Statistical distribution and energy estimation of the wind speed at Saint Martin's Island, Bangladesh

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Abstract

This paper describes some statistical probability distribution functions (pdf) which have been used to measure wind power and energy potential of Saint Martin's Island situated at 13 kilometers apart from the southern-most tip of Bangladesh. One-year measured wind speed data at 60 meters above ground level (agl) are analyzed in the research with some well-known statistical probability density functions (pdf) namely, Normal distribution, Weibull distribution, Gamma distribution, and Rayleigh distribution. The best distribution technique among the four for the wind speed data of the Island has been measured, and the Root Mean Square Error (RMSE) and Mean Bias Error (MBE) are the methods that have been employed throughout the study in order to select the best distribution function. Results for the one-year time-series wind speed data (October 2014-October 2015) recorded every minute of Saint Martin's Island as per RMSE and MBE performance test revealed that, Weibull probability distribution, whose RMSE and MBE values have been measured to be 1.43 and 0.72 respectively proved to be the best distribution function for the wind power estimation and analysis. Rayleigh distribution follows Weibull pdf in this research. The shape (k) and scale parameters (c) for Weibull and Rayleigh pdf have been calculated using Empirical Method (EM) for the research which are 1.70 and 4.90 m/s respectively for Weibull pdf, where the scale parameter for Rayleigh pdf whose defined shape parameter value is 2, is to be measured as 4.92 m/s. Annual Energy Production (AEP) for Saint Martin's Island has been measured as 5997 kWh using frequency distribution (histogram).

Keywords: *Wind speed, statistical probability distribution function, wind energy, performance measurement*

1. Introduction

The offshore area usually shows greater wind energy potential than most other terrestrial locations [1]. As wind energy is an ultimate source of green and clean energy that is well proven in applying to national grid, and even some sort of stand-alone off-grid connections [2], exploration of the wind energy of off-shore areas are and should be characterized in the first kind. In wind turbine design and site planning, the probability distribution of short-term wind speed becomes critically important in estimating energy production [3]. The characterization of wind speed thus is essential in exploitation of the enormous wind power source. Technologies utilized from wind resource for energy conversion systems are well-established worldwide to provide complete security of energy supply [4-7]. Potential site identification and assessment for wind farm development are very important in an early stage of development and planning. In engineering practice, the average wind turbine power $\hat{p}_w$ associated with the probability density function (pdf) of wind speed v is obtained from

$$\hat{p}_w = \int_0^\infty p_w(v) f(v) dv \tag{1}$$

Where $f(v)$ is the pdf of wind speed v and $p_w(v)$ is the power output generated from a turbine describes power output versus wind speed [8].

Minimizing uncertainty in wind resource estimates greatly improves results in the site assessment phase of planning [9], and this can be accomplished by utilizing a more accurate statistical probability distribution function (pdf). It is obvious that the uncertainty in the estimation of power using Equation 1 lies in the choice of the wind speed pdf while the power curve is fairly accurately provided by the wind turbine generator manufacturer. In order to minimize the uncertainty in wind resource characterization and assessment, the application of more accurate pdf should be accomplished to greatly improve results in the site assessment phase of planning [9].

So far, Weibull distribution is the most widely used distribution for characterization of measured wind speeds [10-12]. It consists of two parameters, namely, shape and scale parameters. Except for Rayleigh pdf, which is a special instance of Weibull pdf [13], the other statistical distribution functions have not been used extensively in terms of the analysis of wind energy. But it has been noted that, other statistical distribution functions might be even useful for the analysis of wind speed under different wind regimes [14]. One of the goals of the paper is to investigate the performance of some other distribution functions as well to understand how the evaluated distribution functions work for the wind speed of Saint Martin's Island, Bangladesh in parallel to Weibull and Rayleigh pdf. A number of distribution functions can help understand and verify an optimal model for a successful prediction of the wind at a particular site in practice, which is also a goal of the current research.

2. Data sources

Wind speed observation data of Saint Martin's Island, Bangladesh have been measured at 10 meter above the ground level (agl). 10 meter height test data is the most common all over the world, as it is not viable to establish met tower of more height in many places, and this is expensive too.

