A Numerical Study of Gas Jets in Confined Swirling Air Flow

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ABSTRACT

A numerical investigation of gas jets in confined swirling air flow is presented. Computations were performed using a simplified algebraic Reynolds stress model (ASM). The $k$-$\varepsilon$ turbulence model was also employed in the present simulation for comparison. Emphasis is given to the derivation of an algebraic Reynolds stress model (ASM) for 2D axisymmetrical flows, particularly to the model constants in the algebraic Reynolds stress equations. The code, based on a staggered finite volume approach with the standard $k$-$\varepsilon$ model and first-order numerical schemes built-in, is used to carry out all the computations. The code has been modified in the present work to incorporate the ASM and the second-order numerical schemes. For such a flow compared, the predicted results of gas tangential and axial velocities based on the ASM model are in fairly good agreement with the measurements. The computation shows that the ASM is superior to the $k$-$\varepsilon$ model in capturing the swirling flow fields.

Keywords: ASM, confined swirling air flow, $k$-$\varepsilon$ turbulence model, swirl flow.

1. INTRODUCTION

Swirl flows have been of considerable interest over the past decades because of their occurrence in industrial applications, such as furnaces, utility boilers, internal combustion engines, gas turbine combustors and dust collectors [1-4]. Swirl has been used in combustion systems to enhance the flame stability, the mixing and heat transfer besides prolonging the fuel residence time and abating the pollutants. This is because under appropriate conditions, swirl can be employed to induce a central recirculation zone. The recirculating flow generates additional turbulence in the shear layer between the reverse flow and the surrounding forward flow and helps stabilize the flame in combustors. Swirling turbulent flows are physically complex in nature due to the effect of a swirl-turbulence interaction. The turbulence structure in swirling turbulent flows is generally highly non-isotropic and non-homogeneous.

Computation of swirling flows is a difficult and challenging task. Large velocity gradients appear in these flows, so numerical problems and turbulence modeling play a significant role in their analysis. The commonly used, the $k$-$\varepsilon$ model may not be
suitable for simulating swirling turbulent flows [1]. It is also found that the use of modified $k$-$\varepsilon$ models or even the non-linear $k$-$\varepsilon$ model [5] leads to no significant improvement of the predictions in swirling flows. The second-order moment closure models, i.e., the Reynolds stress model (RSM) and the algebraic Reynolds stress model provide better methods for the simulation of swirling turbulent flows [3, 6, 7], but sometime, the original ASM based on Rodi’s approximation [8] cannot give satisfactory results for certain aspects of swirling flows [5]. The RSM is regarded as a most logical approach to the turbulence closure problem, which does not need any ad hoc modification for extra strain rates. However, in the prediction of swirling flows with the RSM, it is necessary to solve a total of 11 governing differential equations of elliptic type: a continuity equation, three momentum equations, an $\varepsilon$-equation, and six equations for the Reynolds stresses. This leads to much extra computational effort to solve six Reynolds stress transport equations simultaneously [3, 6, 7] and much attention needs to be paid to numerical stability and inlet boundary conditions. It is for this reason that a simplified algebraic Reynolds stress model in axisymmetric cylindrical co-ordinates is employed for simulating strongly swirling flows.

This section provides a review of recent computational work in two or three-dimensional swirling turbulent flows with higher-order turbulence models. Generally, such flows are complex since swirl introduces intense azimuthal streamline curvature and the strong curvature-turbulence interaction affects all six independent stress components. Therefore, higher-order turbulence closure is expected to be significantly advantageous with such flows. Naot and Launder [9] predicted a strongly swirling flow and one with weak swirl using the Quasi-isotropic stress model of Launder et al. [10], but comparisons with eddy-viscosity model results were not reported. Launder and Morse [11] who used the same model found that the predicted stress $\overline{\rho u w}$ was predominantly negative in contrast to the experimental findings. For weak swirl, $\overline{\rho u w}$ had an insignificant direct effect on the mean flow via the associated stress gradient in the momentum equation. The term $\overline{\rho u w}$ is always coupled to the major stress $\overline{\rho u v}$, in the production term, $\overline{\rho u w w/r}$, and the incorrect prediction of the sign of $\overline{\rho u w}$ meant an erroneous representation of the spreading rate. Launder et al. recognized that the root of this defect lay in the pressure strain model used. They demonstrated this by reducing the coefficient of the rapid term of the pressure-strain term by 40%, thereby obtaining broad agreement with experiment.

