Simulation Analysis of Quasi Space Vector PWM in Different Conduction Modes for Inverter Fed Permanent Magnet Brushless DC Motor Drive

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Abstract
It is known that the Space Vector PWM (SVPWM) modulation scheme is superior in driving a three-phase permanent magnet brushless dc (3PPMBLDC) motor through the use of conventional three-phase voltage source inverter (3PVSI). However, implementing SVPWM in an embedded system requires that a high performance chip, such as a DSP chip, be used, not economically suitable for low precision applications such as electric bike motor drive and quad rotor robot motor drive. For these applications, it is acceptable to adopt a 3PVSI with a simpler modulation scheme that can be classified into 3 types, namely, the 120° conduction mode quasi SVPWM (120° QSVPWM), the 150° conduction mode quasi SVPWM (150° QSVPWM), and the 180° conduction mode quasi SVPWM (180° QSVPWM). This paper describes all three modulation schemes and conducts a performance analysis of the 3PVSI s resulting from applying each of the modulation schemes. The performance numbers are obtained through simulations. Eventually, our custom built simulated dynamic model of 3PPMBLDC motor based on the Simulink package in MATLAB software is brought to be verified by our custom built simulated model of 3PVSI with different modulation schemes in order to visualize the torque and mechanical angular velocity(speed) characteristics.

Keywords: Voltage source inverter conduction mode; three phase brushless DC motor drive; space vector voltage source inverter.

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Nomenclature

\( \theta_e \)  
Electrical angular displacement.

\( V_{a0} \)  
Voltage at point \( a \) with respect to point \( 0 \).

\( \theta_m \)  
Mechanical angular displacement.

\( \alpha - \beta \rightarrow v_s \)  
Voltage space vector with respect to the \( \alpha - \beta \) reference frame.

\( S_1, S_2, S_3, S_4, S_5, S_6 \)  
Switching elements \( 1,2,3,4,5,6 \).

\( d \)  
The normalized duty cycle.

\( e_{m}(t) \)  
The peak value of back EMF.

\( K_b \)  
The motor back EMF constant (volt/(rad/s)).

\( e_{a}(t, \theta_e) \)  
The back EMF of the stator windings, phase \( a \).

\( \omega_m(t) \)  
The mechanical angular velocity of the rotor.

\( T_e \)  
The electromagnetic torque.

\( J_m \)  
The moment of inertia of the rotor including mechanical loads attached.

\( B_m \)  
The viscous damping between the rotor and the environment contacted.

\( T_l \)  
The load torque disturbed through the axle of rotor.

\( \omega_e(t) \)  
The rotor electrical angular velocity.

\( p \)  
The number of poles of permanent magnets attached to the rotor.

\( M \)  
The mutual inductance between each phase winding.

\( L \)  
The self inductance of each phase winding.

\( R_p \)  
The resistance of each phase winding.

\( L_p = L - M \)  
The self against mutual inductance difference.

1. Introduction

It is beneficial to use SVPWM as the modulation scheme for the operation of a 3PVSI when driving a 3PPMBLDC motor. However, most studies in the literature cover only the so-called "120° conduction mode" and "180° conduction mode" [3,6,7,8,9,10], which are the modes of which, when applied by 100% duty cycle gating signal to each switching element, the conduction duration allowable to each switching element to conduct for 120° or 180° electrical angular displacement(\( \theta_e \)) respectively, in each switching cycle. Considering the possible combinations of all six switches in the power stage of 3PVSI, it is possible to have one more mode called the 150° conduction mode, a mode that has been mentioned in the literature [1]. In [1], 3PVSI was used to drive the three phase induction motor. However, the study did not cover the use of a 3PPMBLDC motor. No existing work, to the best of our knowledge, demonstrates what happens when one drives a 3PPMBLDC motor using a 3PVSI in this mode. This study then reveals the behaviors of a 3PVSI while carrying out all of three aforementioned...
conduction modes in driving a 3PPMBLDC motor. The torque and mechanical angular velocity (speed) of this motor are examined as well through simulation results. The models of a 3PVSI and a 3PPMBLDC motor are created to explore various aspects of interest. The creation of the models is based on Simulink, packages in MATLAB software. Since SVPWM is computationally intensive, implementing it in an embedded system requires that a high performance processor, such as a DSP chip, be used. This requirement drives up the cost and makes it difficult to employ SVPWM in low precision applications such as electric bike motor drive and quad rotor robot motor drive. In order to accommodate applications with lower precision and computation requirements, this study proposes the concept of the quasi space vector (QSV) upon which three aforementioned modulation schemes are based. QSV is a space vector rotating stepwise at a fixed electrical angular displacement (\( \theta_e \)) whereas a space vector (SV) may rotate continuously in the reference coordinate system.