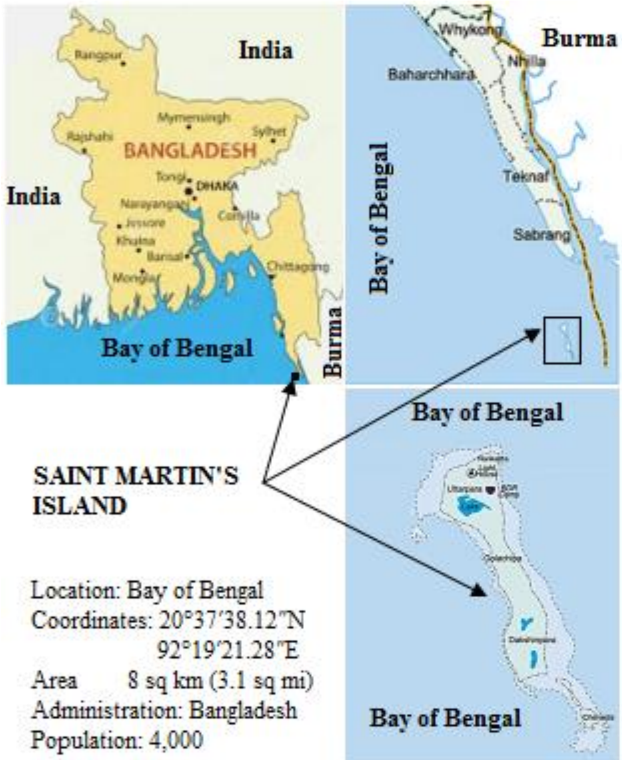

Figure 1 Location of Saint Martin’s Island, courtesy: www.google.com

So, wind speed data taken at 10 meter agl are extrapolated at different elevation using some mathematical models like Power Exponent Law. Modern wind turbines can even use wind at 125 meters height from the ground. That’s why, when record of wind speed exists at different height for a station, power exponent law is considered to be the most common tool to use in order to obtain the extrapolated values wind speed at different heights [15] [16]. Here power exponent law is described for the ease of understanding:

$$\frac{v_z}{v_{z_{ref}}} = \left(\frac{z}{z_{ref}}\right)^{\alpha} \tag{2}$$

Where, $$\alpha = \frac{0.37 - 0.088\ln(v_{ref})}{1 - 0.088\ln(\frac{z_{ref}}{10})} \tag{3}$$

v_z = wind speed at height z , v_{Zref} = wind speed at reference height (z_{ref}), z = height above ground level, z_{ref} = reference height. α = power law exponent. After calculating the wind speed at 60 meters from the above equation, mean wind speed can be measured using the following formula:

$$v_m = \frac{1}{N}\sum_{i=1}^{N} m_i f_i \tag{4}$$

The same formula can be applied to measure the mean wind speed for minutes, hours and days. The peak wind speed occurs as the one-year data of Saint Martin’s Island identifies during the months of June to July [17].

3. Statistical probability density function (PDF)

Four conventional probability density functions were tested to approximate frequency distribution of wind speed data. The statistical functions are shown below.

3.1 Normal distribution

Informally known as bell curve, Normal distribution method is defined by equation 5 given below.

$$f(v) = f\left(v;\mu,\sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} \tag{5}$$

Where, μ is defined as the expected value or mean wind speed, and σ is the standard deviation of the v probability variables.

3.2 Gamma distribution

Gamma function can be interpreted as the following equation:

$$f(v) = f\left(v;\lambda,p\right) = \frac{\lambda^p}{\Gamma(p)} v^{p-1} e^{-\lambda v}, v > 0 \tag{6}$$

Where, $\Gamma(p)$ is the gamma function, v is the expected value (mean), and the σ^2 of the standard deviation of a probability variable with such a distribution is $\mu = \frac{p}{\lambda}, \sigma^2 = \frac{p}{\lambda^2}$

3.3 Weibull probability density function

Weibull probability distribution function is considered to be the best fitting statistical method for analyzing the observed time-series wind speed data. Simplicity and flexibility are the reasons why Weibull pdf becomes widely used for understanding wind characteristics. In general, the Weibull probability density function for fitting a particular wind speed, say v, can be shown in equation below [18],

$$f(v) = \left(\frac{k}{C}\right)\left(\frac{V}{C}\right)^{k-1} exp\left(-\frac{V}{C}\right)^k \quad [v>0, k, c>0] \tag{7}$$