Boysan et al. [1] modelled a swirling two-phase flow in a cyclone separator with an algebraic stress model (ASM) and a skew upstream differencing scheme for the convection terms. They reported that grade-efficiency curves obtained with a stochastic particle tracking technique showed good agreement with experiment. Sloan et al. [12] calculated swirling and re-circulating flows using an algebraic Reynolds stress model (ASM) as well as the standard and modified $k$-$\varepsilon$ models. Many swirling flows such as those of Brum and Samuelsen [13], Yoon [14], Yoon and Lilley [15], Roback and Johnson [16], and Vu and Gouldin [17] were predicted with a hybrid numerical scheme. They reported that the predictions with the modified and standard $k$-$\varepsilon$ models were poor and the ASM was marginally better than the $k$-$\varepsilon$ models. Nallasamy [18] evaluated the application of turbulence models to internal flows by examining the predictions in selected flow configurations. It was found that modified versions of the $k$-$\varepsilon$ model led to improve performance in 2D flows with streamline curvature and heat transfer. In flows with swirl, the $k$-$\varepsilon$ model failed to predict correctly the flow properties and the ASM performed better. The ASM also performed well in flows with regions of secondary flow (noncircular duct flows). Following Launder and Morse, Gibson and Younis [19] predicted Morse’s
swirling jet with the Isotropisation of Production model of Gibson and Launder [20] but with different values of the coefficients $C_1$ and $C_2$ and found a major improvement over the calculations of Launder and Morse. Fu et al. [21] who modelled the strongly swirling flow of Sislian and Cusworth [22], found, however, that the modifications by Gibson and Launder led only to a modest improvement in the solution obtained with the earlier values of the constants.

The swirling flow of Sislian and Cusworth, previously predicted by Fu et al., was also calculated by Kim and Chung [23], with an algebraic Reynolds stress model (ASM). They reported that the ASM predicted the experimental values of axial and tangential velocity distributions better than the standard $k$-$\varepsilon$ model. The $k$-$\varepsilon$ model generally overestimated the maximum velocity and underestimated the spreading rate of the jet. This result seems to contradict the finding of Fu et al. [24] who also used the ASM and the IP stress model of Gibson and Launder to compute the free jets of Ribeiro and Whitelaw [25] and Sislian and Cusworth. Fu et al. did their computations on a $40 \times 35$ grid using the QUICK scheme for convective transport. They recommended that the ASM should not be used in axisymmetric swirling flows where stress-transport processes give rise to significant terms in the all Reynolds-stress components. However, Weber et al. [26], computed confined swirling flows, employing three models of turbulence: Reynolds stress model (RSM), ASM and the $k$-$\varepsilon$ model with the QUICK scheme and found no substantial differences between the RSM and ASM predictions. Cho and Fletcher [27] investigated numerically a turbulent swirling flow in conical diffusers using the $k$-$\varepsilon$ model and an ASM with the QUICK scheme and a non-orthogonal grid system and reported that the ASM predicted the mean velocities more accurately and yielded better predictions of the Reynolds stresses than the $k$-$\varepsilon$ model. Zhang et al. [28], Zhang and Nieh [29], and Nieh and Zhang [30] predicted strongly swirling turbulent flows in a vortex combustor using a simplified version of algebraic Reynolds stress model (ASM) and reported that the predictions of tangential and axial velocities of the gas were in good agreement with their measurements. Comparisons of the predicted mean dynamic properties of strongly swirling flows demonstrated the superiority of the model over the $k$-$\varepsilon$ model.

The main objective of the present investigation is to evaluate the capability of the $k$-$\varepsilon$ turbulence model and the algebraic Reynolds stress model (ASM) to predict swirling aerodynamic field behaviors of gas jets in confined swirling air flow of So et al. [31]. In the present study, numerical computation is carried out using a second order turbulence model together with a second order numerical scheme.

2. MATHEMATICAL FORMULATION

2.1 Governing Equations

The governing equations for constant density, isothermal flows consist of the conservation of mass and momentum. The time-averaged incompressible Navier-Stokes equations in the Cartesian tensor notation can be written in the following form:

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0$$

$$\frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (\tau_{ij} + \tau_{ij})$$

where $\tau_{ij}$ is laminar viscosity. The time-averaged Reynolds stress tensor, $\tau_{ij}$ in the above equation is not known and thus, models are needed to assign values to it or to express it in terms of the solution variables. In the present study, two turbulence closure models are used, namely the standard $k$-$\varepsilon$ model and an algebraic stress model or an algebraic second moment closure (ASM). The $k$-$\varepsilon$ model has already been reviewed in
many references [12, 32, 33] and it will be described only briefly.