2. Conventional 3PVSI driving 3PPMBLDC motor

From Fig. 1, \( S_1, \ldots, S_6 \) are the switching elements (such as MOSFETs or IGBTs) that can have the states of “on” or “off” with the constraint of the shoot-through prohibited condition. The voltage at points \( a, b, c \) with respect to point 0 denoted \( (V_{a0}, V_{b0}, V_{c0}) \), can have three possible values, namely, \( V_{dc}/2, -V_{dc}/2 \), and the value between these two voltage levels. With the load balanced condition of \( V_{an} + V_{bn} + V_{cn} = 0 \), it can be shown that

\[
V_{a0} = \frac{1}{3}[V_{a0} + V_{b0} + V_{c0}],
\]

\[
V_{an} = V_{a0} - V_{n0},
\]

\[
V_{bn} = V_{b0} - V_{n0},
\]

\[
V_{cn} = V_{c0} - V_{n0}.
\]

2.1 Space Vector and Quasi Space Vector Definitions

A space vector (SV) is a rotating vector representing three sinusoidal phase quantities in the same system. It can be rotated continuously in space. This study considers the voltage space vector, \( v_s \), of the three balanced sinusoidal phase voltages, comprising

\[
v_{an} = v_m \cos(\omega t - 2\pi/3),
\]

\[
v_{bn} = v_m \cos(\omega t + 2\pi/3)\]

where \( \omega \) is the electrical angular velocity. We define the two dimensional reference frame, called the \( \alpha - \beta \) reference frame, as the fixed frame. The voltage space vector with respect to this frame can be written as \( \rightarrow v_s \) and can be calculated as in Eq. (5) (phase invariant form):

\[
v_s = \frac{2}{3} (v_{an} + v_{bn} + v_{cn} \rightarrow \alpha \beta).
\]

A depiction of \( v_s \) is shown in Fig. 2.

Quasi Space Vector (QSV) Definition.

QSV is a space vector rotating stepwise by a fixed electrical angular displacement(\( \theta_e \)).
The voltage space vector denoted $\vec{v}_s$, as the vector summation of three sinusoidal phase voltage vectors, $\vec{v}_{an}$, $\vec{v}_{bn}$, $\vec{v}_{cn}$.

2.2 3PVSI in 120°, 150°, and 180° conduction mode square wave outputs

2.2.1 3PVSI in 120° conduction mode

In this mode, the conduction duration allowed for each switching element ($S_1, \ldots, S_6$) of a 3PVSI as shown in Fig. 1 is equal to 120° of $\theta_e$ in each switching cycle. The voltages, $V_{a0}, V_{b0}, V_{c0}, V_{an}, V_{bn}$, and $V_{cn}$ for a 3PVSI with the 120° conduction mode applied by 100% duty cycle gating signal to each switching element can be depicted in Fig. 4. It is clear that there are 3 possible levels of voltages, $V_{an}, V_{bn}$, and $V_{cn}$, each of which is $V_{dc}/2$, $0$, $-V_{dc}/2$ consecutively. From Eq. (5), $\vec{v}_s$ can be displayed in the diagram of Fig. 3. From this diagram, $\vec{v}_s$ rotates discontinuously at the step size of $\pi/3$ radians such as from $V_{61}$ to $V_{12}$. $\vec{v}_s$ in this mode is the QSV. The Fourier series representing $V_{an}$ with the period of $2\pi$ can be written as:

$$V_{an}(\theta_e) = \sum_{m=1}^{\infty} \left( V_{dc} / m\pi \right) \left( \sin \left( \frac{m\pi}{3} \right) \right)^2 \cdot \left[ 1 - 2\cos\left( \frac{m\pi}{3} \right) - 2\cos(m\pi) \right] \sin(m\theta_e).$$  (6)

The fundamental component of $V_{an}$, $HV_{an|_{m=1}}$, is equal to $3V_{dc}/2\pi$.
Fig. 4. The voltages, $V_{a0}, V_{b0}, V_{c0}, V_{a1}, V_{a2}, V_{b1},$ and $V_{c1}$ for a 3PVSI in the $120^\circ$ conduction mode applied by 100% duty cycle gating signals.

