Study of Surface Subsidence due to Underground Opening under Super-critical Condition Using Trap Door Apparatus

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Abstract

Physical model simulations have been performed to determine the effects of underground opening configurations on surface subsidence under super-critical conditions. This paper indicates the importance of the main factors that control the extent of surface subsidence and determines the effects of the geometry of underground openings on the angle of draw, the maximum subsidence and the volume of the subsidence trough. A trap door apparatus has been fabricated to perform the scaled-down simulations of surface subsidence. Gravel is used to represent the overburden in order to exhibit a cohesive frictional behavior. In plan view the excavation dimensions are sufficient to induce maximum possible subsidence. The findings can be used to evaluate the subsidence profile for tunnels and caverns in soft ground. The results show that the angle of draw and the maximum subsidence are controlled by the width (W), length (L), height (H) and depth (Z) of the underground openings. The width of the subsidence trough can be represented by the empirical relations. The relation between opening depth and subsidence trough developed by Rankin is in good agreement with the physical model results for deep openings \( Z/W = 2, 3 \) and \( 4 \). For shallower openings \( Z/W = 1 \), the predicted trough width is less than the physical model simulation. The volume of the subsidence trough is largest for \( Z/W = 2.5 \) and for \( H/W = 0.6 \), and is about 60\% of the volume of the underlying opening.

Keywords: Angle of draw; Tunnels; Subsidence trough; Mines.

1. Introduction

Surface subsidence as a consequence of underground mining and tunneling can impact the environment and surface structures within the mine area [1]. In order to minimize the environmental impact, a reliable subsidence prediction is essential. One key parameter for subsidence analysis and prediction is the angle of draw, which defines the limits of the area affected by subsidence. Determination of the extent of surface subsidence due to underground mining is important for deciding whether a particular structure is located within the subsiding area or not. The area affected by subsidence is controlled predominantly by

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geologic conditions in the overburden and by the mining geometry.

In practice, it is difficult to determine with certainty the precise extent of mining subsidence and hence the angle of draw. However, the angle of draw can be predicted by the observations of the underground openings assisted by analytical solutions, such as the profile function method [2], used to calculate the angle of draw from depth of the excavated opening and the boundary of the subsided area for sub-critical and critical subsidence. Yao et al. [3] introduced an analytical calculation model for the angle of draw by the use of a finite element model from Nottingham University [4]. Their results show that the angle of draw is related to the overburden properties, depth and configurations of the mine openings.

Physical modeling has played an important role in studies related to stability of underground mines and tunnels. A variety of modeling techniques have been developed all over the world to study ground response to underground excavation and tunneling. Terzaghi [5] uses a model, characterized as the trap-door model. According to this model, the deforming arch of a tunnel can be investigated by a downward moving trap-door while the soil above the tunnel can be represented by a layer of granular or slightly cohesive soil. The physical model allowed him to represent a case study and to determine it completely with a limited set of parameters.

Empirically derived relationships are one of the principal methods of predicting mining and tunneling subsidence. Mohammed et al. [6] compared the shape of the settlement trough caused by tunneling in cohesive ground by different approaches: analytical, empirical, and numerical. Their study showed that the finite element method overpredicted the settlement trough width compared with the results from empirical solutions of Peck [7] for soft and stiff clay, but are in excellent agreement with Rankin’s [8] estimation. Even though extensive study has been carried out in an attempt to understand and predict the surface subsidence behavior induced by underground excavations, the effects of opening geometry under super-critical condition have rarely been addressed. The difficulty in predicting the subsidence under super-critical condition is due to the complexity of the post-failure behavior of the overburden.

The objective of this study is to use a trap door apparatus to simulate the surface subsidence under super-critical condition.

Underground opening configurations (width, depth and length) will be varied. The investigation is focused on the angle of draw, maximum subsidence and volume of trough as a function of the opening geometry. In this paper, the results are obtained from the overburden simulated by using gravel (frictional material). The simulations are under super-critical conditions, i.e. in plan view the excavation dimensions are sufficient to induce maximum possible subsidence. The test results are compared with subsidence profile predictions obtained from empirical methods for tunnels in soft ground.