In the standard \( k-\varepsilon \) model the Reynolds stress is linearly related to the mean rate of strain by a scalar eddy viscosity. The standard version relates the turbulent eddy viscosity to the turbulence kinetic energy \( k \) and the dissipation rate \( \varepsilon \) through Boussinesq’s approximation as:

\[
\tau_{ij} = -\frac{2}{3}\delta_{ij}(\rho k) + \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

where \( \mu_t = \rho C_\mu \frac{k^{2/3}}{\varepsilon} \) is the turbulent eddy viscosity and \( \varepsilon \) is the dissipation rate of turbulence kinetic energy (TKE).

The modelled equation of the turbulence kinetic energy (TKE) \( k \) is given by:

\[
\frac{\partial}{\partial t}(\rho u_ik) = \frac{\partial}{\partial x_j} \left( \mu_t \frac{\partial k}{\partial x_j} \right) + G - \rho \varepsilon
\]

Similarly the dissipation rate of turbulence kinetic energy is given by the following equation:

\[
\frac{\partial}{\partial t}(\rho u_i \varepsilon) = \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{\varepsilon}{k} \left( C_{\varepsilon 1} G - C_{\varepsilon 2} \rho \varepsilon \right)
\]

in which \( G \) represents the rate of generation of turbulent kinetic energy while \( \rho \varepsilon \) is its destruction rate. \( G \) is given by:

\[
G = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}
\]

The boundary values for the turbulent quantities near the wall are specified with the wall function method [28].

2.2 Algebraic Reynolds Stress Model (ASM)

The ASM is derived from the Reynolds stress transport equations that relate the individual stresses to the mean velocity gradients, the turbulence kinetic energy \( k \) and its dissipation rate \( \varepsilon \) by way of algebraic expressions that will be described below in some detail. The ASM is obtained by applying certain assumptions to the Reynolds-stress transport equations to allow the convection and diffusive gradient terms to be represented by non-gradient terms. For simplicity in solving the six Reynolds stresses, Rodi’s approximation is used in this study and the Reynolds stress transport can be expressed in algebraic form as follows:

\[
\frac{D\tau_{ij}}{Dt} - T_{ijk,k} = \frac{\tau_{ij}}{\rho k} \left( \frac{Dk}{Dt} - T_{ik,k} \right)
\]

Substitution of equations (5) and (8) into equation (9) gives the desired algebraic expression for \( \tau_{ij} \) as:

\[
-G_{ij} - \Phi_{ij} + \frac{2}{3} \delta_{ij} \rho \varepsilon = \frac{\tau_{ij}}{\rho k} \left( G - \rho \varepsilon \right)
\]

Rodi’s approximation is appropriate only in the Cartesian co-ordinate system and may
not be completely valid under co-ordinate transformation. Boysan et al. [2] suggested that some of the additional non-gradient convective stress terms that arise from the transformation to the cylindrical co-ordinate system might be large relative to the gradient convective stress terms in swirling flow systems. They recommended that the non-gradient convection terms, which are functions of the mean tangential velocity in cylindrical co-ordinates, should be retained or added in the model as extra production terms. The ASM expressions can thus be written as:

\[
\frac{\rho u_i u_j}{\rho_k} (G - \bar{\rho} e) + \psi A_{ij} = G_{ij} + \Phi_{ij} - \frac{2}{3} \delta_{ij} \rho e
\]  

(11)

where \( A_{ij} \) is the “added” convection quantity which has been excluded from Rodi’s approximation and its associated coefficient, \( \psi \), is an arbitrary constant between zero and one. The term \( A_{ij} \) could be

\[
A_{ij} = \begin{bmatrix}
0 & \frac{\rho u_i w}{r} & \frac{\rho v_i w}{r} \\
-\frac{\rho u_i w}{r} & -2 \rho v_i w & \left( \frac{\rho v_i v - \rho u_i w}{r} \right) \\
-\frac{\rho v_i v}{r} & \left( \frac{\rho v_i v - \rho u_i w}{r} \right) & 2 \rho v_i w
\end{bmatrix} \frac{w}{r}
\]  