3PVSI in $150^\circ$ conduction mode

In this mode, the conduction duration allowed for each switching element $(S_1, \ldots, S_6)$ of a 3PVSI as shown in Fig. 1 is equal to $150^\circ$ of $\theta_e$ in each switching cycle. The voltages, $V_{a0}, V_{b0}, V_{c0}, V_{a1}, V_{a2}, V_{b1},$ and $V_{c1}$ for a 3PVSI with the $150^\circ$ conduction mode applied by 100% duty cycle gating signal to each switching element can be depicted in Fig. 7. It is clear that there are 7 possible levels of voltages for $V_{a1}, V_{b1},$ and $V_{c1},$ each of which is $0, \pm V_{dc}/3, \pm V_{dc}/2, \pm 2V_{dc}/3,$ consecutively.

From Eq. (5), $v_s$ can be displayed in the diagram of Fig. 5. From this diagram, $v_s$ rotates discontinuously at the step size of $\pi/3$ radians such as from $V_{b1}$ to $V_{a2}$. $v_s$ in this mode is the QSV. The Fourier series representing $V_{an}$ with the period of $2\pi$ can be written as:

$$V_{an}(\theta_e) = \sum_{m=1}^{\infty} \frac{2V_{dc}}{3m\pi} \left( \sin\left(\frac{m\pi}{6}\right)^2 \right) \cdot \left[3 - 6\cos\left(\frac{m\pi}{6}\right) + 4\cos\left(\frac{2m\pi}{3}\right) - 7\cos\left(\frac{5m\pi}{6}\right)ight] \sin(m\theta_e). \quad (7)$$

The fundamental component of $V_{an}$, $HV_{an_{max}}$, is equal to $(2 + \sqrt{3})V_{dc}/2\pi$.

3PVSI in $180^\circ$ conduction mode

In this mode, the conduction duration allowed for each switching element $(S_1, \ldots, S_6)$ of a 3PVSI as shown in Fig. 1 is equal to $180^\circ$ of $\theta_e$ in each switching cycle. The voltages, $V_{a0}, V_{b0}, V_{c0}, V_{a1}, V_{a2}, V_{b1},$ and $V_{c1}$ for a 3PVSI with the $180^\circ$ conduction mode applied by 100% duty cycle gating signal to each switching element can be depicted as in Fig. 8. It is clear that there are 3 possible levels of voltages for $V_{a1}, V_{b1},$ and $V_{c1}$, each of which is $V_{dc}/2, 0, -V_{dc}/2$ consecutively. From Eq. (5), $v_s$ can be displayed in the diagram of Fig. 6. From this diagram, $v_s$ rotates discontinuously at the step size of $\pi/3$ radians.
such as from $V_1$ to $V_2$. $v_s$ in this mode is the QSV. The Fourier series representing $V_{an}$ with the period of $2\pi$ can be written as:

$$V_{an}(\theta_e) = \sum_{m=0}^{\infty} \frac{4V_{dc}}{3m\pi} \left( \sin\left(\frac{m\pi}{3}\right) \right)^2 \cdot$$

$$\left[ -\cos\left(\frac{2m\pi}{3}\right) - \cos(m\pi) - \cos\left(\frac{4m\pi}{3}\right) \right] \sin(m\theta_e). \quad (8)$$

The fundamental component of $V_{an}$, $HV_{an}|_{m=1}$, is equal to $2V_{dc}/\pi$.

Fig. 5. Diagram depicting the $150^\circ$ conduction mode voltage space vector, $v_s$. 

