Numerical Approach on Parameters of the Thermal Radiation Interaction with Convection in a Boundary Layer Flow at a Vertical Plate with Variable Suction

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Abstract
In this paper the combined free convective dynamic boundary layer and thermal radiation boundary layer at a semi-infinite vertical plate has been studied with variable suction. The fluid is considered to be gray absorbing-emitting. The governing equations of the problem, the coupled unsteady non-linear momentum and energy equation of the combined layer, are reduced to be similar by the usual method of similarity transformation. The similarity equations are solved numerically by adopting a shooting method using a Nachtsheim-Swigert iteration technique. The resulting velocity and temperature distributions are shown graphically for different values of the parameters.

Keywords: thermal radiation, velocity distribution, temperature distribution, similarity analysis, convection, boundary layer, numerical analysis

1. Introduction
The heating of rooms and buildings by the use of radiators is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens etc. surrounded by cooler air, are at least in part, due to free convection. However, the mixed types of problems are very important and have many industrial and technological applications. The problem of reductive transfer in a vertical channel has been studied in recent times as a model for the re-entry problem. This is due to the significant role of thermal radiation in surface heat transfer when convection heat transfer is similar, particularly in free convection problems involving absorbing emitting fluids. One of the initiators of the problem, Goody [1] considered a neutral fluid. Soundalgekar and Takhar [2] studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using the Cogley-Vincntine-Giles equilibrium model. Hossain and Takhar [3] analyzed the effect of radiation using the Rosseland diffusion approximation that leads to non-similar boundary layer equations governing the mixed convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with a uniform free stream velocity and surface temperature. Cess [4] however, considered absorbing-emitting gray fluids with a black vertical plate. His solution was based on a perturbation technique and was applicable for small values of the conduction-radiation interaction parameter. A non- gray analysis on the other hand was made by Bratis and Novotny [5] employing limiting form to approximate the band profile. Novotny et al. [6] studied the same problem employing the method of local non-similarity and the continuous correlation of

Tien and Lowder [7] to account for the band absorption.

A study of the interaction of natural convection with thermal radiation in laminar boundary layer with isothermal horizontal surface in a gray gas was made by Ali et al. [8]. Following Ali et al., Mansour [9(a),(b)] studied the interaction of mixed convection with thermal radiation in laminar boundary layer flow over horizontal, continuous moving sheets with suction and injection. They then studied the interaction of thermal radiation at a semi-infinite plate longitudinally streamlined by visco-elastic fluid. Albraba et al. [10] studied the same problem of free convection interaction with thermal radiation in a hydromagnetic boundary layer taking into account the binary chemical reaction and the less attended Soret and Dufour effects.

Pohlhausen [11] first studied the thermal boundary layer flow past a semi-infinite vertical plate using the momentum integral method. A Similarity solution to this problem was given by Ostrach [12]. Siegel [13] first studied transient free convection flow past a semi-infinite vertical plate by an integral method. Since then many papers have been published in free convection flow past a semi-infinite vertical plate. Some of them are due to Sparrow and Gregg [14]; Szewczyk [15]; Merkin [16]; Eshghy [17] and Acrivos [18]. Sparrow and Gregg; Szewczyk and Menkin all gave numerical solutions to the similarity equations, whereas Eshghy and Acrivos both used an integral method for solving free convection problems.

Soundelgekar et al. [19] studied the free convection flow past a vertical porous plate. The investigation of the flow streaming into a porous and permeable medium with arbitrary but smooth surface was done by Yamamoto et al. [20]. Further analysis for a free convection in a porous medium bounded by an infinite plate was made by Raptis [21 (a), (b)]. Bestmen [22] made analytical efforts to study the free convection flow in a very porous medium with mass transfer and chemical reaction with finite Arrhenious activation. However, Raptis et al. [23] made numerical study of the free convection flow through a very porous medium bounded by a semi-infinite vertical porous plate. Following the work of Raptis et al. [23], Sattar [24] obtained an analytical solution to the same problem by the perturbation technique adopted by Singh and Dikshit [25].

Sattar and Kalim [26] studied the effects of unsteady free convection interaction with thermal radiation in a boundary layer flow. In their work, local solutions were obtained. In the present work, following the work of [26], the problem of unsteady free convection interaction with thermal radiation of an absorbing emitting fluid along a vertical plate with variable suction has been investigated. The investigation is based on a complete similarity analysis unlike the local similar solutions of Sattar and Kalim [26], similarity analysis by employing a time dependent similarity parameter. The similarity solutions are then obtained numerically for very small values of conduction radiation parameter, which are of practical interest from a physical point of view.