(12)

Thus,

\[
\left( \frac{\rho u_i u_j - (2/3) \delta_{ij} \rho k}{k} \right) = \frac{\lambda}{\varepsilon} \left( G_{ij} - (2/3) \delta_{ij} G - \beta A_{ij} \right)
\]

or

\[
\rho u_i u_j = \frac{2}{3} \delta_{ij} \rho k + \frac{\lambda \varepsilon}{\varepsilon} \left( G_{ij} - \frac{2}{3} \delta_{ij} G - \beta A_{ij} \right)
\]

(13)

where the empirical constants \( \lambda \) and \( \beta \) are defined as

\[
\lambda = \frac{1 - C_2}{C_1 - 1 + \frac{G}{\rho e}} \quad \text{and} \quad \beta = \frac{\psi}{1 - C_2}
\]  

(14)

Equation (13) provides an algebraic expression for each of the six Reynolds stresses. These six simultaneous stress-equations are solved along with the equations of turbulence kinetic energy and its dissipation rate. The first implementation of an implicit form of the algebraic relationships conducted by Rodi led to a non-constant coefficient \( C_\mu \) in the eddy-viscosity definition which is a function of the turbulent production and dissipation rate. The implicit nature of the algebraic relationship requires that at each iteration step, an additional inner iteration is needed to solve for the appropriate \( \tau_{ij} \) components. This iterative process can cause solution divergence, and increases numerical overhead because of the extensive matrix inversions required at different iteration levels [32]. The above implicit ASM expressions can be simplified to obtain an explicit set that can be solved easily, as proposed by Zhang et al. and Zhang and Nieh, for application to a strongly swirling flow, e.g. a cyclonic flow. The details of this are given in the following section.

2.3 Simplified ASM for 2D Axisymmetric Flows with High Swirl

Experimental studies have shown that, in an axisymmetric flow with high swirl, generally \( w >> v, u \gg v \) and \( \partial / \partial r >> \partial / \partial x \) in almost the entire flow field. Hence, the terms containing \( \partial u / \partial x, \partial v / \partial x, \partial w / \partial x, \partial v / \partial r \) and \( v/r \) are negligible when compared with the terms containing \( \partial u / \partial r, \partial w / \partial r \), and \( w/r \). The original algebraic equations [12] can therefore be simplified in an explicit form as shown in equation (15) below.

The turbulence kinetic energy and dissipation equations retain the same form as the parallel two-equation model based on Boussinesq hypothesis but different expressions are used for the generation term of the \( k \)-equation.
\[
\rho u v = -\mu_v \frac{\partial u}{\partial x}
\]

\[
\rho w w = -\mu_r \frac{\partial r}{\partial r}
\]

\[
\rho w w = -\lambda \frac{k}{\varepsilon} \frac{\partial w}{\partial x} + \lambda \rho \frac{k}{\varepsilon} \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]
\]

\[
\rho u w = 2 \left( \frac{k}{\varepsilon} + \frac{\kappa}{\varepsilon} \right) \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right]
\]

\[
\rho v w = 2 \left( \frac{k}{\varepsilon} + \frac{\kappa}{\varepsilon} \right) \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right]
\]

\[
\rho w w = 2 \left( \frac{k}{\varepsilon} + \frac{\kappa}{\varepsilon} \right) \left[ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right]
\]

in which

\[
\mu_{r,v} = \frac{b_1 - a_1 b_2}{1 - a_1 a_2} \frac{\partial u}{\partial x}
\]

\[
\mu_{r,\theta} = \frac{b_2 - a_2 b_1}{1 - a_1 a_2} \frac{\partial u}{\partial r}
\]

where

\[
a_1 = \frac{1}{1 + \left( \frac{\lambda}{\varepsilon} \right) \left[ \frac{7}{3} + \beta \right]} \left[ \frac{7}{3} + \beta \right] \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right] + \left( 1 + \beta \right) \frac{w}{r} \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right]
\]

\[
a_2 = \frac{2/3}{1 + \left( \frac{\lambda}{\varepsilon} \right) \left[ \frac{7}{3} + \beta \right]} \left[ \frac{7}{3} + \beta \right] \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right] + \left( 1 + \beta \right) \frac{w}{r} \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right]
\]