Fig. 6. Diagram depicting the $180^\circ$ conduction mode voltage space vector, $v_s$.

Fig. 7. The voltages, $V_{a0}, V_{b0}, V_{c0}, V_{a0}, V_{an}, V_{bn}, V_{cn}$ for 3PVI with the $150^\circ$ conduction mode applied by 100% duty cycle gating signals.
The switching patterns, $V_{A0}, V_{B0}, V_{C0}, V_{n0}$, $V_{an}, V_{bn}, V_{cn}$ for 3PVS\~I with the 180° conduction mode applied by 100% duty cycle gating signals.

The voltages, $V_{a0}, V_{b0}, V_{c0}, V_{n0}, V_{an}, V_{bn}, V_{cn}$ can be tabulated as shown in Table 1.
<table>
<thead>
<tr>
<th>V</th>
<th>(<em>+</em>)</th>
<th>S, S, [V_{dc}/2, V_{dc}/2, V_{dc}/2]</th>
<th>0,0,0</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>(+-)</td>
<td>S, S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
<tr>
<td>V6,1</td>
<td>(+-)</td>
<td>S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
<tr>
<td>V6</td>
<td>(+-)</td>
<td>S, S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
<tr>
<td>V1,2</td>
<td>(+-)</td>
<td>S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
<tr>
<td>V2,1</td>
<td>(+-*)</td>
<td>S, [V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>0,0,0</td>
</tr>
<tr>
<td>V2</td>
<td>(+-)</td>
<td>S, S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
<tr>
<td>V2,4</td>
<td>(+-)</td>
<td>S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
<tr>
<td>V3</td>
<td>(+-)</td>
<td>S, S, S, [6V_{dc}/2, V_{dc}/2, V_{dc}/2]</td>
<td>[2V_{dc}/3, -V_{dc}/3, -V_{dc}/3]</td>
</tr>
</tbody>
</table>

### 2.3 3PVSI in 120°, 150°, and 180° conduction modes, QSVPWM outputs, and switching strategies

The PWM signal used in this study is the center aligned PWM with a triangular wave form as the carrier as shown in Fig. 9. The switching strategies apply the center aligned PWM to the “on” switches in Table 1, taking into account the position of the rotor at that moment, the rotation direction of the motor, and the conduction mode selected. Tables 2, 3, 4 summarize the switching patterns with respect to the rotor position and the direction of rotation.

![PWM signal diagram](image)

**Fig.9.** The center aligned PWM with \(d\) as the normalized duty cycle. The triangle carrier has a period of \(T_s\).

The normalized duty cycle denoted \(d\), is determined by the modified volt-second balance principle:

\[
\frac{d}{dT_s} \int_{0}^{T_s} |v_{s_{ref}}| dt = \int_{0}^{dT_s} |QSV| dt
\]

\[
d = \left|\frac{\int_{0}^{T_s} |v_{s_{ref}}| dt}{\int_{0}^{dT_s} |QSV| dt}\right|
\]

where \(\left|v_{s_{ref}}\right|\) is the magnitude of the reference space vector at any sector \((1, 2, \ldots, 12)\) and \(|QSV|\) is the magnitude of the appropriate QSV in that sector.
Table 2. 120° QSVPWM switching patterns for CCW and CW rotation.