2. Governing equation of the flow:

We consider a two-dimensional unsteady flow in a combined dynamic boundary layer and thermal radiation boundary layer over a vertical plate. The plate is maintained at a uniform temperature T_w and placed vertically in a quiescent fluid of infinite extent at a constant temperature T. There is a variable suction at the plate taken to be a function of time. The fluid is assumed to be gray, emitting and absorbing, but a non-scattering medium. The physical co-ordinates (x,y) are chosen such that the x-axis lies in the plane of the plate and is oriented in the direction of the flow, and the y-axis is normal to it. The radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction. Now, considering the Boussinesque approximation, the two dimensional boundary layer equation related to our problem can be put forward as the equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

equation of momentum:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

and the energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{\alpha}{\kappa}\frac{\partial q_r}{\partial y} \tag{3}$$

The boundary conditions for the present problem are as follows:

$$\begin{Bmatrix} u=o,\ v=0,\ T=T_w \quad \text{at } y=0 \\ u\to 0,\ v\to 0,\ T\to T_\infty \quad \text{at } y\to\infty \end{Bmatrix} \tag{4}$$

The radiative heat flux term is simplified by making use of Rosseland approximation as:

$$q_r=-\frac{4\sigma}{3k}\frac{\partial T^4}{\partial y} \tag{5}$$

Where σ and k are the Stefan-Boltzman constant and mean absorption coefficient.

Since the plate is considered to be of infinite extent, all derivatives with respect to x vanish. Here the governing equations relevant to our problem reduces to:

$$\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{7}$$

and $$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{\alpha}{\kappa}\frac{\partial q_r}{\partial y} \tag{8}$$

3. Mathematical Formulation:

Our aim is to obtain a similarity solution. For this purpose we introduce a similarity parameter δ defined as:

$$\delta=\delta(t) \tag{9}$$

δ is a time dependent length scale. The continuity equation (1) can then be satisfied in terms of this length scale δ as:

$$v=-\frac{v}{\delta}v_0 \tag{10}$$

Here the constant v_0 represents the dimensionless suction or injection parameter. We now introduce the following dimensionless variable:

$$\eta=\frac{y}{\delta}$$
$$u=U_0\delta_* f(\eta) \tag{11}$$
$$\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}$$

Where $\delta_*=\frac{\delta}{\delta_0}$ is the value of δ at $t=t_0$ and U_0 is the uniform constant velocity.

From the equations (9) to (11) we get:

$$\frac{\partial u}{\partial t}=U_0\frac{2\delta}{\delta_0^2}f\frac{d\delta}{dt}-U_0\frac{\eta\delta}{\delta_0}f\frac{d\delta}{dt}$$

$$\frac{\partial^2 u}{\partial y^2}=U_0 f''\frac{1}{\delta_0^2}$$

$$\frac{\partial T}{\partial t}=-\theta'(T_w-T_\infty)\frac{\eta}{\delta}\frac{d\delta}{dt}$$

$$\frac{\partial^2 T}{\partial y^2}=(T_w-T_\infty)\theta''\frac{1}{\delta^2}$$

$$\frac{\partial q_r}{\partial y}=-\frac{4\sigma}{3k}\left[(T_w-T_\infty)^4 12\theta^2\theta'\frac{1}{\delta^2}+(T_w-T_\infty)4\theta^3\theta''\frac{1}{\delta}\right]$$

Substituting the above values in equations (2) and (3) we get respectively:

$$-\frac{\eta\delta f'}{v}\frac{d\delta}{dt}+\frac{2\delta f}{v}\frac{d\delta}{dt}-v_0 f'=f''+G_r\theta \tag{12}$$

$$-\frac{\delta}{v}\frac{d\delta}{dt}\eta\theta'-v_0\theta'=\frac{1}{P_r}\theta''+\frac{R}{P_r}\left[3(C_T+\theta)^2\theta'^2+(C_T+\theta)^3\theta''\right] \tag{13}$$

Where $\frac{\delta}{v}\frac{d\delta}{dt}$ is the dimensionless quantity.

$G_r=\frac{g\beta}{vU_0}(T_w-T_\infty)\delta_0^2$ is the local Grashof number.

$P_r=\frac{v}{\alpha}$ is the Prandtl number.

$R=\frac{16\sigma}{3K\kappa}(T_w-T_\infty)$ is the conduction radiation parameter.