Here, c is the scale parameter unit of which is m/s and k is the dimensionless shape parameter. The cumulative distribution for Weibull pdf can be written as:

$$F(V) = 1 - exp\left(-\frac{V}{C}\right)^k \tag{8}$$

3.4 Rayleigh probability density function

For the Rayleigh probability, it is a special case of the probability density of Weibull function which has k equal to 2. Rayleigh distribution function is being shown in equation 10 where, v_m is mean wind speed.

$$f(v) = \frac{\pi}{2}\left(\frac{v}{{v_m}^2}\right) e^{-\left[\frac{\pi}{4}\left(\frac{v}{v_m}\right)^2\right]} \tag{9}$$

4. Selection of the best distribution method

Two statistical indices have been used for the measurement of the performance of the five statistical distributions analyzed in the current research, Root Mean Square Error (RMSE) and Mean Bias Error (MBE).The root mean square error (RMSE) defines the deviation between the statistically presumed values and the experimental values. The lower the values of RMSE, the more well performance a particular distribution method shows, and vice versa. RMSE should be as close to zero as possible, and it is expressed as [19][20][21]:

$$RMSE = \left[\frac{1}{N}.\sum_{i=1}^{N}(y_i - x_i)^2\right]^{\frac{1}{2}} \tag{10}$$

Mean Bias Error can be estimated by the following formula:

$$MBE = \frac{1}{N}.\sum_{i=1}^{N}(y_i - x_i) \tag{11}$$

MBE can be derived as either positive or negative bias [22]. For fitting a pdf to be the best, RMSE with lower value close to zero is desirable. Generally few errors in the sum can produce a significant increase in the indicator. In the same way, low values of MBE are also desirable, but overestimation of an individual data element will cancel underestimation in a separate observation. It is also possible to have large RMSE values at the same time with a small value of MBE or vice versa [23].

5. Parametric values of the distribution functions

The shape and scale parameters of Weibull pdf have been measured using the following equations:

$$k = \left(\frac{\sigma}{\overline{\overline{v_m}}}\right)^{-1.086} \tag{13}$$

Where, k is the Weibull shape parameter, v_m is the mean wind speed which is measured by equation 3, and σ is standard deviation of wind data which can be calculated using the following equation:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(v_i - v_m)^2} \tag{14}$$

Once k is calculated, scale parameter c can be found from the following equation:

$$c = \frac{v_m}{\Gamma\left(1 + \frac{1}{k}\right)} \quad (15)$$

Where Γ is the standard Gamma function as described by Manwell et al. [8] can be expressed as:

$$\Gamma = \int_0^\infty t^{x-1} e^{-t} dt \quad (16)$$

As Rayleigh pdf is a special case of Weibull pdf with shape parameter *k* value to be 2, the scale parameter for Rayleigh pdf is measured using equation 10. Following is the table showing all of the parametric values used in this paper:

Table 1 Parametric values for probability distribution functions

Distribution Name	Shape Parameter	Scale Parameter	Mean wind Speed (60 meter agl)	Standard Deviation
Weibull pdf	1.70	4.90		
Rayleigh pdf	2	4.92		
Normal distribution			4.37	2.68

6. Wind speed probability distribution analyzed

The following figure illustrates the frequency distribution of the wind speed of the Island**:**

