H-infinity Gain Scheduling Control for Aircraft Trajectory Tracking using Linear Parameter Varying Model

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Abstract

The paper explores an alternative control technique for an aircraft such that the stability and performance are guaranteed. The objective is to develop a controller that tracks reference trajectory of an aircraft. The nonlinear longitudinal dynamics is represented in Linear Parameter Varying (LPV) form utilizing three parameters to bound the nonlinearity. The system is assumed to be affinely dependent on the parameters. The gain scheduled H-infinity control is designed such that the stability can be assured within a specific flight envelope. The controller is then applied on a longitudinal F-16 model to track a given reference trajectory.

1. Introduction

Aircraft modeling and control have always been one of the most challenging problems. Because the dynamics of aircraft are highly nonlinear, the performance depends significantly on flight condition. The flight characteristics are numerically mapped utilizing stability derivatives, which usually are available through flight test.

There are numerous attempts on developing a controller to deal with the nonlinearity of aircraft dynamics. The gain-scheduling PID control was presented in Ref. [1]. The modern control approach, such as, Linear Quadratic Control was developed in [2]. Most of controllers need gain tuning in order to reach the desired performance for a particular aircraft. More importantly, the stability and robustness cannot be guaranteed because of the nonlinearity and uncertainty of the system.

In this work, an alternative control technique that

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is more robust to the change in the model is investigated. The objective is not only to stabilize the system, but also to track a reference trajectory. The aircraft is modeled as a Linear Parameter Varying (LPV) system.

Aircraft dynamics can be parameterized using parameters called stability derivatives that depend on flight condition. Therefore, the model can be represented as a parameter-varying system utilizing stability derivatives. The accuracy of the system depends on how well the parameters (i.e., stability derivatives) are modeled. Stability derivatives are generally very difficult to determine. A rough approximation can be done from geometric data of an aircraft and statistical data (see [3], [4]). However, the best approach is to utilize actual flight data.

Many derivatives are required to represent complete model of aircraft dynamics. This poses more challenge in control problem in a computational aspect. In order to alleviate the computational load, it is proposed here to bound the stability derivatives using affine functions. The parameter bounding technique is also employed to deal with uncertainty of the model and parameters. It is noted that a more complicated function can be utilized to make the system less conservative. However, because the objective of this work is to study the possibility of LPV structure to model aircraft dynamics, a simple function is considered.

There has been a lot of work done in a control design for LPV systems: Ref.[5] develops a guaranteed gain schedule control, Ref.[6] shows an example of a missile control. Although LPV systems require intense computational load, most of computation can be done off-line. In order to assure system robustness, gain scheduling H-infinity controller is considered here.

The paper is organized as follow. Both nonlinear and linearized aircraft dynamics are shown. Then the LPV model for an aircraft is constructed. Each stability derivatives is examined for an entire flight envelope. Finally, the H-infinity gain scheduling controller is designed and applied to an example of trajectory tracking.

2. Aircraft Dynamics

In this section, the nonlinear dynamics of an aircraft is presented. Then, the linearized model is shown. This information is later utilized to form a Linear Parameter Varying Model in Section 3.

Aircraft-particularly high-performance fighter aircraft - are highly nonlinear dynamical systems. The aircraft considered here is an F16-like fighter model, simplified to only the longitudinal ($x, h$) plane. The aircraft motion can be described using six variables as illustrated in Figure 1:

$$x = [x_p \ h \ \theta \ V \ \alpha \ q]$$

where $x_p$ is the position in the horizontal direction, $h$ is Altitude, $\theta$ is pitch angle, $V$ is absolute velocity, $\alpha$ is angle of attack, and $q$ is pitch rate. The coordinate $(x,z)$ is defined on the body frame.

![Figure 1 State of Longitudinal Aircraft Dynamics.](image)
The nonlinear dynamics is given by,

\[
\begin{aligned}
\dot{x}_p &= V \cos(\theta - \alpha) \\
\dot{h} &= V \sin(\theta - \alpha) \\
\dot{\theta} &= q \\
V &= \frac{1}{m} mg \sin(\theta-\alpha) + \cos(\alpha) (T + X) + \sin(\alpha) Z \\
\dot{\alpha} &= q - \frac{V}{m} \cos(\theta-\alpha) \frac{\sin \alpha}{mV} \cdot (T + X) + \frac{\cos \alpha}{mV} \cdot Z \\
\dot{q} &= \frac{1}{J} \left( Q S \cdot (C_m + \frac{2 \pi}{V} C_{mq}) + Z \Delta x \right)
\end{aligned}
\]

where the aircraft parameters are mass \( m \), dynamic pressure \( Q \), wing area \( S \), chord length \( c \), moment of inertia \( J \), aerodynamic center \( x_{ac} \), center of gravity \( x_{cg} \), and thrust \( T \). \( X \) and \( Z \) are forces in \( x \) and \( z \) directions, respectively. \( x \) and \( z \) are defined in body coordinate. Both \( X \) and \( Z \) are functions of angle of attack, elevator deflection, dynamic pressure and wingspan. \( C_m \) and \( C_{mq} \) are stability derivatives which are functions of angle of attack and elevator deflection. The inputs for the system are

\[
u = [\delta_e, T]
\]

where \( \delta_e \) is the elevator deflection and \( T \) is thrust.