$C_T = \frac{T_\infty}{T_w - T_\infty}$ is the temperature difference parameter.

The equations (12) and (13) are similar and expect the dimensionless quantity $\frac{\delta}{v}\frac{d\delta}{dt}$ where t appears explicitly.

Thus, the similarity condition requires that this quantity $\frac{\delta}{v}\frac{d\delta}{dt}$ must be a constant quantity. We therefore suppose that $\frac{\delta}{v}\frac{d\delta}{dt} = C$ (constant)

$$(14)$$

Now integrating (14) with the constant when t = 0,

$\delta = 0$ we obtain $\delta = \sqrt{2cvt}$ (15)

It appears from (15) that the length scale δ is consistent with the usual length scale considered for various non-steady flows.

Thus taking a realistic value of C to be 2 in (14), the equation (12) and (13) finally reduces to:

$$f'' + 2\left(\eta + \frac{v_0}{2}\right)f' - 4f + G_r\theta = 0 \qquad (16)$$

$$\theta'' + 2P_r\left(\eta + \frac{v_0}{2}\right)\theta' = -R\left[3(C_T+\theta)^2\theta'^2 + (C_T+\theta)^3\theta''\right] \qquad (17)$$

Subject to the equations (16) and (17) the boundary conditions (4) now transfer to:

$$\left\{\begin{matrix} f = 0, \theta = 1 \quad \text{at } \eta = 0 \\ f = 0, \theta = 0, \quad \text{as } \eta \to \infty \end{matrix}\right\} \qquad (18)$$

The above equations (16) and (17) thus describe the basic ordinary differential equations of our problem subject to the boundary conditions (18).

4. Method of solution:

Equations (16) and (17) with boundary conditions (18) are solved numerically using a standard initial value solver called the shooting method. For the purpose of this method, we have used the Nacthsheim-Swigert iteration technique (Nachtsheim & Swigert, 1965). In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach, one naturally inquires whether or not there is a systematic way of finding each succeeding (assumed) value of the missing initial condition. The Nachtsheim-Swigert iteration technique thus needs to be discussed elaborately.

The boundary conditions (18) associated with the linear ordinary differential equations (16) and (17) of the boundary layer type are of the two-points asymptotic class. Two-points boundary conditions have values of the dependent variable specified at two different values of the independent variable. Specification of an asymptotic boundary condition implies the value of velocity approaches unity and the value of the temperature approaches zero as the outer specified value of the independent variable is approached. The method of numerically integrating a two-point asymptotic boundary value problem of the boundary layer type, the initial value method, requires that it be recast as an initial value problem. Thus it is necessary to estimate as many boundary conditions at the surface as were given at infinity. The governing differential equations are then integrated with these assumed surface boundary conditions. If the required outer boundary condition is satisfied, a solution is achieved. However, this is not generally the case. Hence a method must be devised to logically estimate the new surface boundary conditions for the next trial integration. Asymptotic boundary value problems such as those governing the boundary layer equations are further complicated by the fact that the outer boundary condition is specified at infinity. The trial integration's infinity is numerically approximated by some large value of the independent variable. There is no a priori general method of estimating this value. Selecting too small a maximum value for the independent variable may not allow solutions to asymptotically approach the required accuracy. Selecting a large value may result in divergence of the trial integration or in slow convergence of surface boundary conditions satisfying the asymptotic outer boundary condition. Selecting too large a value of the independent variable is expensive in terms of computer time.

Nachtsheim-Swigert developed an iteration method, which overcomes these difficulties. Extension of the Nachtsheim-Swigert iteration shell to the above equation system of differential equations (16) and (17) is straightforward. In equation (18) there are two asymptotic boundary conditions and hence two unknown surface conditions $f'(0)$ and $\theta'(0)$. Within the context of the initial-value method and the Nachtsheim-Swigert iteration technique, the outer boundary conditions may be functionally represented as:

$$f(\eta_{\max}) = f[f'(0), \theta'(0)] = \delta_1 \quad (19)$$

$$\theta(\eta_{\max}) = \theta[f'(0), \theta'(0)] = \delta_2 \quad (20)$$

with the asymptotic convergence criteria given by:

$$f'(\eta_{\max}) = f'[f'(0), \theta'(0)] = \delta_3 \quad (21)$$

$$\theta'(\eta_{\max}) = \theta'[f'(0), \theta'(0)] = \delta_4 \quad (22)$$

Expanding in a first order Taylor series using equation (19) to (22) yields:

$$f(\eta_{\max}) = f_c(\eta_{\max}) + f_x \Delta x + f_y \Delta y = \delta_1 \quad (23)$$

$$\theta(\eta_{\max}) = \theta_c(\eta_{\max}) + \theta_x \Delta x + \theta_y \Delta y = \delta_2 \quad (24)$$

$$f'(\eta_{\max}) = f'_c(\eta_{\max}) + f'_x \Delta x + f'_y \Delta y = \delta_3 \quad (25)$$

$$\theta'(\eta_{\max}) = \theta'_c(\eta_{\max}) + \theta'_x \Delta x + \theta'_y \Delta y = \delta_4 \quad (26)$$

