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The use of response surface analysis in obtaining maximum profit in oil palm industry

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Abstract

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This study was conducted to show how to use Response Surface Analysis in obtaining the optimum level of fertilizer needs by oil palm. The ridge analysis was proposed to overcome the saddle point problem. Data from Malaysian Palm Oil Board database was analyzed. The fertilizers considered are N, P, K and Mg. The results from ridge analysis provided several alternatives of the fertilizer combination. Profit analysis was then applied to determine the best combination of fertilizers needed by the oil palm in order to generate maximum profit. It is found that N and K fertilizers were the important fertilizers required by the oil palm. It is also found that the N and K nutrient concentrations of the foliar nutrient composition were higher compared to other nutrients. Three different stations were considered and it was found that the fertilizers needed by the oil palm and foliar nutrient composition were different at the different type of soil series.

Key words : response surface analysis, ridge analysis, profit analysis, oil palm yield, fertilizer

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Response Surface Analysis (RSA) is the technique used to model the relationship between the response variable and treatment factors. The factor variables are sometimes called independent variables and are subject to the control by the experimenter. In particular, the RSA also emphasizes on finding a particular treatment combination, which causes the maximum or minimum responses. The use of analysis for the quadratic response function or RSA is necessary to obtain the optimum level of fertilizer requirements. In response surface analysis, the eigenvalues can be used to determine whether the solution gives a maximum, minimum or saddle point on the response curve. Moreover, the effects of treatment combinations, which have not been carried out in the experiment may still be estimated. This study also proposed the use of ridge analysis as an alternative solution to overcome the saddle point problem. In the oil palm industry, there is a relationship between the response variable namely oil palm (*Eleais guineensis* Jacq.) yield and the four fertilizer treatments, namely nitrogen (N), phosphorus (P), potassium (K) and magnesium (Mg). The expected yield can be described as a continuous function of the application rate factor. A continuous second-degree-function is often a sufficient description of the expected yield over the range of factor levels applied (Verdooren, 2003). If the fertilizer application rates are greater or smaller than the optimum application rates, they may result in a reduction in yields. Fertilizers are wasted if the amount of applied is more than the optimum rate. The purpose of implementing this RSA technique is to determine the optimum levels of fertilizer usage in order to optimize oil palm yields. Although it has been a common practice in modeling oil palm yield using the response surface analysis, this study focused on experimental location. Conclusions cannot be drawn if the stationary point is saddle. Hence, this study proposed the use of ridge analysis to offer an alternative solution for the problem mentioned above.

Mohammed, Foster, Zakaria and Chow (1986) analyzed fertilizer trials which was carried out over a range of environments in Peninsular Malaysia. Their yield response functions for specific soil series had been used for fertilizer recommendation formulation. Yield response equation which took into account curvilinear responses to each fertilizer treatment, and two and three factor interactions between these treatments were fitted to the experimental plot data. Analysis of variance indicated the significance of the individual variables in these equations. Chan, Lim and Alwi (1991) studied the fertilizer efficiency in oil palm in different locations in Peninsular Malaysia. The yield response and environmental factors affecting the fertilizer application were investigated by Mohammed, Zakaria, Dolmat, *et al*. (1991), was found that the environmental factors contributed negatively to the efficiency of urea fertilizer.

Goh, Hardter and Fairhust (2003) and Verdooren (2003) conducted an experiment to determine the optimum levels of fertilizer inputs that gives the optimum yields. Statistical techniques involved in his study were regression analysis and analysis of variance. He concluded that fertilizer experiments with at least three quantitative levels can be used to derive an estimate of the agronomic and economic optimum rate but it was much better to include five quantitative levels based on the central composite design to obtain a reliable estimate for the optimum with a small standard error. Figure 1 illustrates the response curve function of FFB yield to the fertilizers rate for stations S1 and S2. It appears that there is only one maximum point in station S1, whereas more than one maximum point can be seen in station S2.