Because the objective is to represent the system using Linear Parameter Varying (LPV) formulation, the linearized form of the nonlinear system has to be computed for the entire flight envelope. The linearized model around an operating point e.g. trim condition, can be written as

\[
\dot{x}(t) = A_x x(t) + B_x u(t)
\]

and

\[
A_x = \begin{bmatrix}
0 & 0 & A_{13} & A_{14} & A_{15} & 0 \\
0 & 0 & A_{23} & A_{24} & A_{25} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & A_{33} & A_{34} & A_{35} & A_{36} \\
0 & 0 & 0 & 0 & A_{43} & A_{45} & A_{46} \\
0 & 0 & 0 & 0 & A_{53} & A_{54} & A_{55} \\
0 & 0 & 0 & 0 & A_{63} & A_{64} & A_{65} & A_{66}
\end{bmatrix}
\]

\[
B_u = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
B_{31} & B_{32} & B_{33} \\
B_{41} & B_{42} & B_{43} \\
B_{51} & B_{52} & B_{53} & B_{54} \\
B_{61} & B_{62} & B_{63} & B_{64} & B_{65}
\end{bmatrix}
\]

and

\[
A_{13} = -V \sin(\theta - \alpha) \\
A_{15} = V \sin(\theta - \alpha) \\
A_{24} = \sin(\theta - \alpha) \\
A_{25} = -V \cos(\theta - \alpha) \\
A_{33} = X_u \frac{z}{V} \\
A_{34} = X_u \frac{z}{V} \\
A_{35} = \frac{z}{V} \\
A_{36} = \frac{z}{V} \\
A_{44} = \frac{z}{V} \\
A_{45} = X_u \frac{z}{V} \\
A_{46} = X_u \frac{z}{V} \\
A_{53} = \frac{z}{V} \\
A_{54} = \frac{z}{V} \\
A_{55} = \frac{z}{V} \\
A_{56} = \frac{z}{V} \\
A_{63} = \frac{z}{V} \\
A_{64} = \frac{z}{V} \\
A_{65} = \frac{z}{V} \\
A_{66} = \frac{z}{V}
\]

Parameter \( X_u \) indicates the change of force \( X \) with respect to velocity in \( x \) direction, \( u \). Similarly, \( X_u \) indicates the change of force \( X \) with respect to angle of attack, \( \alpha \) while \( Z_u \) is the change of force \( Z \) with respect to elevator deflection. Parameter \( M \) is the pitching moment, hence, \( M_u \) is derivative of pitching moment with respect to rate of change of angle of attack, \( \dot{\alpha} \). Other parameters are denoted in the same manner. These parameters are known as stability derivatives or aerodynamics derivatives.

The full-state sensor output is assumed here. The assumption can be relaxed by integrating an estimator into the control. Therefore, the matrices \( C_{yx} \) and \( D_{yx} \) are identity matrix and zero matrix respectively. Details
on the derivation is presented in Ref. [7].

It can be seen that the parameters in matrix $A_i$ depend on trim condition, and many stability derivatives (e.g., $Z_o, M_a, Z_a$). These stability derivatives also depend on the trim condition of an aircraft. The linearized model is only valid on a particular trim condition.

3. Linear Parameter Varying Models

Linear Parameter Varying models or LPV can be described as a linear system with parameters that depend on the state. For a linearized system $S$ dependent on parameter $p$, which is a $n \times 1$ vector,

$$S = S(p) = S(p_1, p_2, ..., p_n)$$

The LPV system is expressed as a linear sum of the effects of each parameter,

$$S = S_0 + S_1 p_1 + S_2 p_2 + ... + S_n p_n$$

Figure 2: Relation between angle of attack ($\alpha$), altitude ($h$) and parameters $A_{z_2}$, $A_{z_5}$, $A_{45}$, $A_{65}$, $A_{46}$, $A_{56}$, and $B_{n i}$

Figure 3: $A_{z_2}$ as a function of altitude and angle of attack along with the approximating plane.