\[
b_1 = \frac{1}{1 + \left( \frac{\lambda}{\varepsilon} \right) \left[ \frac{7}{3} + \beta \right]} \left[ \frac{7}{3} + \beta \right] \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right] + \left( 1 + \beta \right) \frac{w}{r} \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right]
\]

\[
b_2 = \frac{2/3}{1 + \left( \frac{\lambda}{\varepsilon} \right) \left[ \frac{7}{3} + \beta \right]} \left[ \frac{7}{3} + \beta \right] \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right] + \left( 1 + \beta \right) \frac{w}{r} \left[ \frac{w}{r} + \frac{2}{3} \frac{\partial w}{\partial r} \right]
\]

The model constant, \( \lambda \), was found to be 0.135. If \( C_s \) is taken as 0.55 and \( y \) varies between 0.0 and 1.0, equation (14) gives the value of \( \beta \) between 0.0 and 2.2.

### 2.4 Common Form for the Equations

All the governing partial differential equations can be re-organised and expressed in a standard form that includes the convection, diffusion, and source terms for 2-D axisymmetric flows as follows:

\[
\frac{\partial}{\partial x} \left( \rho u \frac{\partial \phi}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho u \frac{\partial \phi}{\partial r} \right) - \frac{\partial}{\partial r} \left( r \Gamma \frac{\partial \phi}{\partial r} \right) = S_\phi
\]

where \( \phi, \Gamma, \Gamma, \) and \( S_\phi \) represent the generalised variables, the exchange coefficients, the convection terms in x and r directions, and the source terms, respectively.

### 3. Solution Procedure

In the present computation the time-averaged Navier-Stokes equations, equation (1), the TKE equation, equation (5); the TKE dissipation rate equation, equation (6); and the averaged Navier-Stokes equations, equation (11) and (12); the TKE equation, equation (13); and the source terms for swirling flows, lower order non-linear and coupling features of the governing equations for swirling flows, lower underrelaxation factors ranging from 0.005 to 0.1 are chosen for the three velocity components to ensure that the stability and convergence of the iterative calculation. About 10,000 iterations were needed to achieve satisfactory convergence for each calculation case, which requires about 3 hr of computer time.
Boundary Conditions

**Inlet:** Experimental data at the entry are used [31].

**Outlet:** Conditions at the exit plane are usually not known beforehand and it is assumed that in case of the exit plane normal to the \( x \) direction, the axial gradients of all variables except \( u \) are zero here. Values of \( u \) at the exit are initially assumed to be the same as those immediately upstream of the exit plane and subsequently scaled appropriately to satisfy overall mass conservation. Hence, at the exit plane it may be written as

\[
\left( \frac{\partial \phi}{\partial x} \right)_{\text{exit}} = 0.0
\]

**Axis of symmetry:** At the axis of symmetry, the radial and tangential components of velocity, and radial gradients of other variables are set to zero.

**Wall:** At a wall, the no slip condition is applied and the values of \( u, v \) and \( w \) are set to zero. The flow near the wall is influenced by molecular viscosity rather than by turbulence. The wall function method of Patankar and Spalding (1970) which uses algebraic formulations to link the quantities at the wall to that further away.


The strongly swirling, confined, constant density flow of So et al. [31] was chosen to validate the \( k-\varepsilon \) model and the ASM because this flow was comprehensively mapped and documented, particularly in relation to turbulence quantities. Figure 1 shows the flow arrangement with a strongly swirling outer airflow (swirl number, \( S = 2.25 \)) directed into a uniform duct of radius \( R = 62.5 \text{ mm} \) with a small central nonswirling air jet of diameter \( d_j = 8.7 \text{ mm} \). The purpose of this jet was to delay the onset of reverse flow along the centerline beyond the measurement section that extended to \( 40d_j \). Without this jet, a reverse flow reflecting a 'vortex breakdown', was found to occur at \( 12d_j \) from the inlet. Measurements using laser doppler velocimetry (LDV) were obtained at 10 axial stations.