The Malaysian Palm Oil Board (MPOB) provided data from the MPOB Database of oil palm fertilizers treatments, which have been carried out from three oil palm estates. All the data from each estate has been collected, recorded and compiled by MPOB researchers in the Research Database Center. All treatments were based on a factorial design with at least three levels of N, P and K fertilizer rates. Although different types of fertilizer were used in the treatments, the rates quoted in the final analysis will be equal to the

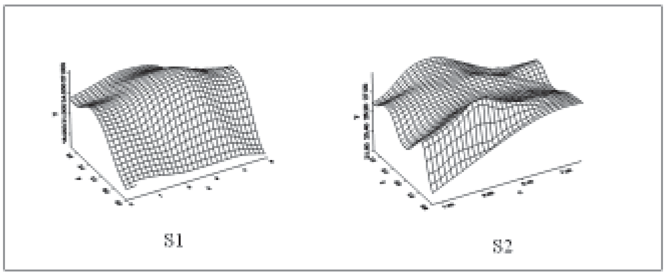

Figure 1. The response surface plots for the fertilizer treatments for stations S1 and S2.

amounts of ammonium sulphate (AS), muriate of potash (KCI), Christmas Island Rock Phosphate (CIRP) and kieserite (Kies). Cumulative yields obtained over a period of two to five years in each trial were analyzed. The data of this study were based on experiment and collected for a certain period of time and differed for each experiment. There was 232, 405 and 324 observations analyzed for stations S1, S2 and S3 respectively. Fresh fruit bunches (FFB) yield data used in this study were measured in tonnes per hectare per year or the average of FFB yield in one year. Foliar analysis was only done once a year and the samples were taken either in March or July every year. The type of FFB yield data and foliar analysis data is continuous, and fertilizer input is in coded form (0, 1, 2, and 3).

Research Methodology

The purpose of using response surface analysis is to determine the optimum level of palm oil yield using fertilizer information. Canonical analysis was used to investigate the shape of the predicted response surface. The brief mathematical explanation of the response surface analysis is written here and analysis of response surface was conducted using Statistical Analysis System (SAS) version 6.12 package and the subsequent results are demonstrated.

Response Surface Analysis

In response surface analysis, it is assumed that the true functional relationship

$$y = f(\mathbf{x}, \beta) \qquad (1)$$

is, in fact, unknown. Here $(x_1, x_2, ..., x_p)$ are centered and scaled design unit. The genesis of the first order approximating model, or the model that contains first order terms and low order interaction terms, is the notation of the Taylor series approximation. In general, second order response surface models can be written as

$$\begin{aligned} y = {} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p + \beta_{11} x_1^2 \\ & + ... + \beta_{pp} x_p^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + ... \\ & + \beta_{p-1,\,p} x_{p-1} x_p + \varepsilon, \end{aligned} \qquad (2)$$

where β_i is an unknown parameter and ε is a random error.

Consider the true functional relationship with second order polynomial as

$$\hat{y} = b_0 + \mathbf{x}^{\mathrm{T}}\mathbf{b} + \mathbf{x}^{\mathrm{T}}\hat{\mathbf{B}}\mathbf{x}, \qquad (3)$$

where b_0, $\mathbf{b}$ and $\hat{\mathbf{B}}$ contains estimates of the intercept, linear and second order coefficients, respectively. In fact, $\mathbf{x}^{\mathrm{T}} = [x_1, x_2, ..., x_p]$, $\mathbf{b}^{\mathrm{T}} = [b_1, b_2, ..., b_p]$ and $\hat{\mathbf{B}}$ is the k x k symmetric matrix

$$\hat{\mathbf{B}} = \begin{bmatrix} b_{11} & \frac{b_{12}}{2} & \Lambda & \frac{b_{1p}}{2} \\ \mathrm{M} & b_{12} & \Lambda & \frac{b_{2p}}{2} \\ \mathrm{M} & & \mathrm{O} & \mathrm{M} \\ \frac{b_{1p}}{2} & \frac{b_{2p}}{2} & ... & b_{pp} \end{bmatrix}, \qquad (4)$$

One can differentiate in (4) with respect to x and obtain

$$\frac{\partial \hat{y}}{\partial \mathbf{x}} = \mathbf{b} + 2\hat{\mathbf{B}}\mathbf{x}, \qquad (5)$$

where **b** is the estimate of the linear coefficients of the second order coefficients. Allowing the derivative to be set to **0**, we can solve for the stationary point, $\mathbf{x}_s$ of the system. As a result, we obtain the solution $\mathbf{x}_s$ as

$$x_s = \frac{-\hat{\mathbf{B}}^{-1}\mathbf{b}}{2}, \quad (6)$$

the point $\mathbf{x}_s$ is the stationary point of the system.