<table>
<thead>
<tr>
<th>Sect or no.</th>
<th>Range of $\theta_e$</th>
<th>Switching pattern for CCW rotation</th>
<th>Switching pattern for CW rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-2\pi/6 \leq \theta_e &lt; -11\pi/6$ or $0 \leq \theta_e &lt; 7\pi/6$</td>
<td>$v_{1.2}(+*-)$</td>
<td>$v_{4.5}(-*+)$</td>
</tr>
<tr>
<td>2</td>
<td>$-11\pi/6 \leq \theta_e &lt; -10\pi/6$ or $\pi/6 \leq \theta_e &lt; 2\pi/6$</td>
<td>$v_{1.2}(+*+)$</td>
<td>$v_{4.5}(-*+)$</td>
</tr>
<tr>
<td>3</td>
<td>$-10\pi/6 \leq \theta_e &lt; -9\pi/6$ or $2\pi/6 \leq \theta_e &lt; 3\pi/6$</td>
<td>$v_{2.3}(<em>+</em>)$</td>
<td>$v_{5.6}(<em>-</em>+)$</td>
</tr>
<tr>
<td>4</td>
<td>$-9\pi/6 \leq \theta_e &lt; -8\pi/6$ or $3\pi/6 \leq \theta_e &lt; 4\pi/6$</td>
<td>$v_{2.3}(<em>+</em>)$</td>
<td>$v_{5.6}(<em>-</em>+)$</td>
</tr>
<tr>
<td>5</td>
<td>$-7\pi/6 \leq \theta_e &lt; -\pi$ or $5\pi/6 \leq \theta_e &lt; \pi$</td>
<td>$v_{3.4}(-*+)$</td>
<td>$v_{6.7}(-++)$</td>
</tr>
<tr>
<td>6</td>
<td>$-\pi \leq \theta_e &lt; -5\pi/6$ or $\pi \leq \theta_e &lt; 7\pi/6$</td>
<td>$v_{4.5}(-*+)$</td>
<td>$v_{6.7}(-++)$</td>
</tr>
<tr>
<td>7</td>
<td>$-5\pi/6 \leq \theta_e &lt; -4\pi/6$ or $7\pi/6 \leq \theta_e &lt; 8\pi/6$</td>
<td>$v_{5.6}(-*+)$</td>
<td>$v_{6.7}(-++)$</td>
</tr>
<tr>
<td>8</td>
<td>$-4\pi/6 \leq \theta_e &lt; -3\pi/6$ or $8\pi/6 \leq \theta_e &lt; 9\pi/6$</td>
<td>$v_{5.6}(*-+)$</td>
<td>$v_{6.7}(-++)$</td>
</tr>
<tr>
<td>9</td>
<td>$-3\pi/6 \leq \theta_e &lt; -2\pi/6$ or $9\pi/6 \leq \theta_e &lt; 10\pi/6$</td>
<td>$v_{5.6}(*-+)$</td>
<td>$v_{6.7}(-++)$</td>
</tr>
<tr>
<td>10</td>
<td>$-2\pi/6 \leq \theta_e &lt; -\pi/6$ or $10\pi/6 \leq \theta_e &lt; 11\pi/6$</td>
<td>$v_{6.1}(+*+)$</td>
<td>$v_{3.4}(-*+)$</td>
</tr>
<tr>
<td>11</td>
<td>$-\pi/6 \leq \theta_e &lt; 0$ or $11\pi/6 \leq \theta_e &lt; 2\pi$</td>
<td>$v_{6.1}(+*+)$</td>
<td>$v_{3.4}(-*+)$</td>
</tr>
</tbody>
</table>

Table 3. 150° QSVPWM switching patterns for CCW and CW rotation.

<table>
<thead>
<tr>
<th>Sect or no.</th>
<th>Range of $\theta_e$</th>
<th>Switching pattern for CCW rotation</th>
<th>Switching pattern for CW rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-2\pi/6 \leq \theta_e &lt; -11\pi/6$ or $0 \leq \theta_e &lt; \pi/6$</td>
<td>$v_{1.2}(+*-)$</td>
<td>$v_4(-++)$</td>
</tr>
<tr>
<td>2</td>
<td>$-11\pi/6 \leq \theta_e &lt; -10\pi/6$ or $\pi/6 \leq \theta_e &lt; 2\pi/6$</td>
<td>$v_{2}(++*)$</td>
<td>$v_{4.5}(-*+)$</td>
</tr>
</tbody>
</table>

Table 4. 180° QSVPWM switching patterns for CCW and CW rotation.