5. RESULTS AND DISCUSSION

The purpose of this study was to examine the interaction between swirl-induced curvature and turbulence, but with reliable data in a strongly swirling flow. Since the swirl was extremely strong, the swirl-related stresses and strain were likely to contribute strongly to the overall flow behavior. Before discussing the predictions, it is instructive to draw attention to some peculiar features of the flow resulting from the intensive swirling motion. Confined swirling flows are classified into 'super-critical' (no reverse flow at exit) and 'sub-critical' (reverse flow at exit) flows. The former is primarily governed by upstream processes while the latter is highly sensitive to perturbations far downstream. The difference between these two types of the flows was demonstrated by experimental studies of the influence of an outlet constriction which showed that supercritical flows with low swirl intensity responded minimally to changes in exit conditions while the flow over the entire duct length in highly swirling sub-critical flows responded strongly to geometric perturbations at the exit. As a consequence, Hogg and Leschziner [36], and Jones and Pascau prescribed the \( u \)-profiles at the exit (\( x/d_j = 40 \)) as well as at the inlet explicitly, while zero-gradient conditions were applied to all other properties. In the present computation, zero-gradient conditions were applied to all properties but the location of the exit was extended to \( x/d_j = 115 \) or about \( 1 \text{ m} \) from the inlet to ensure that no negative (reversed) flow existed at the exit. However, it appears that the subcritical phenomenon as described above was not found in this prediction.

A 60x40 grid with non-uniform distribution in the \( x \)-direction was used and found to be grid-independence from the preliminary study by using double the grid. Hogg and Leschziner, and Jones and Pascau also used 24x24 and 50x33 grid points in the calculation of this flow, respectively. In order to reduce uncertainties in the inlet profiles of the mean
Figure 1. Flow arrangement with air jet in ducted swirling flow.

Figure 2. Sensitivity of different exit locations to (a) axial and (b) tangential velocity profiles.
flow field, the inlet boundary conditions were specified at \( x/d_j = 1.0 \) for which experimental data was available, apart from the radial velocity \( v \) which is derived from the continuity equation. The turbulent kinetic energy was obtained by a sum of the mean square of fluctuations, \( \frac{1}{2} (u'^2 + v'^2 + w'^2) \). The rate of energy dissipation at inlet was estimated using the relationship similar to that used for the flow of Roback and Johnson. The length scale was chosen to be 0.01-0.5 times of the vortex tube radius, and the sensitivity of the results to this range will be presented later in this sub-section.

Calculated results and associated comparisons with experimental data are shown in Figure 2 to 6. Figure 2 conveys the sensitivity of the solution to the extended exit distance located at \( x = 1 \) m and \( x = 1.5 \) m. As anticipated, predictions with the two duct lengths are identical. This means that the assumption of zero-gradients for the flow properties at the exit was reasonable, and the exit location, \( x = 1 \) m, was used in the computations. The sensitivity of the computed solution to the chosen level of inlet dissipation was examined in view of the uncertainty about this level. A rational approach is to relate \( \varepsilon \) to the length scale since \( \varepsilon = k/\eta l \) in which \( l \) is varied in the range of 0.01R to 0.5R, R being the radius of the confining tube. A strong variation in \( \varepsilon \) is associated with corresponding changes in the level of \( u'^2 \), \( v'^2 \) and \( w'^2 \) (or \( k \)), which influence, more indirectly, the shear stresses and hence the mean velocity. Attention is restricted here to an examination of the sensitivity of the turbulence models to \( \varepsilon \) that is of primarily interest, as shown in Figure 3a and 3b for axial and tangential velocity profiles respectively. An increase in \( l \) led to a 50% increase in normal stresses. Therefore, as expected, the link between \( \varepsilon \) and the normal stresses are strong. The figures show that a suitable \( l \) is between 0.02R and 0.05R. The length scale used is less than that used in the Roback and Johnson case because of the higher swirl number so that the dissipation rate was higher (the lower the length scale the higher the heat dissipation rate). A value \( l = 0.03R \) is selected on the basis of the prediction of the decay of centreline axial velocities and used in the present computations. This level of \( l \) is not only observed to give close agreement with the data but is physically more realistic for both turbulence models.

The effect of inlet radial velocities on the solutions was also investigated. Since measured data of radial velocity were not provided, the inlet radial velocity used in the calculation was obtained using the continuity equation or alternatively by setting \( v = 0 \). The results obtained with the inlet radial velocity profiles from both the methods are shown in Figure 4a and 4b for axial and tangential velocity profiles respectively. It is seen that the results from inlet radial velocities adopted by the continuity equation yield better agreement with measurements than those with \( v = 0 \). As a consequence, the inlet radial velocity profile obtained using the continuity equation is employed throughout the calculations.

