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Contributed Paper

The Semi-parametric Prediction in Nonlinear-autoregressive Model for Annual Ring Width of *Pinus eldarica*

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ABSTRACT

The aim of this research is to find the regularity in frequencies of diameter increment, and determine the relationship between tree age and fluctuations in radial increment of Pine trees (*Pinus eldarica*). In this direction, three normal pine trees from Kelardashat site (north of Iran) were selected. Disks were cut down at breast height, and the annual ring width in radial axis (1974-2008) was determined by Binocular system. The additive functional-autoregressive model is considered and semi-parametric method is proposed to forecast regression function. Furthermore, the method is applied for annual ring width prediction. The results showed that the nonlinear time series model is an efficient model for prediction of annual ring width.

Keywords: semi-parametric estimation, nonlinear time series models, conditional least squares method, annual ring width

1. INTRODUCTION

Due to increasing wood consumption and the development of pulp and paper production, plantations of fast growing tree species managed with short rotations have a growing importance for the sustainability of industrial wood raw material. The softwoods, especially fast growing pine species, have a higher priority for plantation forestry due to their adaptation capability to wide ecological conditions and various usage areas. Therefore, Parks and Forestry officials have imported approximately 48 foreign fast-grown softwood species into Iran since 1956 and planted them in different ecological conditions [1]. *Pinus eldarica* was one of the softwood species planted in many parts of Iran and has shown a good adaptation to environmental condition.

Dendrochronology (or tree-ring dating) is the scientific method of dating tree rings (also called growth rings) to the exact year they were formed in order to analyze atmospheric conditions during different periods in history. Dendrochronology is useful for determining the timing of events and rates of change in the environment (most prominently climate) and also in works of art and architecture, such as old panel paintings on wood, buildings, etc [2-5].

In recent years, a combination of parametric forms and nonlinear functions has been used to make a more efficient model in various branches of sciences, particularly in applied statistics, econometrics and climatology and forest studies. The nonparametric regression models with linear autoregressive error considered as the following form:

$$Y_t = g(X_t) + U_t \text{ with } U_t = \theta U_{t-1} + \varepsilon_t,$$

where (X{t},Y{t}) is stationary bivariate time series and θ is an unknown parameter such that |θ|<1. Also g(.) is an unknown function and {ε(t)} is a sequence of independent errors with zero mean and finite variance [6]. To estimate the above parameter, they used a semi-parametric method

From 1995 to 2005, an autoregressive conditional heteroskedasticity model, which is somehow with dependent errors, such as:

$$Y_t = g(Y_{t-2}) + \beta Y_{t-1} + e_t,$$

where {et} is assumed to be stationary and depends on Y(t-1), both β and g are identifiable, in econometrics was suggested [7, 8, 9, 10, 11].

The following functional-autoregressive model is proposed in this paper:

$$Y_t = \mu + f(Y_{t-2}) + \beta Y_{t-1} + e_t,\ t = 1, \ldots n,$$

Where {e{t} } is a sequence of independent and identity-distributed (i.i.d) random variables with mean zero and variance σ^2. Also e(t) and Y(t-1) are independent for each t and μ is constant.

We first suppose that f(.) has a parametric framework, namely a parametric model as

$$f(x) \in \{g(x, \theta); \theta \in \Theta\}.$$

Where Θ⊆R{p} is a parametric space and g(x,θ) is known, also f(x) and g(x,θ) are bounded and continuous with respect to x, away from 0 in a neighborhood of the point x

The regression function f(.) is estimated by f(x)=g(x,θ) where θ is an estimator of θ. In the next step we adjust the estimator f(x) by f(x)=g(x,θ)ζ(x)

In this paper, the estimator of the regression function f(x)=g(x,θ) is first regarded as a crude guess of f(x) .When this initial parametric approximation is adjusted by nonparametric multiplier ζ(x), we get the semi-parametric form g(x,θ)ζ(x

We use a combination of parametric method and nonparametric adjustment. The parameters and nonparametric adjustment are estimated by using conditional least squares method and smooth-kernel method respectively

We shortly explain the contents of the manuscript. In section 2, the annual ring width in radial axis 1974 to 2008 was determined. In section 3, the least squares estimation is considered to estimate parameters β, μ,σ^2 and θ. Also in this section the semi-parametric regression estimator is introduced by a natural consideration of the local L2-fitting criterion. In section 4, finally, this model is used to forecast the annual ring width data in radial axis (1974-2008). The data were determined by Binocular system in Kelardashat site (north of Iran).

2. MATERIALS AND METHODS

In this research, three normal *Pinus eldarica* trees were randomly selected from a plantation at Garagpas-Kelardasht site, which is located in the western part of the Mazandran province in the north of Iran. These trees have been formed for 35 years in this site. The annual rainfall and annual average of temperature (1974-2008 years) were 434.9 mm and 9.6 °C, respectively. October and November are high-rain months and June and July are low-rain months. The temperature in June, July and August reaches its maximum level. *Pinus eldarica* is mixed with *Pinus sylvestris*, *Pinus nigra* and *Picea abies* at the Garagpas-Kelardasht site. The Pinus eldarica trees were cut for this study in January 2009. A 5-cm-thick disc was removed from each tree at breast height level for evaluation of anatomical properties. Width of the every annual ring was measured using a Normal Binocular and Lintab 5 ring width measuring system (Rinnteech Company, Germany).