<table>
<thead>
<tr>
<th>Sect or no.</th>
<th>Range of $\theta_e$</th>
<th>Switching pattern for CCW rotation</th>
<th>Switching pattern for CW rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-2\pi/6 \leq \theta_e &lt; -11\pi/6$ or $0 \leq \theta_e &lt; \pi/6$</td>
<td>$v_{1}(++-)$</td>
<td>$v_{4.5}(-*+)$</td>
</tr>
<tr>
<td>2</td>
<td>$-11\pi/6 \leq \theta_e &lt; -10\pi/6$ or $\pi/6 \leq \theta_e &lt; 2\pi/6$</td>
<td>$v_{2}(++*)$</td>
<td>$v_{4.5}(-*+)$</td>
</tr>
<tr>
<td>3</td>
<td>$-10\pi/6 \leq \theta_e &lt; -9\pi/6$ or $2\pi/6 \leq \theta_e &lt; 3\pi/6$</td>
<td>$v_{2}(++*)$</td>
<td>$v_{4.5}(-*+)$</td>
</tr>
<tr>
<td>4</td>
<td>$-9\pi/6 \leq \theta_e &lt; -8\pi/6$ or $3\pi/6 \leq \theta_e &lt; 4\pi/6$</td>
<td>$v_{3}(-+-)$</td>
<td>$v_{6}(+-+)$</td>
</tr>
<tr>
<td>5</td>
<td>$-8\pi/6 \leq \theta_e &lt; -7\pi/6$ or $4\pi/6 \leq \theta_e &lt; 5\pi/6$</td>
<td>$v_{3}(-+-)$</td>
<td>$v_{6}(+-+)$</td>
</tr>
<tr>
<td>6</td>
<td>$-7\pi/6 \leq \theta_e &lt; -\pi$ or $5\pi/6 \leq \theta_e &lt; \pi$</td>
<td>$v_{4}(+-+)$</td>
<td>$v_{6}(+-+)$</td>
</tr>
</tbody>
</table>
3. The dynamic modeling of 3PPMBLDC motor

Trapezoidal back EMF modeling. A 3PPMBLDC motor in this study is classified as the surface-mounted-magnet, conventional stator. It has the trapezoidal back EMF in each stator winding with the electrical angular displacement difference in each phase. The peak value of the back EMF denoted \( e_m(t) \), is derived as shown in Eq. (10) [2] where \( K_b \) is the motor back EMF constant (volt/(rad/s)). We define \( f_a(\theta_e) \), \( f_b(\theta_e) \), and \( f_c(\theta_e) \) as shown in Eq. (11) and (12). Where \( \theta_e \) is the rotor electrical angular displacement. The back EMFs of the stator windings, phase a\( (e_a(t, \theta_e)) \), phase b\( (e_b(t, \theta_e)) \), and phase c\( (e_c(t, \theta_e)) \) can be expressed as in Eq. (13):

\[
e_m(t) = K_b\omega_m(t), \tag{10}
\]

\[
f_a(\theta_e) = f_a(\theta_e - \frac{2\pi}{3}), \quad f_c(\theta_e) = f_a(\theta_e + \frac{2\pi}{3}) \tag{12}
\]

\[
e_a(t, \theta_e) = f_a(\theta_e)e_m(t), \quad e_b(t, \theta_e) = f_b(\theta_e)e_m(t) \quad \text{and} \quad e_c(t, \theta_e) = f_c(\theta_e)e_m(t). \tag{13}
\]

The electromagnetic torque, rotational equation of motion, the rotor mechanical and electrical angular velocities and displacements.

The electromagnetic torque denoted \( T_e \) can be expressed as shown in Eq. (14) where \( \omega_m(t) \) is the mechanical angular velocity of the rotor:

\[
T_e(t, \theta_e) = \frac{e_a(t, \theta_e)i_a + e_b(t, \theta_e)i_b + e_c(t, \theta_e)i_c}{\omega_m(t)}
\]

\[
= K_b[f_a(\theta_e)i_a + f_b(\theta_e)i_b + f_c(\theta_e)i_c] \tag{14}
\]

The rotational equation of motion. The rotational equation of motion can be expressed as in Eq. (15) where \( J_m \) is the moment of inertia of the rotor including mechanical loads attached. \( B_m \) is the viscous damping between the rotor and the environment contacted. \( T_l \) is the load torque
disturbed through the axle of rotor Eq. (15) is as follows:

\[
J_m \frac{d\omega_m}{dt} + B_m \omega_m = T_c - T_i. \tag{15}
\]