Stationary Point

The sign of the stationary point is determined from the signs of the eigenvalues of the matrix $\hat{\mathbf{B}}$. It turns out that the relative magnitudes of these eigenvalues can be helpful in the total interpretation. For example, let the $k \times k$ matrix **G** be the matrix whose columns are the normalized eigenvectors associated with the eigenvalues of $\hat{\mathbf{B}}$. We know that $\mathbf{G}^T\hat{\mathbf{B}}\mathbf{G} = \mathbf{\Lambda}$, where $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues of $\hat{\mathbf{B}}$ as main diagonal elements. If we translate the model of (3) to a new center, namely the stationary point, and rotate to axis corresponding to the principle axis of the contour system, we have

$$\mathbf{v} = \mathbf{x} - \mathbf{x}_s \text{ and } \mathbf{w} = \mathbf{G}^T\mathbf{v}, \quad (7)$$

This translation gives

$$\begin{aligned}\hat{y} &= b_0 + (\mathbf{v} + \mathbf{x}_s)^T \mathbf{b} + (\mathbf{v} + \mathbf{x}_s)^T(\mathbf{v} + \mathbf{x}_s) \\ &= [b_0 + \mathbf{x}_s^T\mathbf{b} + \mathbf{x}_s^T\mathbf{x}_s] + \mathbf{v}^T\mathbf{b} + \mathbf{v}^T\mathbf{v} + 2\,\mathbf{x}_s^T\mathbf{v} \\ &= \hat{y}_s + \mathbf{v}^T\hat{\mathbf{B}}\mathbf{v}, \quad (8)\end{aligned}$$

because $2\mathbf{x}_s^T\mathbf{B}\mathbf{v} = -\mathbf{v}^T\mathbf{b}$ from (6). The rotation gives

$$\begin{aligned}\hat{y} &= \hat{y}_s + \mathbf{w}'\mathbf{G}'\hat{\mathbf{B}}\mathbf{G}\mathbf{w} \\ &= \hat{y}_s + \mathbf{w}'\Lambda\mathbf{w}, \quad (9)\end{aligned}$$

The *w*-axes are the principle axes of the contour system. Equation (9) can be written as

$$\hat{y} = \hat{y}_s + \sum_{i=1}^{p} \lambda_i w_i^2, \quad (10)$$

where $\hat{y}_s$ is the estimated response at the stationary point, and λ_1, λ_2, λ_3, ..., λ_p are the eigenvalues of $\hat{\mathbf{B}}$. The variables w_1, w_2, w_3, ..., w_p are known as canonical variables.

The signs of λ values determine the nature of the stationary points, and the relative magnitude of eigenvalues will help us to gain a better understanding of the response system. If all the λ values are negative, the stationary point is a point of maximum response. If all λ values are positive, the stationary point is a point of minimum response, and if the λ values have mixed signs, the stationary point is a saddle point (Myers and Montgomery, 1995, Christensen, 2001). If the estimated surface is found at a maximum or minimum point, the analysis performed by model fitting and the canonical analysis may be sufficient (Myers and Montgomery, 1995; Christensen, 2001; Box and Draper, 1987 and SAS, 1992). If the stationary point is a saddle point then the ridge analysis is proposed to ensure the stationary point will be inside the experimental region. The result is a set of coordinates for the maximum or minimum point, along with the predicted response at each computed point on the path. The method of ridge analysis solved the estimated ridge for the optimum response and increased the radius from the center of the original design.

Ridge Analysis

The main purpose of ridge analysis is to ensure that the stationary point is inside the experimental region. The output of the analysis is the set of coordinates of the maximum (or minimum) along with the predicted response,, at each computed point on the path. This analysis provides useful information regarding the roles of the design variables inside the experimental region. Ridge analysis may provide some guidelines regarding where future experiments should be made in order to achieve conditions that are more desirable. However, ridge analysis is generally used when the practitioner feels that the point is near the region of the optimum.

