development of consolidation models for titanium metal matrix composites. Two models have been developed. The first one employs micromechanical finite element methods to model explicitly matrix, fibre and void. In other words, finite element meshes are generated to represent explicitly the matrix and fibre in the coated fibre composite. The model has been used to develop understanding of stress-state sensitivity in consolidation. This type of model can provide useful insight, at material level, into composite consolidation. However, it is computationally prohibitive to simulate the whole array of coated fibres which are compacted to produce a component. It has been developed for validation and comparisons. The second model is considered to be of appropriate form for simulations of practical component manufacturing processes. It uses the Carmai and Dunne constitutive equations [1-2] for densification of matrix coated fibre composites under symmetric pressures. To enable simulations of practical manufacturing processes, the constitutive equations have been generalised to multiaxial stress states using the results of micromechanical finite element studies and been implemented into finite element software at the continuum level within a finite deformation framework. The validity of the generalisation has been assessed by means of comparison of predicted results with those obtained from micromechanical finite element calculations. The ability of the model to predict the observed behaviour has been tested by comparing the simulation results with experimental results. The model predictions compare well with the measured results.

Keywords: Fibre reinforced composite material, Consolidation, Material modelling

1. Introduction
The use of titanium metal matrix composite (Ti-MMC) in aerospace applications has substantially increased in the last few decades. It can be fabricated using the matrix-coated fibre method in which bundles of pre-coated fibres are aligned into a die or canister which is subjected to high temperature and pressure. Consolidation process conditions have significant effects on the mechanical properties of the final composite component. It is therefore necessary to choose carefully the processing variables to obtain optimal conditions for consolidation. Computer simulation, alternative to costly and time consuming trial and error experiments, can allow the prediction of optimum temperature, pressure and time required to obtain fully dense composite material and to enable manufacturing process design. The paper addresses the development of the models for consolidation of matrix coated fibre composite. Two

* Department of Production Engineering, Faculty of Engineering, King Mongkut’s Institute of Technology North Bangkok.
approaches are presented. The first approach makes use of micromechanical finite element modelling in which fibre, matrix and void are modeled explicitly. The model is able to predict the time required to produce a fully dense composite under given temperature and pressure for a simple fibre-matrix system. It also can provide an understanding of the effects of stress, strain and strain rate on the deformation and porosity evolution during consolidation. However, this type of model is not practical to be used to simulate the composite component forming processes. The second approach is more fundamental. It employs physically-based constitutive equations for consolidation of matrix-coated fibre composites developed by the present authors [1-2]. They have been derived only for the case of isostatic loadings. Several steps are still required before they can be used in the simulation and hence design of manufacturing processes. This, therefore, is the subject of this paper. The paper firstly describes the development of micromechanical finite element models together with their use in investigating the stress-state sensitivity of consolidation. This study provides useful information for the multiaxial generalisation of constitutive equations for consolidation. Next, generalisation of constitutive equations for consolidation to make them suitable for general, multiaxial stress states is described. This is essential for process simulation, since practical processes always lead to multiaxial stress states. Implementation of the multiaxial constitutive equations is necessary to enable practical process simulation. The implementation of the constitutive equations into a finite deformation, finite element model is also described. Finite deformation is necessary, since the strains and rigid body rotation in practical processes can be large. The multiaxial, constitutive equations implemented into finite deformation, finite element software are then verified by comparisons of predictions with independent, micromechanical calculations and experimental data.

2. Explicit Micromechanical Finite Element Model

An explicit micromechanical finite element models have been developed for square array packing using ABAQUS finite element software and are shown schematically in figure 1. The coated fibres are assumed to be perfect cylinders of infinite length so plane strain conditions hold on planes perpendicular to the fibres axis. The matrix is assumed to obey power-law creep behaviour while the fibre is assumed to be rigid. A multiaxial power-law creep equation has been implemented into ABAQUS using a UMAT subroutine. Since the fibre is assumed to be rigid, the finite element mesh is developed only for the matrix. The matrix of the micromechanical finite element model consists of two dimensional plane strain, 4 noded, fully integrated elements (CPE4). All nodes lying on the fibre-matrix boundary are fixed in both the x and y directions and those on the lower unit-cell boundary are allowed to move in the x direction only while all nodes lying on the left unit-cell boundary are allowed to move in the y direction only. Movable boundaries are used for the application of the distributed load, simulating the pressure state during the consolidation process. The movement of the right movable boundary is restricted to be along the x-axis while the upper movable boundary can only move in the direction of the y-axis. Sticking friction is assumed between matrix-matrix interfaces.