Where $x = f'(0), y = \theta'(0)$ and the x and y subscripts

Indicate partial differentiation, e.g. :

$$f'_x = \frac{\partial f'(\eta_{\max})}{\partial f'(0)}, f'_y = \frac{\partial f'(\eta_{\max})}{\partial \theta'(0)}$$

The subscript "c" indicates the value of the function at $\eta_{\max}$ determined from the trial integration. Solutions of these equations in a least squares sense requires determining the minimum value of:

$$E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 \quad (27)$$

Differentiating E with respect to x:

$$\left(f_x^2 + \theta_x^2 + f_x'^2 + \theta_x'^2\right)\Delta x + \left(f_x f_y + \theta_x \theta_y + f'_x f'_y + \theta'_x \theta'_y\right)\Delta y = -\left(f_c f_x + \theta_c \theta_x + f'_c f'_x + \theta'_c \theta'_x\right) \quad (28)$$

Differentiating E with respect to y:

$$\left(f_y^2 + \theta_y^2 + f_y'^2 + \theta_y'^2\right)\Delta y + \left(f_x f_y + \theta_x \theta_y + f'_x f'_y + \theta'_x \theta'_y\right)\Delta x = -\left(f_c f_y + \theta_c \theta_y + f'_c f'_y + \theta'_c \theta'_y\right) \quad (29)$$

We can write (28) as a (29) in system of linear equations in the following form as:

$$A_{11}\Delta x + A_{12}\Delta y = d_1 \quad (30)$$

$$A_{21}\Delta x + A_{22}\Delta y = d_2 \quad (31)$$

Where

$$A_{11} = \left(f_x^2 + \theta_x^2 + f_x'^2 + \theta_x'^2\right)$$

$$A_{22} = \left(f_y^2 + \theta_y^2 + f_y'^2 + \theta_y'^2\right)$$

$$A_{12} = A_{21} = \left(f_x f_y + \theta_x \theta_y + f'_x f'_y + \theta'_x \theta'_y\right)$$

$$d_1 = -\left(f_c f_x + \theta_c \theta_x + f'_c f'_x + \theta'_c \theta'_x\right)$$

$$d_2 = -\left(f_c f_y + \theta_c \theta_y + f'_c f'_y + \theta'_c \theta'_y\right)$$

From equations (30) and (31) we have

$$\Delta x = \frac{\det A_1}{\det A}, \Delta y = \frac{\det A_2}{\det A}$$

Where,

$$\det\ A = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}, \det\ A_1 = \begin{vmatrix} d_1 & A_{12} \\ d_2 & A_{22} \end{vmatrix}$$

$$\det\ A_2 = \begin{vmatrix} A_{11} & d_1 \\ A_{21} & d_2 \end{vmatrix}$$

Adopting the numerical technique aforementioned, the solutions of the linear ordinary differential equations (13) and (14) with boundary conditions (15) are obtained, together with sixth order implicit Runge-kutta initial value solver. Here we applied the shooting method for different values of pertinent parameters. In the process of integration, the skin friction coefficient $f'(0)$ and the heat transfer rate $\theta'(0)$ are also calculated.

5. Numerical results:

In this paper we have considered the problem of laminar free convection flow along with thermal radiation of an absorbing-emitting fluid along a vertical plate. The flow is also considered to be unsteady. Similarity equations of these problems are obtained by introducing a similarity parameter, taken to be a time dependent length scale. The suction velocity is also taken to be a function of time. Under these conditions, the solutions to the problem are finally solved numerically, which is applicable for small values of conduction radiation interaction parameter R. For the purpose of discussing the numerical solution, the effects of various parameters on the flow behavior have been calculated for different values of suction parameter v_0, the conduction radiation interaction parameter R, temperature difference C_T, Prandtl number P_r and Grashof number G_r. Since these are free parameters of interest in the present problem, values of these parameters are respectively taken to be:

$v_0 = 0.5, 0.8, 1.0, 1.5$; R = 0.0, 0.1, 0.3, 0.5; $C_T = 0.0, 0.5, 1.0, 1.5$; $P_r = 0.71, 1.0, 2.0, 3.0$ and $G_r = 3.0, 5.0, 8.0, 10.0$

The effects of temperature difference C_T on the velocity profiles are shown in Fig.1. It can be seen from the figure that the velocity profiles increase with increasing values of C_T . An increase in C_T correspondingly increases the effects on free convection. Thus from Fig.1 we conclude that as the free convection increases, velocity also increases.