3. SEMIPARAMETRIC METHOD IN THE ADDITIVE NONLINEAR AUTOREGRESSIVE MODEL

We consider the following model:

$$Y_t = \mu + f(Y_{t-2}) + \beta Y_{t-1} + e_t,\ t = 1, \ldots n,$$

We want to estimate regression function f(x) that can be formed as g(x,θ) where g(x,θ) is known function of x and unknown parametr θ. It is clear that

$$E_{\beta,\theta,\mu}[Y_t | Y_{t-1}, Y_{t-2}] = \mu + f(Y_{t-2}) + \beta Y_{t-1} = \mu + g(Y_{t-2}, \theta) + \beta Y_{t-1}$$

For the model (2.1), θ , β and μ should be well estimated with conditional least squares errors method as follows

$$Q_n(\theta, \beta, \mu) = \sum_{j=2}^{n} \{(Y_t - \mu - g(Y_{t-2}, 0) + \beta Y_{t-1})^2\},$$

and

$$(\hat{\theta}_n, \hat{\mu}_n, \hat{\beta}_n) = \arg\min Q_n(\theta, \mu, \beta),\ \theta \in \Theta,\ |\beta| < 1.$$

In fact, θ{n} , β{n} and μ{n} are the common conditional least squares (CLS) estimators based on data, Y{n}

Now, we estimate the adjustment factor ζ(x) in f(x)=g(x,θ)ζ(x). So we obtain the estimator ζ(x) of ζ(x) by minimizing the object function

$$q(x, \xi) = \frac{1}{h_n} \sum_{j=2}^{n} k\left(\frac{Y_{i-2}}{h_n}\right)\{f(Y_{i-2}) - g(Y_{i-2}, \theta_n)\xi\}^2,$$

with respect to ξ(x.(

Here f(.) is unknown autoregression function. Also K(.) and h_{n} are kernel and bandwidth respectively. This kernel estimator is a special case of the local polynomial estimator which was proposed by Hardle and Tsybakov [11].

Therefore, we get a nonparametric estimator of ξ(x) as

$$\hat{\xi}(x) = \frac{\sum_{j=2}^{n}\left[K\left(\frac{Y_{j-2} - x}{h_n}\right) g(Y_{j-2}, \hat{\theta}_n) f(Y_{j-2})\right]}{\sum_{j=2}^{n}\left[K\left(\frac{Y_{j-2} - x}{h_n}\right) g^2(Y_{j-2}, \hat{\theta}_n)\right]},$$

Then the estimator of f(x) could be

$$\hat{f}(x) = g(x, \hat{\theta})\, \hat{\xi}(x)$$

Using

$$\sum_{j=2}^{n} K\left(\frac{Y_{j-1} - x}{h_n}\right) g(Y_{j-1}, \hat{\theta}_n) f(Y_{j-2}) \approx \sum_{j=2}^{n} K\left(\frac{Y_{j-1} - x}{h_n}\right) g(Y_{j-1}, \hat{\theta}_n)(Y_j - \mu - \beta Y_{j-1}),$$

One can obtain,

$$\tilde{\xi}(x) = \frac{\sum_{j=2}^{n}\left[K\left(\frac{Y_{j-2} - x}{h_n}\right) g(Y_{j-2}, \hat{\theta}_n)((Y_j - \mu - \beta Y_{j-1})\right]}{\sum_{j=2}^{n}\left[K\left(\frac{Y_{j-1} - x}{h_n}\right) g^2(Y_{j-2}, \hat{\theta}_n)\right]}$$

Finally, the auto-regression function estimator is

$$\tilde{f}(x) = g\,(x, \hat{\theta}).\ \tilde{\xi}\,(x)$$

The asymptotic behaviors of the estimator and consistency of regression function are investigated by Zhouxi .Y et al. [11] by using this paper it can be proved that: f(x)→f(x) as n→∞ and f(x)-f(x)→0 in probability.

4. EMPIRICAL APPLICATION

To illustrate the suitability of our methodology to the annual ring width data, we used an additive functional autoregressive model to forecast the annual ring width of Kelardasht site in the north of Iran, from 1974 to 2008. Using the presented semi-parametric method, we forecasted regression function. We computed the average squared error (ASE) for the efficiency of the proposed estimation method:

$$ASE = \frac{1}{n}\sum\{\tilde{f}(x_i) - f(x_i)\}^2.$$

The square root of ASE is denoted by MSE. Figure 2 shows the curves of observation (the annual ring width data in three trees of *Pinus Elderica*) from 1974 to 2008

The mean of annual ring width data and its semi-parametric estimator selected bandwidth respectively are shown in Figure 2. The red and blue lines are the curves of observation and the semi-parametric predictor respectively.