The rotor mechanical and electrical angular velocities and displacements. The rotor electrical angular velocity denoted \( \omega_e(t) \), is defined as in Eq. (16) where \( p \) is the number of poles of permanent magnets attached to the rotor:

\[
\omega_e(t) = \frac{d\theta_e}{dt} = \left( \frac{p}{2} \right) \frac{d\theta_m}{dt} = \frac{p}{2} \omega_m(t). \tag{16}
\]

The state space equation of 3PPMBLDC motor. Under the assumptions of \( i_a + i_b + i_c = 0 \), the mutual inductance between each phase winding is equal to the others and is defined as \( M \). The self inductance of each phase winding is equal to the others and defined as \( L \). The resistance of each phase winding is equal to the others and defined as \( R_p \). We define \( L_p = L - M \), the state space equation of 3PPMBLDC motor which can be written as in Eq. (17).

\[
\begin{bmatrix}
\frac{di_a}{dt} \\
\frac{di_b}{dt} \\
\frac{di_c}{dt} \\
\frac{d\theta_a}{dt} \\
\frac{d\theta_b}{dt} \\
\frac{d\theta_c}{dt}
\end{bmatrix} =
\begin{bmatrix}
-R_p & 0 & 0 & -f_a(\theta)eK_v & 0 & 0 \\
0 & -R_p & 0 & 0 & -f_b(\theta)eK_v & 0 \\
0 & 0 & -R_p & 0 & 0 & -f_c(\theta)eK_v \\
0 & 0 & 0 & -R_p & 0 & 0 \\
0 & 0 & 0 & 0 & -B_m & 0 \\
0 & 0 & 0 & 0 & 0 & -B_m
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{L_p} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{L_p} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{L_p} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{L_p} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{-1}{J_m} & -1 \\
0 & 0 & 0 & 0 & 0 & \frac{-1}{J_m}
\end{bmatrix}
\begin{bmatrix}
V_{ar} \\
V_{br} \\
V_{cr} \\
T_c \\
T_i \\
T_i
\end{bmatrix}
\] \tag{17}

4. Simulation Results and Discussion

The torque, speed, current and back EMF characteristics of 3PPMBLDC motor when driven by 3PVSI in the 120°, 150° and 180° QSPWM are investigated with the following motor parameters: \( V_{dc} = 36 \) volt, \( R_p = 0.5 \Omega \), \( L_p = 0.005 H \), \( p = 46 \) poles, \( J_m = 2 \text{ kgm}^2 \), \( B_m = 0.2 N \text{m/}(\text{rad/s}) \) and \( K_v = 2.45 \text{ volt/}(\text{rad/s}) \). The sampling frequency for PWM generation during the simulation is 1 kHz. The simulation duration is 5 seconds. The spectral plots of \( \omega_m \), and \( T_e \) are considered during the period in which the 3PPMBLDC motor is in the steady state (as observed from the time-domain simulated plots) of which the start time is from the 3rd second to the end of simulation. The fundamental frequency is set to 1 Hz with 2 cycles to cover the signals of interest until the end of the simulation. The maximum frequency of the spectrum is 500 Hz.

Table 5. Parameters for simulations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC link power supply</td>
<td>36</td>
<td>Volt</td>
</tr>
<tr>
<td>Phase winding resistance (( R_p ))</td>
<td>0.5</td>
<td>Ohm</td>
</tr>
</tbody>
</table>
From Fig. 10, 16, and 22, it can be seen that with the same 100% duty cycle, driving 3PPMBLDC motor with 150° conduction mode yields the highest steady state $\omega_m$ of 11.52 rad/s. This is because each QSV in the 150° conduction mode stays closer to the next one by an amount of $\pi/6$ rad that produces the rotating magnetomotive force which is aligned much closer to the right angle to the rotor magnetomotive force, resulting in a better tangential torque acting upon the rotor.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig10.png}
\caption{$\omega_m$ plot when we use a 3PVSI in the 120° conduction mode and 100% duty cycle in CCW direction.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig11.png}
\caption{Spectral plot of $\omega_m$ when we use a 3PVSI in the 120° conduction mode and 100% duty cycle in CCW direction.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig12.png}
\caption{$T_e$ plot when we use a 3PVSI in the 120° conduction mode and 100% duty cycle in CCW direction.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig13.png}
\caption{Spectral plot of $T_e$ when we use a 3PVSI in the 120° conduction mode and 100% duty cycle in CCW direction.}
\end{figure}
Fig. 14. \( i_a, i_b, i_c \) plot when we use a 3PVSI in the 120° conduction mode and 100% duty cycle in CCW direction.