Predictions using the \( k-\varepsilon \) model and the ASM are compared with experimental data in Figure 5a and 5b. The radial profiles of axial velocity of Figure 5a indicate that the \( k-\varepsilon \) model predicts a fast decay of the centre-line velocity while the ASM slightly over-estimates the centre-line velocity. Generally, the ASM predictions of the axial velocity profiles are closer to the measurements than those with the \( k-\varepsilon \) model. The radial profiles of tangential velocity in Figure 5b show that the \( k-\varepsilon \) model leads to a rapid decay of the profiles to a solid-body rotation while the ASM gives slight under-prediction. Overall, the ASM performs better than the \( k-\varepsilon \) model. An examination of Figure 5a-5b and figures of other related results (not shown here) reveals that predictions with the ASM are as good as those of Hogg and Leschziner [36] and those with Jones and Pascau [7], both predicted using Reynolds stress models, despite the difference
Figure 3. Sensitivity of length scales to predicted velocity profiles, (a) axial velocity and (b) tangential velocity.
Figure 4. Effect of inlet radial velocities on (a) axial and (b) tangential velocity profiles.
Figure 5. Comparison between predicted velocity profiles and measurements, (a) axial velocity and (b) tangential velocity.
Figure 6. Comparison of centre-line axial velocity profiles with measurements.

in exit boundary conditions.

Figure 6 shows the axial variation of centreline axial velocity for the two turbulence models using the second-order upwind scheme (SOU). It is seen that the ASM results agree very well with measurements up to 4-5 jet diameters and are slightly overpredicted for further downstream values. The ASM predicts the trend of the data reasonably well while the $k$-$\varepsilon$ model fails to capture the trend of measurements. It is likely that predictions with the ASM would be improved significantly by the use of the correct value of length scale.

7. CONCLUSION

Numerical investigations have been carried out to predict of gas jets in confined swirling flow using the $k$-$\varepsilon$ model and the ASM and comparison with experimental data. The computations of the flows showed that the ASM performs better than the $k$-$\varepsilon$ model in capturing the mean flow behavior, superiority rooted in the response of the ASM to turbulence interaction because of higher-order turbulence model. The inlet conditions for $k$ and $\varepsilon$ played a crucial role in achieving accuracy in the predictions, apart from turbulence modelling details. Also, a suitable length scale should be used for predicting the swirling flows.

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**Nomenclature**

\(C_{\varepsilon 1}, C_{\varepsilon 2}\) constants in the dissipation rate equation

\(C\) convection term

\(C_{\mu}\) constant in the k-\(\varepsilon\) turbulence model

\(D\) pipe or tube diameter

\(d\) centre core jet diameter

\(G\) stress generation

\(k\) turbulence kinetic energy

\(l\) turbulence characteristic length scale

\(\overline{p}\) mean pressure

\(P_{\tau}\) production rate of stress component

\(r\) radial co-ordinate; radius

\(R\) radius of pipe or tube

\(t\) time

\(t_\alpha\) viscous stress tensor

\(S\) swirl number (inlet angular momentum/outlet axial momentum)

\(u_i\) fluctuating velocities in direction \(x_i\)

\(\rho u_i\) Reynolds stresses

\(\bar{u}\) time-averaged velocity in x-direction

\(\bar{v}\) time-averaged velocity in r-direction

\(\bar{w}\) time-averaged velocity in \(q\)-direction

\(\chi\) axial co-ordinate

**Greek Symbols**

\(\beta, \lambda\) turbulence model constants

\(\delta_{ij}\) Kronecker delta tensor

\(\varepsilon, \varepsilon_u\) dissipation, local dissipation tensor

\(\phi\) generalised dependent variable

\(\Phi_{ij}\) local pressure-strain or redistribution term

\(\Gamma_{\phi}\) exchange coefficient

\(\mu, \mu_{ij}\) dynamic viscosity, eddy-viscosity or turbulent viscosity

\(\rho\) density

\(\theta\) circumferential co-ordinate

\(\sigma_{\phi}\) Schmidt or Prandtl numbers for the scalar \(\phi\)

\(\tau_{ij}\) Reynolds stress tensor
Subscripts
$e$ effective
$t$ turbulence
$i,j,k$ Cartesian indices

Superscripts and Overbars
$\cdot$ fluctuating quantity in time-averaging
$\bar{}$ mean quantity