Consider the fitted second-order response surface model in (3), which maximize subject to the constraint $\mathbf{x}^T\mathbf{x} = \mathbf{H}^2$, where $\mathbf{x}^T = [x_1, x_2, ..., x_p]$ and the center of the design region is taken to be $x_1 = x_2 = ... = x_p = 0$. Using Lagrange multipliers, differentiate, $\mathbf{J} = b_0 + \mathbf{x}^T\mathbf{b} + \mathbf{x}^T\mathbf{x} - \kappa(\mathbf{x}^T\mathbf{x} - \mathbf{H}^2)$ with respect to the vector **x**. The derivative of **J** with respect to **x** is given by $\frac{\partial \mathbf{J}}{\partial \mathbf{x}} = \mathbf{b} + 2\hat{\mathbf{B}}\mathbf{x} - 2\kappa\mathbf{x}$, and the constrained stationary point is determined by setting $\partial\mathbf{J}/\partial\mathbf{x} = \mathbf{0}$. This gives the result

$$\left(\hat{\mathbf{B}} - \kappa \mathbf{I}\right)\mathbf{x} = -\frac{1}{2}\mathbf{b}, \qquad (11)$$

As a result, for any fixed value of κ, a solution **x** of (11) is a stationary point on $\mathbf{H} = (\mathbf{x}^T\mathbf{x})^{1/2}$. However, the appropriate solution **x** is that which results in a maximum $\hat{y}$ on **H** or a minimum $\hat{y}$ on **H**, much depending on which is desired. The appropriate choice of κ depends on the eigenvalues of the $\hat{\mathbf{B}}$ matrix. Myers and Montgomery (1995) provided important rules on selecting the value of κ.

(i) If κ exceeds the largest eigenvalue of $\hat{\mathbf{B}}$, the solution **x** in (11) will result in an absolute maximum for $\hat{y}$ on $\mathbf{H} = (\mathbf{x}'\mathbf{x})^{1/2}$.

(ii) If κ is smaller than the smallest eigenvalue of $\hat{\mathbf{B}}$, the solution **x** in (11) will result in an absolute minimum for $\hat{y}$ on $\mathbf{H} = (\mathbf{x}'\mathbf{x})^{1/2}$.

Some mathematical insight into (i) and (ii) above is provided in Appendix A. Then we also examined the relationship between **H** and κ. The analyst desires to observe results on a locus of points. As a result, the solution of (11) should fall in the interval [0, $\mathbf{H}_b$], where $\mathbf{H}_b$ is a radius approximately representing the boundary of the experimental region. The value **H** is actually controlled through the choice of κ value. In the working region of κ namely $\kappa > \lambda_m$ or $\kappa < \lambda_1$, (where λ_1 is the smallest eigenvalue of and λ_m is the largest eigenvalue of $\hat{\mathbf{B}}$), **H** is a monotonic function of κ.

Computer Application

The SAS package provided the easy way to perform response surface analysis via PROC RSREG procedure (SAS, 1992). The RSREG procedure allows one of each of the following statements;

PROC RSREG option;
    MODEL response = independents/options;
    RIDGE option;
    WEIGHT variable;
    ID variables;
    BY variables;

The PROC RSREG and MODEL statements are required. The MODEL statement lists the dependent variable (oil palm yield) followed by an equal sign, and then lists independent variables namely, N, P, K and Mg fertilizers. Independent variables specified in the MODEL statement must be variables used in the data set.

A RIDGE statement specifies that the ridge of optimum response is being computed. The ridge starts at given point $\mathbf{x}_0$, and the point on the ridge at radius *r* from $\mathbf{x}_0$ is a collection of factor settings that optimizes the predicted response at that radius. The ridge analysis can be used as a tool to help interpret an existing response surface or to indicate the direction in which further experimentation should be performed. A BY statement can be used with PROC RSREG to obtain separate analyses on observations in groups defined by the BY variable. When it is stated in the programming, the procedure expects the input data set to be sorted in order of the BY variables. The ID statement names variables that are to be transferred to the created data set, which contains statistics for each observation. The WEIGHT statement names a numeric variable in the input data set.