Fig. 15. \( e_a, e_b, e_c \) plot when we use a 3PVSI in the 120° conduction mode and 100% duty cycle in CCW direction.

Fig. 16. \( \omega_m \) plot when we use a 3PVSI in the 150° conduction mode and 100% duty cycle in CCW direction.

Fig. 18. \( T_e \) plot when we use a 3PVSI in the 150° conduction mode and 100% duty cycle in CCW direction.

Fig. 19. Spectral plot of \( T_e \) when we use a 3PVSI in the 150° conduction mode and 100% duty cycle in CCW direction.

Fig. 20. \( i_a, i_b, i_c \) plot when we use a 3PVSI in the 150° conduction mode and 100% duty cycle in CCW direction.
Fig. 21. $e_a, e_b, e_c$ plot when we use a 3PVSI in the 150° conduction mode and 100% duty cycle in CCW direction.

Fig. 22. $\omega_m$ plot when we use a 3PVSI in the 180° conduction mode and 100% duty cycle in CCW direction.

Fig. 23. Spectral plot of $\omega_m$ when we use a 3PVSI in the 180° conduction mode and 100% duty cycle in CCW direction.

Fig. 24. $T_e$ plot when we use a 3PVSI in the 180° conduction mode and 100% duty cycle in CCW direction.

Fig. 25. Spectral plot of $T_e$ when we use a 3PVSI in the 180° conduction mode and 100% duty cycle in CCW direction.

Fig. 26. $i_a, i_b, i_c$ plot when we use a 3PVSI in the 180° conduction mode and 100% duty cycle in CCW direction.

Fig. 27. $e_a, e_b, e_c$ plot when we use a 3PVSI in the 180° conduction mode and 100% duty cycle in CCW direction.
Fig. 28. $\omega_m$ plots when we use a 3PVSI in the 120°, 150°, 180° conduction mode and 50% duty cycle in both CW and CCW directions.

Fig. 29. $T_e$ plots when we use a 3PVSI in the 120°, 150°, 180° conduction mode and 50% duty cycle in both CW and CCW directions.

On the other hand, the 150° conduction mode QSPWM produces a slightly higher \%THD of $\omega_m$. This is because there are more step transitions (12 QSVs for the 150° conduction mode QSPWM) in a rotation cycle. From Fig. 12, 18, 24, the maximum $T_e$ that can be achieved for the 150° and for the 180° conduction mode QSPWMs is almost the same and higher than in the 120° conduction mode QSPWM. In addition, the 180° conduction mode QSPWM produces the lowest \%THD of $T_e$ whereas the 120° conduction mode QSPWM produces the highest \%THD of $T_e$ because each stator phase winding in the 120° conduction mode QSPWM is idle for at least $2\pi/3$ in each switching cycle as can be seen in Fig. 3 when $V_{an} = 0$, causing an abrupt increase in the value of $T_e$ while each stator winding in the 180° conduction mode QSPWM is always energized, thus avoiding sudden changes in the value of $T_e$.

5. Conclusions

This paper proposes three modulation schemes for a conventional 3PVSI that require less computation and are suitably acceptable for low precision applications such as electric bike motor drive and quad rotor robot motor drive. They are based on the QSVs. The appropriate QSV closest to the right angle to the rotor position is selected as the switching pattern to follow as in Tables 2, 3, 4 depending on the conduction mode selected. It is clear from the simulation results that driving 3PPMBLDC motor by 3PVSI with the 150° conduction mode QSPWM yields the highest $\omega_m$ with a compromisable \%THD. In addition, the maximum $T_e$ produced by this conduction mode QSPWM is almost the highest. Therefore, in terms of $\omega_m$, $T_e$ and \%THDs, it is promising to choose the 150° conduction mode QSPWM as an alternative modulation scheme for 3PVSI driving 3PPMBLDC motor.

6. References