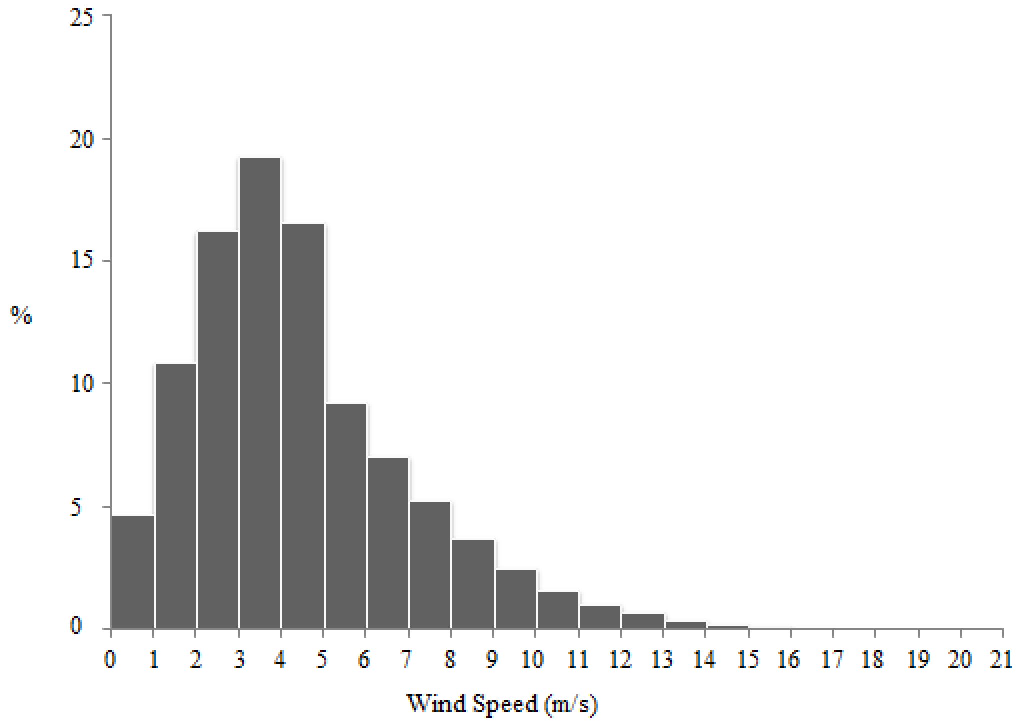

Figure 2 Frequency Distribution for Wind Speed of Saint Martin's Island (Oct '14 – Oct '15)

Following is the figure showing probability distributions of the wind speed**:**

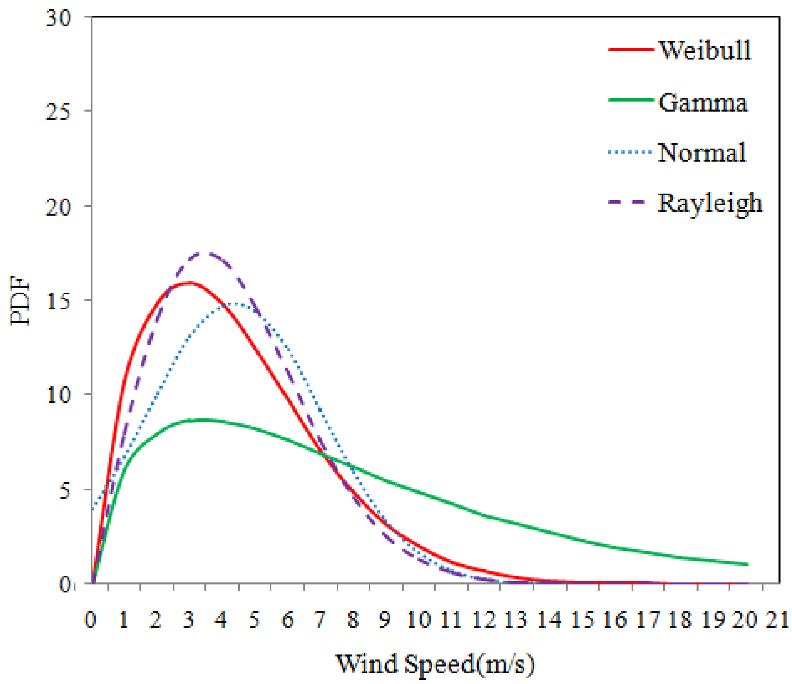

Figure 3 Probability Distribution for Wind Speed of Saint Martin's Island (Oct '14 – Oct '15)

7. Results and discussions

Table.2 shows the Root Mean Square Error and Mean Bias Error values for Saint Martin's Island:

Table 2 RMSE and MBE Values for Saint Martin's Island

Test Methods	Statistical Distribution			
	Weibull	Gamma	Normal	Rayleigh
RMSE	1.43	3.51	2.41	1.76
MBE	0.72	2.43	1.27	0.91

For better understanding of the Table 2 data, the following figures can be shown:

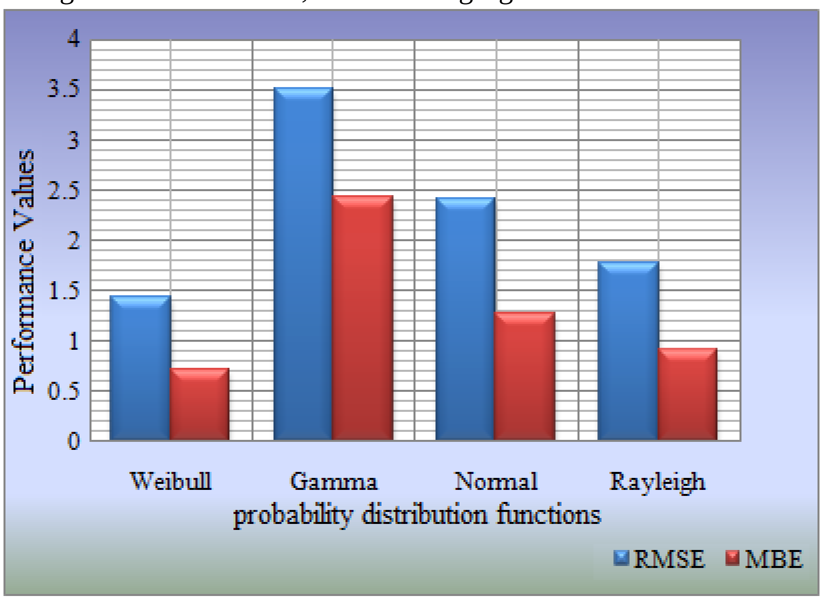

Figure 4 RMSE and MBE values for different pdf

From the table and figure above, it is now becoming easy for selecting the best distribution method for the wind speed in the Island. The ranks of the pdf as per the best performance can be summarized in the following table:

Table 3 Ranking of the five probability distribution functions for RMSE and MBE method

Methods for Performance Test	Statistical Distribution			
	First	Second	Third	Fourth
RMSE	Weibull	Rayleigh	Normal	Gamma
MBE	Weibull	Rayleigh	Normal	Gamma

For both RMSE and MBE methods, the results remain same which indicates that the data to be much consistent. The table below shows the annual energy production (AEP) of Saint Martin's for all of the five distribution functions:

Table 4 Annual Energy Production of Saint Martin's Island for the wind turbine model of G114 2.5 MW WTG

AEP (kWh) from pdf				AEP from Actual Data
Weibull	Gamma	Normal	Rayleigh	
7088	12388	7618	6972	5997

The annual energy production (AEP) of energy for Saint Martin's Island has been calculated for all of the distribution functions. Weibull distribution methods have proved to be the best for different performance analysis (RMSE and MBE).

8. WTG model G114 2.5 MW power curve

A wind power project needs to have a proper understanding of the Wind Energy Conversion System (WECS) when the concern of the selection of a perfect wind turbine generator arises. There are several classes of wind machines (WTGs) with the same rated capacity but different operating characteristics may be available in the market [19]. This is why the knowledge of the wind criteria that would be fit for a particular machine is very important. Power response for a wind turbine (WTG) can be specified by the power curve it gives. In general, wind turbine can be thought of having three very important wind speed categories – (i) cut-in speed V_I, at which a turbine starts producing power, below which the turbine is idle, (ii) rated speed V_R, at which a turbine start producing constant power called P_R at some increasing wind speed up to (iii) cut-out speed V_O when the turbine must be shut down.

For Saint Martin's Island having low wind speed if compared to very good potential area, a wind turbine of the model G114 2.5MW is analyzed here from theoretical point of view, and not for commercial perspective. This wind machine can start producing energy at 2 m/s (cut-in speed) with a rated wind speed and cut-out wind speed to be 11 m/s and 25 m/s respectively. The turbine produces the following power curve using the manufacturer's specifications:

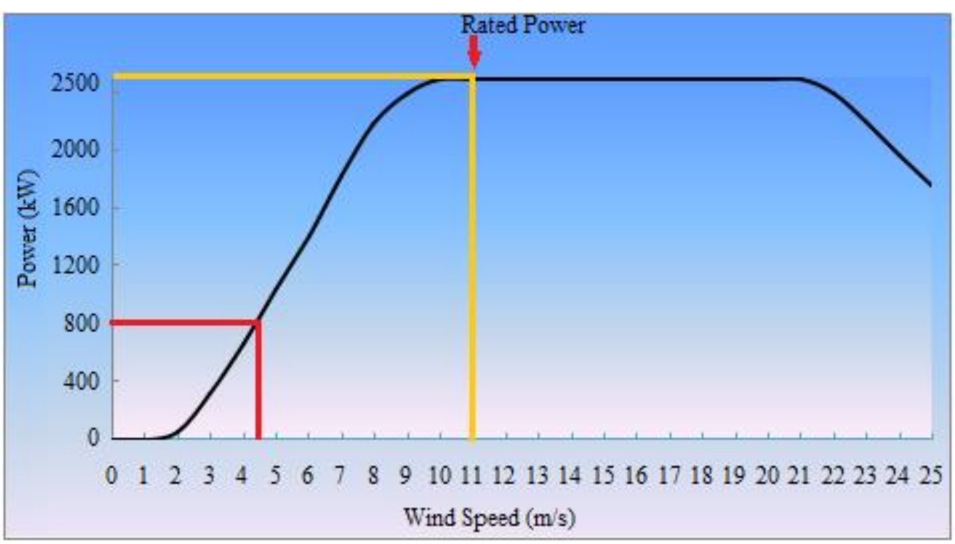

Figure 5 G114 2.5MW Wind Turbine Power Curve

In a normalized view, the probability distribution of any kind, described in this paper, probability distribution, power (kW) and energy (kWh) have been shown in 3D shape. The power and energy estimation can be equated as per the percentile values of the probability of the five distribution functions. Probability, power and energy for every distribution functions as well as those three for frequency distribution is now shown in the following figures:

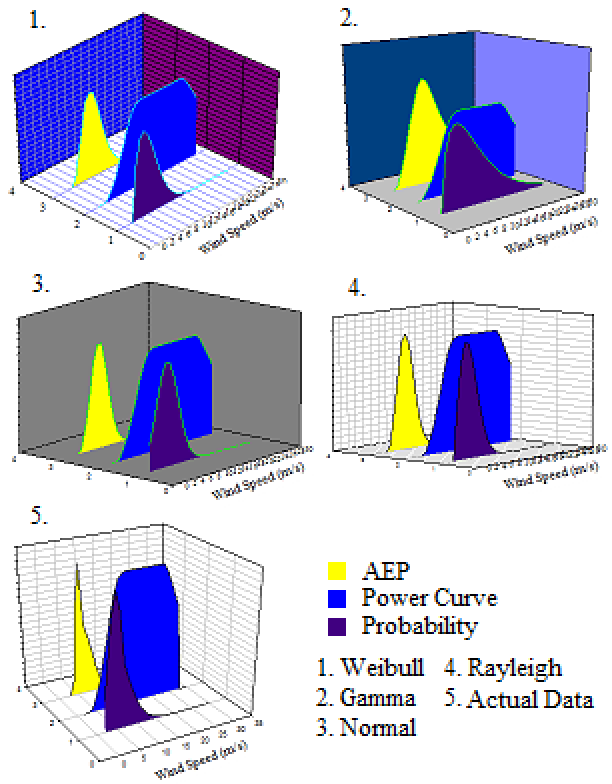

Figure 6 Normalized power and energy curve for statistical probability distribution functions.

9. Conclusion

Statistical distribution for a number of different methods were employed to remote and a very small Saint Martin's Island, Bangladesh, where wind speed has enough potential. This research has been undertaken in order to justify which of the statistical distribution methods is suitable for a particular area. Each of the distribution functions was examined with Root Mean Square Error (RMSE) and Mean Bias Error (MBE) methods for the understanding of the performance. As per the test done in the study, the following points can be concluded:

i) One year time-series wind speed data of Saint Martin's Island, Bangladesh have been statistically distributed for four probability distribution functions – Normal, Gamma, Weibull and Rayleigh pdf.

ii) Frequency distribution (histogram) has been measured from the actual one-year data.

iii) Parameters k and c for Weibull pdf have been found to be 1.70 and 4.90 m/s respectively, and c for Rayleigh pdf to be 4.92 m/s. Mean wind speed and standard deviation have been measured as 4.37 m/s and 2.68 respectively.

iv) Weibull pdf becomes the best when Root Mean Square Error (RMSE) method is the index of performance analysis.

v) When Mean Bias Error (MBE) method is analyzed, again Weibull pdf is to be the best distribution technique.

vi) Rayleigh pdf resides after Weibull pdf for both RMSE and MBE methods in measuring performance.

vii) AEP has been estimated for Saint Martin's which has been shown in Table 4.

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