2. Trap Door Apparatus

A trap door physical model has been designed and developed to simulate subsidence of overburden in three dimensions and to assess the effect of the geometry of underground openings on the surface subsidence. The physical model (Fig. 1) comprises three main components: the sample container, the mine opening simulator, and the surface measurement system. The sample container is filled with materials, in this case gravel, used to simulate overburden. The testing space is 0.95×0.95×0.60 m$^3$. The mine opening simulator is an array of wooden blocks with the dimensions of 50×50×100 mm$^3$. The wooden blocks are arranged in ten columns with five blocks for each column. Fifty small blocks can be gradually and systematically moved down to simulate
underground openings with different geometries and hence inducing the subsidence of the gravel. The mine opening simulator is installed underneath the sample container. The measurement system of the surface subsidence includes a sliding rail with a laser scanner. To measure the surface subsidence under various underground opening geometries, the laser scanner is moved horizontally in two directions. The precision of the measurements is one micron. The results are recorded and plotted as three-dimensional profiles.

**Fig.1.** Trap door apparatus used for physical model testing.

### 3. Properties of Gravel

Clean gravel is used to simulate the overburden in the physical model. The gravel is well rounded with average particle size of 5 mm. The material is subjected to grain size analysis and direct shear testing. The grain size analysis is performed to determine the percentage of various size particles, and to classify the material (Fig. 2). The test method and calculation follow the ASTM (D422-63) [9] standard practice. To classify the gravel in accordance with ASTM (D2487-06) [10] the uniformity coefficient ($C_u$) and the coefficient of curvature ($C_c$) are determined as follows:

\[
C_u = \frac{D_{60}}{D_{10}} \quad (1)
\]

\[
C_c = \frac{D_{30}^2}{(D_{10} \times D_{60})} \quad (2)
\]

where $D_{60}$ is particle size at 60% finer, $D_{30}$ is particle size at 30% finer and $D_{10}$ is particle size at 10% finer. The uniformity coefficient is 1.62 and the coefficient of curvature is 1.34. The gravel is classified as poorly graded soil or GP.

The direct shear test is performed to determine the cohesion and friction angle of the gravel sample. A circular shear box with 190.5 mm diameter and 152.4 mm thick is used. The test method and calculation follow the ASTM (D5607-08) [11] standard practice. The constant normal stresses are 0.08, 0.16, 0.24, and 0.32 MPa. Each specimen is sheared once under the predefined constant normal stress using a direct shear device (SBEL DR44). The shearing rate is 0.02 MPa/s. The shear force is continuously applied until a total shear displacement of 8 mm is reached. The applied normal and shear forces and the corresponding normal and shear displacements are monitored and recorded.

**Fig.2.** Grain size distribution of tested gravel.

The peak and residual shear strengths as a function of the normal stress. The shear strength ($\tau$) is calculated based on the Coulomb’s criterion (Jaeger et al. [12]; section 4.5) as follows:
\[ \tau_p = c_p + \sigma_n \tan \phi_p \] for peak shear strength (3) 
\[ \tau_r = c_r + \sigma_n \tan \phi_r \] for residual shear strength (4)

where \( \sigma_n \) is the normal stress, \( c_p \) is the peak cohesion, \( c_r \) is the residual cohesion, \( \phi_p \) is the peak friction angle and \( \phi_r \) is the residual friction angle. The cohesion and friction angle for the peak shear strength are 39 kPa and 37°, and for the residual shear strength they are 23 kPa and 37°.

4. Physical Model Testing

For each series of simulations the sample container is filled with the clean gravel to a pre-defined thickness. The thickness of the gravel layer represents the opening depth or the thickness of overburden. The gravel is lightly packed and the top surface is flattened before beginning the test.

The underground opening is simulated by systematically pulling down the wooden blocks underneath the sample container. The opening width (W) can be simulated from 50 mm to 250 mm with an increment of 50 mm. The opening length (L) can be simulated from 50 mm up to 500 mm with 50 mm increments. All blocks equivalent to the predefined W and L are simultaneously moved down. The effect of the mining sequence is not investigated in this paper. The opening height (H) is selected from 10, 20, 30, 40, to 50 mm. The overburden thickness (Z) is varied from 50 to 200 mm (at 25 mm intervals). Fig. 3 shows the test parameters and variables defined in the simulations. While the underground opening is simulated, the settlement of the top surface of the gravel occurs. The laser scanner measures the surface profile of the gravel before and after the subsidence is induced. The effects of opening length (L) and opening height (H) are assessed by simulating the \( L/W \) from 1, 2, 3, 4 to 5 and \( H/W \) from 0.2, 0.4, 0.6, 0.8 to 1, where \( W = 50 \) mm. The effect of opening depth (Z) is investigated here by varying \( Z/W \) from 1 to 3 to 4. The simulation results are focused on the variation of the angle of draw, the maximum surface subsidence and the volume of the trough as affected by the opening geometry. Each opening configuration is simulated at least 3 times to verify the repeatability of the results.