In Fig.2, the effects of the conduction radiation interaction parameter R on the velocity profiles are shown. From this figure, it is observed that as the interaction of the thermal radiation intensifies (increase in R), the velocity increases with an accompanying increase in the velocity gradient at the wall.

In Fig.3, the effects of suction on the velocity profiles are shown. It can be noted that velocity profiles decreases with the increase of the suction parameter. From this figure it can also be noted that boundary layer thickness decreases with the increase of suction parameter.

In Fig.4, the effects of the Prandtl number on the velocity profiles are shown. It can be seen that the velocity profiles decrease due to increasing values of the Prandtl number.

In Fig.5, the effects of the Grashof number on the velocity are displayed. It is apparent from the figure that the increasing values of Grashof number enhance the velocity.

In Fig.6, the effects of the Prandtl number on the temperature profiles are seen. The temperature in this case reduces with the increase of Prandtl number, with reduction of the thermal boundary layer thickness.

In Fig.7, the temperature profiles are viewed for different values of v_0 . In this case the temperature reduces with the increase of v_0 .

Fig.8 shows the effects of C_T on the temperature profiles. The temperature profile is found to increase with the increase of the parameter C_T .

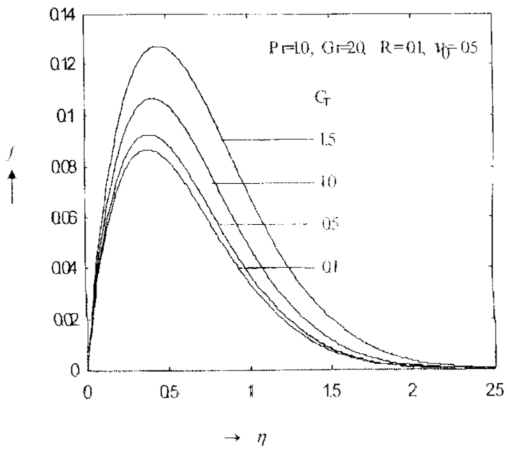