Results and Discussion

Analysis of the fertilizer treatments using response surface analysis was conducted in three stations. The discussion on the findings will be divided into the canonical analysis and the ridge analysis for the fertilizer treatments. The stationary point was identified to determine its turning points; either a maximum, a minimum or a saddle. The ridge analysis was introduced if the stationary point was a saddle point.

Canonical analysis for fertilizer treatment

The summary of the response surface analysis, which provides the values of parameter estimate, parameter testing, the MSE, RMSE and R^2, is presented in Table 1. The R^2 value represents the variance explained by the exploratory variables or factors where stations S1, S2 and S3 recorded the values as 0.5802, 0.7613 and 0.5972, respectively. The average of FFB yield for each station is 23.74, 26.69 and 22.74 tonnes/hec./year, respectively. The individual t-test for parameter estimated

Table 1. The estimated parameter, T value, prob. > |T|, MSE, RMSE and R^2 values for stations S1, S2 and S3.

Station	Parameter	Estimated parameter	T value	Prob > |T|	MSE	RMSE	R^2
	Intercept	12.6408	5.921	0.0000			
	N	2.0138	3.586	0.0004	15.7966	3.9744	0.5802
	P	5.1217	5.635	0.0000			
	K	-0.2369	-0.374	0.7087			
	Mg	1.2143	1.441	0.1509			
	N^2	-0.2342	-4.054	0.0001			
	P^2	-1.2657	-7.521	0.0000			
S1	K^2	-0.1553	-1.793	0.0743			
	Mg^2	-0.1779	-1.083	0.2799			
	P*N	0.2516	2.851	0.0048			
	K*N	0.0867	1.422	0.1563			
	K*P	0.2821	2.708	0.0073			
	Mg*N	-0.0066	-0.079	0.9370			
	Mg*P	0.0957	0.684	0.4945			
	Mg*K	0.1237	1.403	0.1619			
	Intercept	23.6739	16.899	0.0000			
	N	2.6871	6.464	0.0000	7.0654	2.6581	0.7613
	P	-0.0021	-0.0025	0.9980			
	K	-0.1027	-0.247	0.8057			
	Mg	-2.5519	-3.069	0.0031			
	N^2	-0.2379	-5.031	0.0000			
	P^2	-0.0691	-0.365	0.7160			
S2	K^2	-0.0192	-0.405	0.6866			
	Mg^2	0.3988	2.109	0.0388			
	P*N	0.0521	0.779	0.4385			
	K*N	0.0334	0.999	0.3215			
	K*P	-0.0958	-1.433	0.1567			
	Mg*N	0.0658	0.985	0.3282			
	Mg*P	0.1624	1.215	0.2289			
	Mg*K	0.1493	2.233	0.0289			
	Intercept	15.2392	2.400	0.0192			
	N	-0.8306	-0.523	0.6027	4.3348	2.0820	0.5972
	P	3.3242	1.109	0.2716			
	K	5.0898	2.591	0.0018			
	Mg	-4.1955	-1.043	0.3006			
	N^2	0.0261	0.162	0.8715			
	P^2	-0.4142	-0.723	0.4723			
S3	K^2	-0.9678	-3.949	0.0002			
	Mg^2	0.7918	0.782	0.4372			
	P*N	-0.0258	-0.142	0.8875			
	K*N	0.2035	1.700	0.0093			
	K*P	0.0308	0.133	0.8950			
	Mg*N	0.4877	2.074	0.0042			
	Mg*P	-0.5417	-1.200	0.2350			
	Mg*K	0.2077	0.702	0.4851			

was performed to investigate the significantly contribute of the variables to the model. If the prob. > |T| is less than 0.05 it means that the parameter estimated is statistically significant. The variables of fertilizers N and P, and the second order interactions of N^2, P^2, P*N and K*P are found significant in station S1. For station S2, the major fertilizers N and Mg, and the second order interactions N^2, Mg^2 and Mg*K are found significant. Meanwhile, station S3 showed that the K fertilizer and the second order fertilizer interactions K^2, K*N, and Mg*N to be statistically significant to the model.