Figure 1 Schematic diagram showing the explicit micromechanical finite element model.
3. Influence of Asymmetric Pressure Application on the Densification

Analyses have been carried out for the micro-mechanical finite element model to study the effects of asymmetric pressure application on densification. Two different pressure magnitudes have been applied to the movable boundaries, leading to an asymmetric pressure distribution within the unit cell. Figure 2 shows comparisons of relative density evolution over time predicted by the micromechanical finite element model for six different pressure couples which are summarized in Table 1. The 50/50 MPa pressure arrangement has been included as a reference as it is just the symmetric pressure application. The hydrostatic stresses for pressure couples 20/80, 30/70, 40/60 and 50/50 MPa are the same but different equivalent stresses occur in each case. The equivalent stresses for pressure couples 50/50, 30/57.73 and 20/56.90 MPa are the same but the hydrostatic stresses are different. All simulations have been carried out for consolidation temperature of 900°C and with 25% volume fraction of fibres.

Table 1 Pressure arrangements used in the consolidation analyses.

<table>
<thead>
<tr>
<th>T1 (MPa)</th>
<th>T2 (MPa)</th>
<th>( \sigma_m ) (MPa)</th>
<th>( \sigma_e ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>80</td>
<td>33.33</td>
<td>72.11</td>
</tr>
<tr>
<td>30</td>
<td>70</td>
<td>33.33</td>
<td>60.82</td>
</tr>
<tr>
<td>40</td>
<td>60</td>
<td>33.33</td>
<td>52.92</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>33.33</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>57.73</td>
<td>29.23</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>56.90</td>
<td>25.63</td>
<td>50</td>
</tr>
</tbody>
</table>

The analyses show that the equivalent stress has little effect on the densification rate, for constant hydrostatic stress. Furthermore, the hydrostatic effects are shown to be significant for the rate of densification of matrix-coated fibre composites, and it is concluded that it is they that determine the densification rate, rather than the equivalent stress. This study provides useful information for the multiaxial generalisation of constitutive equations for consolidation, which is described in subsequent sections.

4. Multiaxial Constitutive Equations for Composite Consolidation

Physically-based constitutive equations for consolidation of matrix coated fibres systems have been developed by the present authors. The model has been developed for square array packing of coated fibres subjected to symmetric in-plane compressive loads as shown in Figure 3. They made used of a variational method in which velocity fields for the fibre matrix coating are assumed, and Hill’s minimum principle used to derive constitutive equations for deformation which minimise the power functional [3]. The constitutive equations so derived have been validated for the case of isostatic loading by comparison of the predicted results with those produced from micromechanical finite element models. The resulting equations give good representations of the densification behaviour of the Ti-6Al-4V coated fibres under symmetric pressure. Further details can be found in [1-2].
The rate of plastic deformation needs to be defined to describe the deformation behaviour of the matrix coated fibre composites. The rate of plastic deformation of a compressible material can be decomposed into two parts as follows:

\[
\dot{D}_P = \dot{D}_{cr} + \dot{\varepsilon}_{sw} I
\]  

(8)

where \(D\) is the relative density and \(\dot{\varepsilon}_{kk}\) is the dilatation rate.

The rate of plastic deformation of a compressible material can be decomposed into two parts as follows:

\[
\dot{\varepsilon}_{kk} = \dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33}
\]  

(7)

Equation (1) for monolithic porous materials can be decomposed into two parts which are re-written in tensor form as

\[
\dot{D} = - D \dot{\varepsilon}_{kk} + \frac{1}{3} \dot{\varepsilon}_{sw} I
\]  

(9)

where \(D\) is the relative density and \(\dot{\varepsilon}_{sw}\) is the swelling term for a monolithic, processes always lead to multiaxial stress states. It is therefore necessary to generalise the resulting constitutive equations to make them suitable for general, asymmetric loadings.

The generalisation can be made by adopting the generalised models for consolidation of metal powders [4-7]. The model of Duva and Crow [7], for example, takes the form

\[
\dot{\varepsilon} = \frac{\partial \phi}{\partial \sigma} = AS^{n-1} \left( \frac{3}{2} a \sigma_i \sigma_j + \frac{1}{3} b \delta_{ij} \sigma_m \right)
\]  

(1)

where

\[
S^2 = a \sigma_v^2 + b \sigma_m^2
\]  

(2)

\[
\sigma_v^2 = \frac{3}{2} \sigma_{ij} \sigma_{ij}
\]  

(3)

\[
\sigma_{ij}' = \sigma_{ij} - \delta_{ij} \sigma_m
\]  

(4)

\[
\sigma_m = -\frac{1}{3} \sigma_{kk}
\]  

(5)

\(S\) is an effective effective stress [7-8] for the porous creeping material. The coefficients \(a\) and \(b\) are functions of current relative density, \(D\), which is equivalent to the solid volume fraction of the porous material, and the creep exponent, \(n\). The densification rate can be obtained from

\[
\dot{D} = - D \dot{\varepsilon}_{kk}
\]  