The linear effects of each parameter on the system matrices $(A, B, C, D)$ can similarly be written as,

$$A = A_0 + A_1 p_1 + \cdots + A_n p_n$$
$$B = B_0 + B_1 p_1 + \cdots + B_n p_n$$
$$C = C_0 + C_1 p_1 + \cdots + C_n p_n$$
$$D = D_0 + D_1 p_1 + \cdots + D_n p_n$$

As mentioned in the previous section, the linearized aircraft model depend greatly on stability derivatives that vary with flight condition. This fits the parameter varying system framework. However, because the number of stability derivatives in system matrices is quite large, to consider all derivatives as parameters would pose a problem on the computational complexity. The number of Linear Matrix Inequalities (LMIs) that is posed in the $H\infty$ design is $2^n$, where $n$ is the number of dependent parameters. Therefore, there are $2^9 = 512$ LMIs to be solved.

The trim condition of an aircraft can be parameterized by two variables angle of attack, $\alpha$ and
altitude \( h \). Because stability derivatives depend on a trim condition, all stability derivatives can be expressed in functions of the two variables, \( \alpha \) and \( h \).

A number of linearized models are generated for a given flight envelope (different values of angle of attack and altitude). The angle of attack in interest ranges from 0° to 10° with increment of one while the altitude varies from 10,000 feet to 20,000 feet with increment of 1,000 feet. Therefore, 121 linearized models are utilized in this work. Parameters of matrix \( A_u \) and \( B_u \) (Equation 3) are recorded for each trim condition. The relationship between each stability derivative and the state variables \( h \) and \( \alpha \) is determined. It is found that following parameters \( A_{ij}, A_{ij}, A_{ij}, \) and \( A_{ij} \) are constant for the flight envelope that is considered here. Examples of the parameters are shown in Figure 2.

As seen in Figure 2, relations between \((\alpha, h)\) and most parameters except \( A_{ij} \) are very close to linear. It is proposed here to approximate these relations using linear planes. Figure 3 shows the parameter \( A_{ij} \) along with a linear plane that is utilized in approximation. Even though the approximation removes some guarantees given by LPV approach, the effects can be minimized given a sufficiently conservative controller. This will be discussed in the next section.

Therefore, parameters \( A_y \) and \( B_y \) can be expressed by linear functions as follows,

\[
A_{ij} = ra_{ij} \alpha + sa_{ij} h + A0_{ij} \tag{4}
\]
\[
B_{ij} = rb_{ij} \alpha + sb_{ij} h + B0_{ij} \tag{5}
\]

where \( ra_{ij} \) and \( sa_{ij} \) are slope of \( A_y \) with respect to \( \alpha \) and \( h \) respectively, while \( rb_{ij} \) and \( sb_{ij} \) are slope of \( B_y \) with respect to \( \alpha \) and \( h \) respectively. \( A0_{ij} \) and \( B0_{ij} \) are are constant for each \((i, j)\).

Equations 4 and 5 can replace every parameter in Equation 3 except \( A_{ij} \) (which equals to \( X_a/V \)) that is highly nonlinear with respect to \( \alpha \) and \( h \) as illustrated in Figure 2. From Equation 3, \( A_{ij} \) equals to \( X_a/V \) that is the ratio between derivative of force in \( x \) direction with respect to angle of attack and velocity.

All stability derivatives considered here but \( X_a \) by replacing them with linear planes in function of \( \alpha \) and \( h \) as discussed. Because the relation between \( A_{ij} \) is so complex that linearizing it would result in large error. Therefore, the LPV model must consider \( A_{ij} \) as another parameter in the model. The nonlinearity of \( A_{ij} \) dealt with in Section 4 using bounding technique.

Therefore, the aircraft longitudinal model as described earlier is parameterized by three parameters: 1) angle of attack \( \alpha \) (rad), 2) altitude \( h \) (10^4 ft), and 3) \( A_{ij} \) which is the ratio between the derivative of in-plane force with respect to angle of attack \( X_a \) and velocity \( V \).

\[
P = \{ \alpha, h/10^4, A_{45} \}
\]

The system is represented in the LPV form, and the solution requires only 23 Linear Matrix Inequalities (LMIs). Likewise, the operating limits of three parameters are utilized to generate a three dimensional “box.” The different flight conditions that are previously referred as flight envelope are now depicted as a box. The objective is to control an aircraft that is in any point in the box.