Fig.1 Velocity profiles for different values of R, P_r, G_r and v_o,

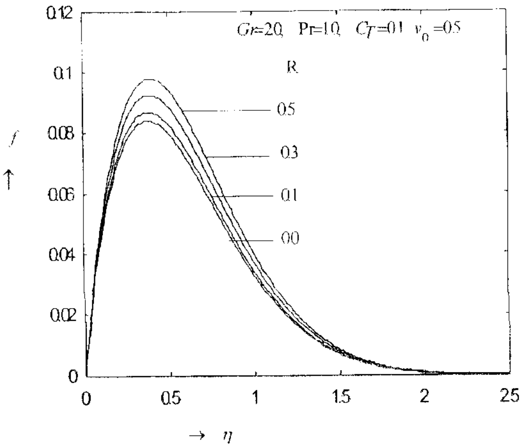

Fig.2 Velocity profiles for different values of C_T, P_r, G_r and v_o,

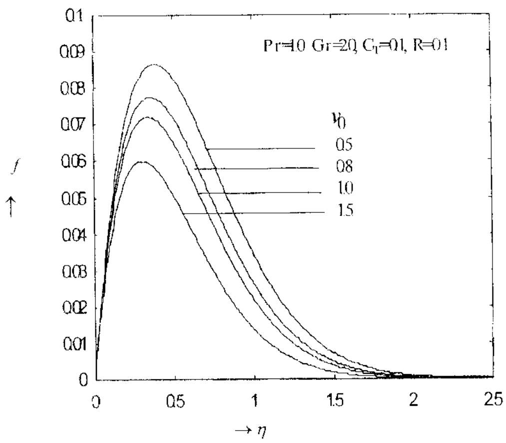

Fig.3 velocity profiles for different values of C_T, P_r, G_r and R

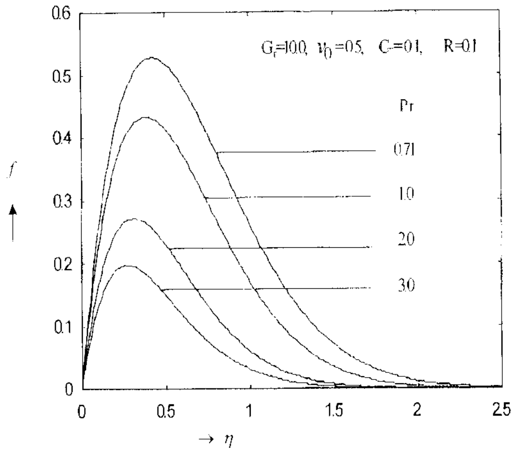

Fig.4 Velocity profiles for different values of R, C_T, G_r and v_o,

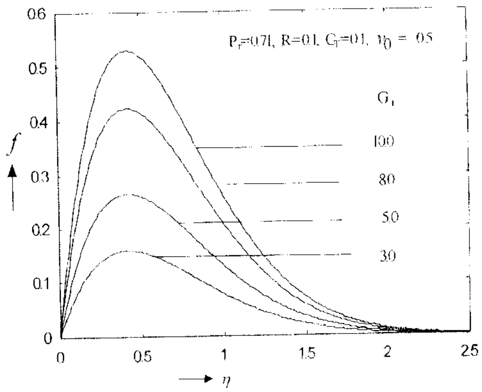

Fig.5 Velocity profiles for different values of C_T, P_r, v_0 and R

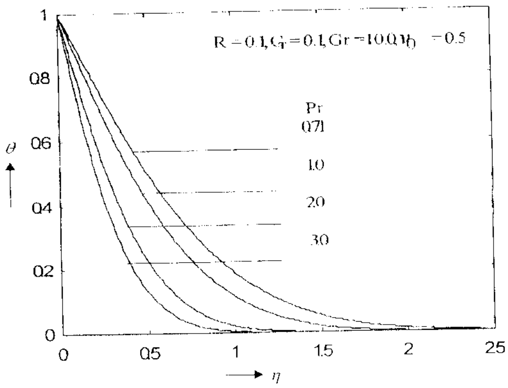

Fig.6 Temperature profiles for different values of C_T, G_r, v_0 and R

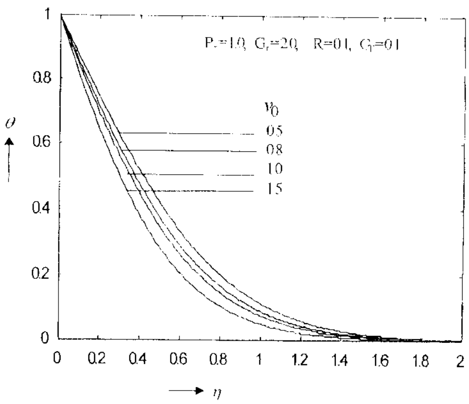

Fig.7 Temperature profiles for different values of C_T, P_r, G_r and R

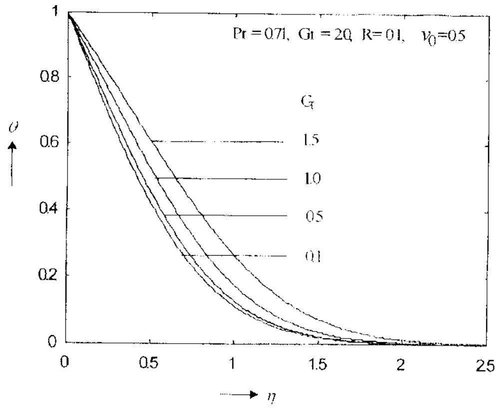

Fig.8 Temperature profiles for different values of G_r, P_r, v_0 and R

6. Nomenclature:

x,y,z	Cartesian coordinates
u,v,w	Components of velocity
t	time
η	Similarity variable
υ	Coefficient of kinematics viscosity
θ	Dimensionless temperature
ρ	Density of fluid
β	Coefficient of volume expansion
g	Gravitational acceleration
G_r	Grashof number
C_p	Specific heat at constant pressure
P_r	Prandtl number
q_r	Local radiative flux
τ_w	Local shear stress
v_0	Suction parameter
T	Temperature
T_w	Plate temperature
T_∞	Free steam temperature
κ	Heat diffusivity coefficient
k	Permiability coefficient
U_∞	Free steam velocity
α	Thermal diffusivity
δ	Time dependent length scale

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