All the important results, including the eigenvalues, critical values, predicted FFB yield values at the stationary points and concluding remarks of each stationary point are presented in Table 2. As shown in the Table 2, station S1 has all negative values of λ (λ_1 = -0.3439, λ_2 = -0.9395, λ_3 = -2.3165 and λ_4 = -4.5217), thus the stationary point is a maximum point. The results from these findings indicated that 5.148 kg of N, 2.7054 kg of P, 3.2195 kg of K and 2.8126 kg of Mg fertilizer were needed to achieve the maximum level of FFB yield of 30.1826 tonnes per hectare per year. In S2 station, the eigenvalues of the eigen vector are λ_1 = 1.4988, λ_2 = 0.0634, λ_3 = -0.6883 and λ_4 = -3.1878. The signs of the eigenvalues are mixed, thus the stationary point is shown to be a saddle point. This also occurred in station S3, which the predicted FFB yield at stationary point is 23.08 tonnes per hectare per year, and the critical values of N, P, K and Mg fertilizers are 5.21, 1.84, 0.44 and 3.39 kg, respectively.

The canonical analysis indicated that the predicted response surface was shaped as saddle at station S2. The eigenvalue of the N fertilizer 1.4988, shows that the valley orientation of the saddle point was less curved than the hill orientation with the Mg concentration eigenvalues of -3.1878. The negative signs of the eigenvalues for K and Mg fertilizers indicated the directions of downward curvature. The largest eigenvalue (in absolute) for the Mg fertilizer, means that the Mg fertilizer was more pronounced and the curvature of the response surface was in the associated direction. The surface was more sensitive to the changes in Mg, compared to fertilizers of K and P. As the results of stations S2 and S3 were saddle points, the ridge analysis was performed.

Ridge analysis for fertilizer treatment

The estimated responses of the FFB yield at certain radii and the fertilizer levels for stations

Table 2. The eigenvalues, predicted FFB yield at the stationary points, and critical values of fertilizer level.

Station/	fertilizer	Eigenvalue (λ)	Critical value	Predicted FFB yield at stationary point	Concluding remarks
S1	N	-0.3439	5.1480	30.1826	Maximum point
	P	-0.9395	2.7054		
	K	-2.3165	3.2195		
	Mg	-4.5217	2.8126		
S2	N	1.4988	6.3486	29.9374	Saddle point
	P	0.0634	2.9778		
	K	-0.6883	2.0066		
	Mg	-3.1878	1.6929		
S3	N	1.3554	5.2134	23.0854	Saddle point1
	P	-0.1517	1.8404		
	K	-0.8699	0.4391		
	Mg	-4.2806	3.3921		

S2 and S3 are presented in Table 3. As mentioned earlier, ridge analysis is used to find the optimum value of FFB yield when the canonical analysis indicated the stationary point was a saddle point. At station S2, the estimated FFB yield was 28.2359 tonnes per hectare per year at radius 0.0, and corresponded with 3.6400 kg of N, 1.8200 kg of P, 3.6400 kg of K and 1.6200 kg of Mg fertilizer. When the radius was increased from 0.0 to 0.5, the estimated FFB yield also increased to 29.8253 tonnes per hectare per year. An increase in fertilizer inputs was also detected to 5.3372 kg of N, 1.8192 kg of P, 3.8738 kg K and 2.1270 kg of Mg fertilizer. When the radius reached its maximum value, the estimated FFB yield was recorded at 31.0969 tonnes per hectare per year, and it corresponded with 5.6169 kg of N, 1.9442 kg of P, 4.5206 kg of K and 3.2781 kg of Mg fertilizer.

The estimated FFB yield at radius 0.1 for station S3 is 22.97 tonnes per hectare per year. Therefore it needed 4.52 kg of N, 2.22 kg of P, 3.40 kg of K and 1.78 kg of Mg fertilizer to achieve this level. The increase in radius from 0.1 to 0.5 has resulted in an increase in the estimated FFB yield to 23.61 tonnes per hectare per year, and corresponded with 5.33 kg of N, 1.88 kg of P, 3.28 kg of K and 1.83 kg of Mg fertilizer. The maximum value of the estimated FFB yield was 24.89 tonnes per hectare per year, when the radius reached the maximum value of 1.0. The fertilizer levels required were also increased to 6.31 kg of N, 1.41 kg of P, 3.27 kg of K and 1.84 kg of Mg fertilizer.