![Fig.3. Variables used in physical model simulations and analysis.](image-url)
5. Results

The results are presented in terms of the angle of draw (\(\gamma\)) and the maximum subsidence (\(\delta_{\text{max}}\)). The angle of draw is a parameter used for defining the position of the limit of subsidence at the surface. The angle of draw is the angle between a vertical line from the edge of the underground opening and a line from the edge of the opening to the point of zero surface subsidence. The point of maximum surface subsidence is located in the center of the trough.

Fig. 4 shows the angle of draw as a function of the opening length-to-width ratio (\(L/W\)). The angle of draw increases with increasing \(L/W\) ratio and tends to approach a limit when \(L/W\) is beyond 3. This is probably because the effect of the opening ends decreases and eventually disappears when the \(L/W\) is beyond 3. Fig. 5 shows the angle of draw as a function of the opening height-to-width ratio (\(H/W\)). The results indicate clearly that the angle of draw increases with increasing \(H/W\) ratio. This is because under super-critical conditions, the material can collapse (flow) into the opening more and induce larger angle of draw and trough width. Under the same \(L/W\) and \(H/W\) ratios, increasing the \(Z/W\) ratio can reduce the angle of draw. This is due to that the settlement in the physical model has created new voids in the gravel (overburden) above the opening. The maximum subsidence-to-opening width ratio increases with increasing \(L/W\) ratios. The maximum subsidence however tends to be constant as the \(L/W\) is beyond 2 because the effect of the opening ends is reduced. The \(\delta_{\text{max}}/W\) ratio increases with increasing \(H/W\) ratio because the material can collapse into the opening more when the volume of opening becomes larger, particularly for \(Z/W = 1\). The \(\delta_{\text{max}}/W\) ratios tend to be independent of the opening length, when \(L/W\) is 2 or greater. The maximum subsidence tends to decrease as the opening depth increases.

![Fig. 4. Angle of draw (\(\gamma\)) as a function of the opening length-to-width ratio (\(L/W\)), where \(Z = 50-200\) mm, \(W = 50\) mm, \(H = 10-50\) mm and \(L = 50-250\) mm.](image)

This is because of the interlocking of the gravel particles above the opening. However, the magnitudes of maximum subsidence and the angle of draw are likely to vary for different overburden types.
The angle of draw and the maximum subsidence are related to the overburden properties, depth and configurations of the underground opening. It is postulated here that the maximum subsidence values and the angle of draw would be greater if finer gravels had been used in the simulation. This agrees with numerical simulation by Yao et al. [3].

The angularity and the particle size of the overburden can affect to the surface subsidence magnitude and volume. This is because of the interlocking of the gravel particles above the opening. The surface subsidence is less when particles of overburden have more angularity and larger particle size. This is identified by the test results of Sakulnitichai et al [13]. They found the maximum span increases with decreasing joint spacing \((S_V:S_H)\) ratio where \(S_V\) is the vertical joint spacing and \(S_H\) is the horizontal joint spacing. This means that the subsidence magnitude and the volume reduce for a larger block sizes.

### 6. Empirical Subsidence Calculations

The empirical method given by Peck [7] is used to predict the subsidence trough profile. Results obtained from this empirical method are compared with the physical simulation results.

Peck’s equation representing the assumed trough shape is

\[
\delta = \delta_{\text{max}} \exp \left( -\frac{x^2}{2i^2} \right) 
\]

where \(\delta\) is the surface settlement, \(\delta_{\text{max}}\) is the maximum vertical settlement, \(x\) is the transverse distance from the tunnel centerline, and \(i\) is a measure of the width of the settlement trough, defined as the distance from the center to the point of inflection of the curve (corresponding to one standard deviation of the normal distribution curve), and is determined by the ground conditions. The solution that has been proposed by Rankin [8] for calculating the trough width at an inflection point \((i)\) is

\[
i = k \cdot Z 
\]

where \(k\) is a dimensionless constant, depending on soil type \((k = 0.5\) for clay; \(k = 0.25\) for cohesionless soils), and \(Z\) is the depth of the opening.
Peck [7] proposes that depth of the opening $Z_c$ is measured from the gravel surface to the mid-height of the opening. However, it is found here that a closer agreement between the test results and the predictions was obtained if the depth is measured to the roof of the opening. As a result, this study will consider $Z_r$ as the depth to the opening roof.