Given the position reference points \((x_{ref}, h_{ref})\), the performance index is defined in order to minimize the error between aircraft position and reference trajectory using limited control command \((\delta_e, T)\) as

\[
z(t) = \begin{bmatrix}
x - x_{ref} \\
h - h_{ref} \\
\delta_e \\
T
\end{bmatrix}
\]
The system can then be written as follow,

\[
\dot{x}(t) = A_w x(t) + \left[ \begin{array}{c} B_w \\ B_u \end{array} \right] \left\{ \begin{array}{c} w \\ u(t) \end{array} \right\} \tag{7}
\]

where \( \omega \) is the augmented disturbance term, which consists of actual process noise and the reference trajectory \( (x_{ref}, h_{ref}) \):

\[
u(t) = \left\{ \begin{array}{c} w_{\text{noise}} \\ x_{\text{ref}} \\ h_{\text{ref}} \end{array} \right\}^T
\]

Moreover, the delay due to actuator dynamics is integrated into the system. In the frequency domain, the delay can be expressed as,

\[
\tau \ddot{u}(t) + \dot{u}(t) = u(t) \tag{10}
\]

The system in Equation (12) is a proper formulation for the H-infinity control framework.

4. H-infinity Gain Scheduling Controller

In the last section, the system can be constructed using \( S_p, S_r, S_p, S_r \) by identifying parameter values at
each time step. The system matrices are then defined as above, with linear dependence on the parameters $h$, $a$, and $A_{45}$.

Synthesizing a controller requires finding a gain matrix $K$ which satisfies the performance objective, namely to minimize the effects of disturbances $d$ on the augmented state $z$:

$$
\begin{align*}
\min_{\|d\| \leq 1} \frac{\|z\|}{\|d\|} &= \min_{\|d\| \leq 1} \max_{p} \frac{\|z\|}{\|d\|} \\
\text{(13)}
\end{align*}
$$

Again, the system is as follow,

$$
\begin{align*}
\dot{x}(t) &= A_h x(t) + B_h w \\
\dot{y}(t) &= C_h y(t) \\
\text{(14)}
\end{align*}
$$

The objective is to design a controller that can guarantee stability for all allowable state $x(t)$ and input $u(t)$. The inputs are limited to a maximum of 33,000 lbs thrust and elevator deflection in of 25° in either direction.

In order to deal with ranges of linearized model, a dynamic controller has to be utilized. At each time step a new controller is computed based on the current status of the system. The flight condition here can be expressed through the parameter, $p$, as discussed in the last section.

Constructing the limits of the flight regime into an operating box, a gain matrix can be found for each of the limiting values as illustrate in Figure 4. For example, $K_j$ represent the control gain in the case where $A_{45}$ is maximum while $h$ and $a$ are minimum. The box is bounded within the following limits,

$$
\begin{align*}
H_e &\quad \text{the control gain for any point in the flight envelop lies inside a box and can be expressed by a linear combination of gain for each corner of the box. Given the weighting for the } i^{th} \text{ corner, } W_i, \text{the final gain matrix becomes,} \\
K &= \frac{W_1 K_1 + W_2 K_2 + \ldots + W_8 K_8}{W_1 + W_2 + \ldots + W_8} \quad \text{(15)}
\end{align*}
$$

The weighting, $W_i$, is determined by the solving a set of linear equations. Here, the weighting is solved using MATLAB function “polydec.”

The set of controllers, $K_1$ to $K_8$, is found using the MATLAB function “hinfgs,” which utilizes the systems $S_0$, $S_1$, $S_2$, and $S_3$ to find an optimal controller for the given LMI problem. The runtime is 10.9 seconds using Matlab 6.5 running on 3GHz PentiumIV CPU. It is noted that the controller design can be done off-line. The on-line computation consists of 1) Weighting determination using command “polydec” as mentioned and 2) Evaluation of Equation (15).
5. Simulation Results

In Figure 5, the reference altitude is plotted against the actual trajectory. It can be seen that trajectory matches almost perfectly in this case.

In order to evaluate tracking performance through the range of acceptable operating points, the sinusoidal trajectory is given. The result is shown in Figure 6. The controller is able to track the reference, though it suffers from some lag in altitude. The desired x position is tracked almost identically, and the aircraft remains stable.

6. Conclusion

The F-16 like longitudinal model is formulated as a Linear Parameter Varying system. In order to reduce the intensity of computation, all of the stability derivatives except $X_a /V$ are represented by linear planes in function of angle of attack $\alpha$ and altitude $h$. The parameters are bounded so that they covers all possible values of the derivatives, hence, the stability can be guaranteed. The dynamics are described using three parameter: $\alpha$, $h$, and $X_a /V$. The performance is defined by tracking ability and control effort of the aircraft. Finally, the gain scheduling H-infinity controller is determined. The control gain for each corner of the bounding box is computed off-line. The control gain at each time instant is linear combination of control gains at the corner. The simulations verify that the aircraft is able to track reference trajectories and maintain stability.

References