(6)
homogeneous material containing voids. But in the present work, the swelling strain is just the dilatation rate for the matrix-fibre-void system obtained from the Carmai and Dunne constitutive equations [2]. It is given by

\[ \dot{\varepsilon}_{kl} = \frac{-2X}{R \cos \gamma} \]  

(10)

in which \( R \), \( \gamma \), and \( X \) are geometrical quantities relating to the unit cell shown in figure 3(b). In particular, \( X \) is the macroscopic deformation of the cell.

\[ \lambda = (TX(R - X))^n \times A \frac{\partial}{\partial f} \left[ \int P(\varepsilon_{,c}) \right] r dr d\theta 
+ \beta_r \left[ \int P(\varepsilon_{,c}) \right] r dr d\theta 
+ \beta_f \left[ \int P(\varepsilon_{,c}) \right] r dr d\theta \]  

(11)

where

\[ P(\varepsilon_{,c}) = \frac{n+1}{n} W(\varepsilon_{,c}) \frac{1}{\lambda^{n+1}} \lambda^n \]  

(12)

Hence, for the composite material considered here, the total plastic deformation rate is given by

\[ \dot{D}^p = \frac{3}{2} AS_{n-1} \sigma + \frac{1}{3} \varepsilon_{,kl} \]  

(13)

where \( \varepsilon_{,kl} \) is given in equation (10) and the coefficient \( a \) is the same as that in [4-7], i.e.

\[ a = \frac{1 + \frac{2}{3} (1 - D)}{D^{2n+1}} \]  

(14)

In order to determine \( \lambda \) (and hence \( \varepsilon_{,kl} \)), the pressure \( T \) imposed on the repeating cell, as shown in figure 3(a), is required. In general, the repeating cells will not be subjected to purely hydrostatic stress states. However, it was shown in an earlier section that the densification rate is dominated by the hydrostatic stress, and independent of the equivalent stress. Hence, the pressure \( T \) is calculated from

\[ T = \sigma_m = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \]  

(15)

where \( \sigma_1 \), \( \sigma_2 \) and \( \sigma_3 \) are principal stresses. The implementation of the multiaxial model into finite element software is described in the next section.

5. Finite Element Implementation of the Multiaxial Constitutive Equations

The multiaxial constitutive equations have been implemented into the finite element software ABAQUS within a large deformation formulation using a UMAT subroutine. ABAQUS supplies to the UMAT subroutine the deformation gradient at the beginning and the end of each time step, \( F_t \), and \( F_{t+\delta t} \). The user is required to supply the Cauchy stress at the end of the time step. The algorithms, firstly, need to define the rate of deformation gradient, \( \dot{F} \) which can be calculated, for small time steps, as

\[ \dot{F} = \frac{1}{\delta t} (F_{\text{v+\delta t}} - F_t) \]  

(16)

Then, the velocity gradient, \( L \), is

\[ L = \dot{F} F^{-1} \]  

(17)

The total rate of deformation, \( D \), and the spin tensor, \( W \), are given by

\[ \dot{D} = \frac{1}{2} (L + L^T) \]  

(18)

\[ \dot{W} = \frac{1}{2} (L - L^T) \]  

(19)

The dilatation rate, \( \dot{\varepsilon}_{,kl} \), and the densification rate, \( \dot{D} \), can be calculated from equations (10) and (6) respectively. The relative density at the end of each time step is determined using the first order Euler integration scheme

\[ D^{\text{v+\delta t}} = D^v + \dot{D} \delta t \]  

(20)
The co-rotational stress rate \( \dot{\sigma} \) is given by

\[
\dot{\sigma} = \frac{E}{(1 + \nu)} \dot{D}^e + \frac{E\nu}{(1 + \nu)(1 - 2\nu)} \text{tr}(\dot{D}^e) I \tag{21}
\]

where \( E \) is Young’s modulus, \( \nu \) is Poisson’s ratio and \( \dot{D}^e \) is the rate of elastic deformation

\[
\dot{D}^e = D - \dot{D}^p \tag{22}
\]

\( \dot{D}^p \) is the rate of plastic deformation as given in equation (9). The stress rate, \( \dot{\sigma} \), is calculated as

\[
\dot{\sigma} = \dot{\nu} + \mathbf{W} \cdot \dot{\sigma} - \dot{\sigma} \cdot \mathbf{W} \tag{23}
\]

Finally, the stress, \( \sigma \), for each time increment can be determined by utilising the first order Euler integration scheme

\[
\sigma^{t+\delta t} = \sigma^t + \dot{\sigma} \delta t \tag{24}
\]

6. Model Validation

The generalised multiaxial constitutive equations for consolidation, implemented into finite element software, presented above, are used to predict the behaviour of some simple densification test cases with a range of applied stress states. The validity of the multiaxial generalisation of the constitutive equations is assessed by comparing the results obtained with those obtained from explicit micromechanical finite element calculations and experimental results.