Rankin [8] provide empirical solutions for “$i$” for both cohesionless material and cohesive soil. His solution is used here to compare with the settlement profiles obtained from the physical model simulations. The results indicate that the solution for cohesionless soils is in good agreement with the measurement results. Fig. 6 shows the settlement trough profiles predicted by the solution of Rankin [8]. The good agreement is obtained for $Z/W = 2, 3$ and 4 while for $Z/W = 1$, the predicted trough widths are less than the physical model results, as shown in Fig. 7.

7. Volume of Surface Settlement

For a single tunnel, the volume of surface settlement for the individual tunnel is assumed equal to the volume of lost ground. This concept was used by Peck [7] to correlate field measurements of trough width for several cases. In all cases in the calculations, the ground surface is assumed at the bottom of the building footing and the influence of building footing and building stiffness is ignored. The volume of subsidence trough per unit length ($V_s$) is obtained from

$$V_s = 2.5i \cdot \delta_{\text{max}} \text{ (Peck [7])}$$

(7)

where $\delta_{\text{max}}$ is the maximum vertical settlement and $i$ is the width of settlement trough, defined as the distance from the center to the point of inflection of the curve, which is obtained from Rankin’s solution. An attempt is made here to determine the effects of opening depth and height on the volume of the subsidence trough. Fig. 8 plots the subsidence trough volume normalized by the opening volume as a function of opening depth-to-width ratio ($Z/W$) and opening height-to-width ratio ($H/W$). The physical model results show clearly that the trough volume is less than the opening volume. This holds true for all opening depths used here ($Z/W = 1$ to 4). The largest trough volume is obtained for $H/W = 3$ and $Z/W = 1$, which is about 60% of the opening volume.

The subsidence trough volume tends to decrease as the opening depth increases, particularly for short opening ($L/W = 2$). This observation disagrees with the concept proposed by Peck [7]. This may be explained by the fact that the gravel particle sizes used here are relatively large (10-20% of the opening width), compared to some actual cases. It is postulated here that the subsidence trough volume-to-opening volume ratio ($V_s/V_o$) would be greater if finer gravels had been used in the simulation.

Fig. 6. Comparison of the model simulation subsidence trough and the troughs calculated by different empirical formulae, where $Z_r = 200 \text{ mm}$, $W = 100 \text{ mm}$, $H = 50 \text{ mm}$ and $L = 200 \text{ mm}$. 

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Fig. 7. Surface settlement troughs for different values of $Z/W$, where $H = 50$ mm and $L = 250$ mm.

Fig. 8. Volumetric ratio ($V_s/V_o$) as a function of opening depth ratio (a) and opening height ratio (b).

8. Conclusions

This paper focuses on the prediction of surface subsidence induced by underground openings. The surface subsidence has been estimated using physical models and empirical calculations. From the above analysis the following conclusions can be drawn:

The physical model test results clearly indicate that the angle of draw and the maximum subsidence are controlled by the geometrical characteristics of underground openings and by overburden thickness. The
extent of the mining subsidence affected area is defined by the limit angles, which are controlled predominantly by geological conditions of the overburden strata and the mining configurations.

The angle of draw and maximum subsidence increase with increasing $L/W$ ratio and tends to approach a limit when $L/W$ equals 3. For the same underground opening geometry ratio, increasing the $Z/W$ ratio reduces the angle of draw.

The empirical solution of Rankin for cohesionless soil gives good predictions for calculating the settlement trough width $i$ when compared with the results of the physical model simulations. The results show that there is good agreement for $Z/W = 2, 3$ and 4, while for $Z/W = 1$, the curve of Rankin’s solution is smaller than the physical model simulation.

To evaluate the width of the settlement trough, using the distance $Z_r$ from the surface to the roof of the underground opening gave better predictions than using the distance $Z_c$ from the surface to the center of the tunnel.

The volume of subsidence trough observed from the physical model is always less than the opening volume. The maximum trough volume is about 60% of the opening volume, which is observed for $Z/W = 2.5$ and for $H/W = 0.6$. The subsidence trough volume decreases rapidly as the opening depth ratio ($Z/W$) increases beyond 3.

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10. References