Table 1, respectively, shows the observed - forecasted values for the annual ring width data of kelardasht site in the north of Iran, from 1974 to 2008. The descriptive statistics of annual ring width of pine wood are shown in Table 2. There are significant differences between growth year and ring width, such that the annual ring width values were increasing by increasing age of tree (radial axis). The mean of annual ring width was 3.74 mm. The MSE and ASE values of regression function for the presented model are computed in Table 3. We see that the presented semi-parametric method for a functional autoregressive model is proved to be more efficient.
Therefore, the forecasted model is as

$$Y_t = 2.2001 + 2.79\exp(-Y_{t-2}) + .8011\ Y_{t-1} + e_t,$$

Such that the kernel was considered as Gaussian function and h(n)=.09011.

Ultimately, the autocorrelation of errors is checked (Figure 4). In this figure, the errors of presented model show weak correlation, therefore the goodness of fit is confirmed.

Figure 1. Annual ring width in pine wood.

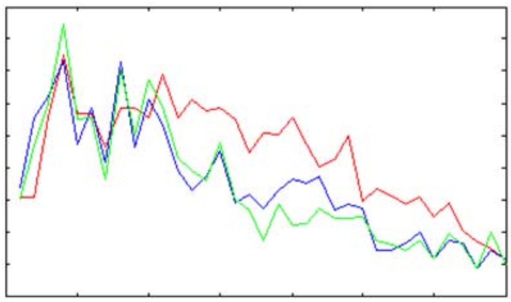

Figure 2. Exact values of annual ring width for three trees (*Pinus eldarica*) of Kelardasht site (The north of Iran).

Table 1. The observed - forecasted mean -values for the annual ring width data of kelardasht site in the north of Iran, from 1974 to 2008.

Time	1974	75	76	77	78	79	80	81	82	83	84	85
Exact-value	3.17	4.43	5.91	7.75	5.30	5.68	4.14	6.71	5.18	6.12	6.03	4.59
Forecasted value	3.21	4.39	6.30	7.73	3.52	3.90	4.91	5.39	5.83	8.34	6.66	4.87
Time	86	87	88	89	90	91	92	93	94	95	96	97
Exact-value	4.43	4.36	5.03	3.8	3.45	3.18	3.74	3.82	3.51	3.51	3.14	3.41
Forecasted value	4.47	6.33	4.47	6.64	3.51	3.02	4.21	3.13	4.07	3.62	3.34	3.74
Time	98	99	000	001	002	003	004	005	006	007	008	009
Exact-value	2.72	2.18	2.07	2.00	2.28	1.61	2.21	1.77	1.15	1.65	1.08	-
Forecasted value	2.74	2.71	2.97	2.57	2.96	2.33	2.53	2.02	1.56	1.67	1.112	-

Table 2. The central tendency and dispersion of mean -values for the annual ring width data of Kelardasht site in the north of Iran (from 1974 to 2008).

N	Mean	Std	Min	Max	Sum
35	3.7463	1.66	1.08	7.75	131.12

Table 3. MSE for estimating regression function for the annual ring width data of Kelardasht site in the north of Iran, (1974 to 2008).

N	Ase	$\hat{\sigma}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\mu}$	MSE
35	1.0113	1.23	0.8011	2.79	2.2001	1.0056

Table 4. Descriptive statistics for the errors of presented model.

N	Mean	Min	Max	Std-deviation	Skewness
35	0.0091	-0.09	1234	0.9812	0.1021

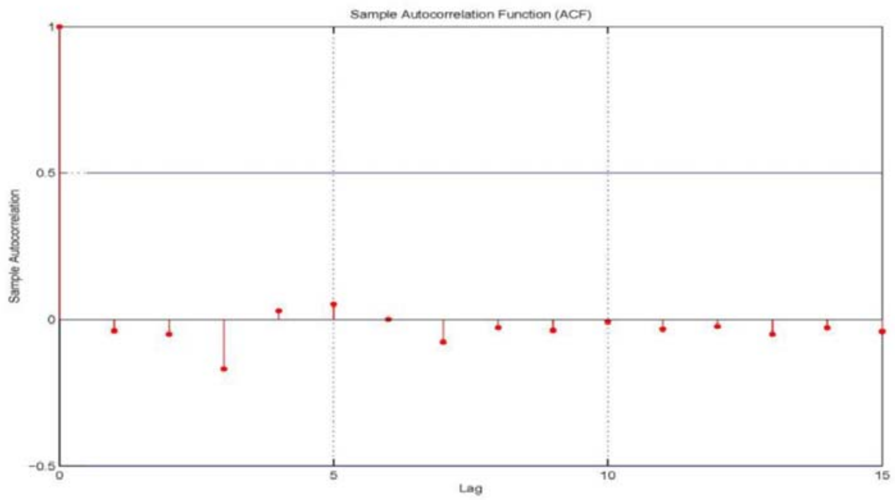

Figure 3. Autocorrelation of the errors in the presented model (for 1-15 lags).

CONCLUSION

In this research, the mean of annual ring width *Pinus eldarica* trees in Kelardasht site (The North of Iran) was predicted by additive nonlinear autoregressive model. In this model the parameters and regression functions were estimated by semi-parametric methods. The MSE criterion is also applied to verify the accuracy of the suggested model. The results of the study indicate the accuracy of the suggested model.

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