6.1 Comparisons with Explicit Micromechanical Finite Element Calculations

The first analysis of densification behaviour of the Ti-6Al-4V coated fibres has been carried out for a symmetric pressure application in order to validate the implementation of the generalised multitaxial constitutive equations into ABAQUS. The predictions obtained from the multiaxial constitutive model have been compared with those obtained from an independent numerical implementation of the constitutive equations for symmetric loads using Fortran, and the explicit micromechanical finite element model. The consolidation behaviour of matrixcoated fibre composites has been simulated by imposing constant, symmetric pressures of 50 MPa onto a single plane strain element in both the \( x \) and the \( y \) directions. The consolidation temperature is 900°C with a volume fraction of fibres of 25%. The coated fibres are assumed to be perfectly circular and uniformly distributed within the element.

Figure 4 shows comparisons of relative density evolution with time curves obtained from the multiaxial constitutive model, the explicit micromechanical finite element model, and the numerical implementation of the constitutive equations for symmetric loading. The prediction obtained from the multiaxial constitutive model shows good agreement with that obtained from the independent numerical implementation. Differences can be seen at relatively high density when compared with the prediction obtained from the explicit micromechanical finite element model. The total consolidation time predicted by the multiaxial constitutive model is less than that predicted by

![Figure 4](image-url)
6.2 Comparisons of Predicted and Experimental Results

An experiment was carried out to investigate the consolidation behaviour of Ti-6Al-4V matrix coated fibre under uniaxial constrained compression loading and to compare with the model prediction. A specimen was consolidated at a constant temperature of 900°C and under a constant pressure of 20 MPa. The volume fraction of fibres is 33%. The current height of the specimen was obtained directly from the LVDT. The specimen was modelled using a single plane strain element. The movements of left and right boundaries of the model were constrained in the horizontal direction. The bottom boundary of the model was constrained in the vertical direction. The top boundary of the model is subjected to a vertical compressive load. The material behaviour was described by the multiaxial constitutive equations presented above. The creep parameter $A$ and exponent $n$ for the matrix material is required. They were obtained from constant load creep tests of PVD Ti-6Al-4V conducted by Warren and co-workers [10]. The volume fraction of fibres was set to be 33%. The initial relative density was set to

![Figure 5](image1.png)

**Figure 5** Comparison of relative density evolution with time obtained from the multiaxial constitutive model and the explicit finite element model at constant temperature of 900°C, 25% volume fraction of fibres for pressure couple 30/70 MPa.

The second analysis has been carried out for asymmetric pressure application for pressure couple 30/70 MPa with fibre volume fraction of 25% and at a constant temperature of 900°C. The coated fibres are assumed to be perfectly circular and uniformly distributed within the element. Figure 5 shows comparisons of relative density evolution with time obtained from the finite element multiaxial constitutive model and the explicit micromechanical finite element model for pressure couple 30/70 MPa. The results show good agreement and therefore lend confidence in the generalisation of the model for complex stress states.

![Figure 6](image2.png)

**Figure 6** Graph showing comparisons between predicted and experimental relative density evolution with time curves of Ti-6Al-4V/ SiC composite with 33% volume fraction of fibres consolidated at a constant temperature of 900°C under a constant pressure of 20 MPa.
0.7854, assuming square array packing. Since no consolidation takes place before the load is applied, all simulations start when the desired consolidation temperature has been reached and the loads have been applied to the specimens. Figure 6 shows a comparison of predicted and measured results. Good agreement is achieved.

7. Conclusion
A micromechanical finite element model has been developed to investigate the influence of stress states in consolidation as well as for validation and comparisons. It has been shown that the rate of densification is largely controlled by hydrostatic stress, and is largely independent of equivalent stress. The results of the micromechanical studies have been used to generalised the physically-based constitutive equations for composite consolidation derived for the case of symmetric pressure loading to multiaxial stress state. The total deformation of the consolidating composite is expressed as the sum of a conventional deviatoric creep term, together with a dilatational term, which was derived using a variational method in a previous paper [1-2]. The equations contain only two material parameters which are just the conventional creep coefficient and exponent for the fibre coating material (in this case, Ti-6Al-4V). The embodiment of the constitutive equations within a large deformation framework, and their implementation within finite element software, have been presented. The validity of the generalisation has been assessed by means of comparison of predicted results with those obtained from explicit, micromechanical finite element calculations and experimentally measured data. Good comparisons have been achieved.

References