Table 3. The estimated FFB yield and fertilizer level at certain radii for stations S2 and S3.

Station	Radius	Fertilizer Level (kg/palm/year)				Estimated FFB yield (ton/hectare/year)
		N	P	K	Mg	
	0.0	3.6400	1.8200	3.6400	1.6200	28.2359
	0.1	4.0036	1.8121	3.6367	1.8200	28.6748
	0.2	4.3673	1.8053	3.6425	1.8256	29.0506
	0.3	4.7298	1.8003	3.6652	1.8450	29.3638
S2	0.4	5.0810	1.8008	3.7274	1.9124	29.6169
	0.5	5.3372	1.8192	3.8738	2.1270	29.8253
	0.6	5.4454	1.8477	4.0349	2.4012	30.0304
	0.7	5.5045	1.8741	4.1720	2.6447	30.2560
	0.8	5.5479	1.8986	4.2952	2.8671	30.5079
	0.9	5.5844	1.9218	4.4104	3.0766	30.7880
	1.0	5.6169	1.9442	4.5206	3.2781	31.0969
	0.0	4.3250	2.2750	3.4750	1.7400	24.4803
	0.1	4.5706	2.3088	3.4393	1.7512	24.7287
	0.2	4.8201	2.3354	3.4304	1.7757	24.9778
	0.3	5.0655	2.3536	3.4351	1.8127	25.2337
	0.4	5.3021	2.3635	3.4470	1.8623	25.5011
S3	0.5	5.5273	2.3664	3.4628	1.9225	25.7835
	0.6	5.7406	2.3636	3.4806	1.9909	26.0841
	0.7	5.9427	2.3564	3.4994	2.0650	26.4050
	0.8	6.1351	2.3461	3.5186	2.1431	26.7479
	0.9	6.3192	2.3333	3.5380	2.2241	27.1141
	1.0	6.4966	2.3188	3.5574	2.3068	27.5043

Profit Analysis

In addition to the statistical analysis, an economic analysis should be carried out to determine the point at which the total profit of the oil palm yield is at the highest level (Nelson, 1997). The economic analysis is purposely focused on gaining the optimum level of fertilizer which can produce maximum profit. As discussed earlier, ridge analysis will give several optimum solutions based on the estimated FFB yield and the fertilizer level of the N, P, K and Mg at certain radii. Thus, an economic analysis is required to obtain the optimum profit in oil palm yield modeling.

To obtain the maximum profit in oil palm yield production, four types of fertilizers are considered, namely, nitrogen (N), phosphorus (P), potassium (K) and magnesium (Mg). These fertilizers are the most needed in oil palm yield. Let the cost of fertilizer at a certain radius, C_i be given as;

$$C_i = a_i N_p + b_i P_p + c_i K_p + d_i Mg_p \quad \text{for } i = 1, 2, .., j, \qquad (17)$$

Where *a, b, c* and *d* are the weights for N, P, K and Mg fertilizers (measured in kg per palm per year) respectively, derived form the ridge analysis, and N_p, P_p, K_p and Mg_p are the prices of fertilizer N, P, and K respectively. Since the FFB yield is measured in tonnes per hectare per year, we also converted the cost and total profit (TP) into RM per hectare per year. The total income per hectare per year at a certain radius, H_i, is then given as

$$H_i = E_{ri} * Y_p \quad \text{for } i = 1, 2, ..., j, \qquad (18)$$

where E_{ri} is expected FFB yield at radius *i*, and Y_p is the yield price.

Therefore, the total profit *TP* can be formulated as;

$$TP_i = H_i - C_i \quad \text{for } i = 1, 2, ..., j, \qquad (19)$$

Thus, we can determine the optimum fertilizers, which can be used to achieve a high *TP*.

Based on the fertilizer prices in January 2005, the price of the ammonium sulphate (AS) was RM720 per ton, christmas island rock phosphate (CIRP) was RM 440 per ton, murate of potash (MOP) was RM1040 per ton and kieserite (Mg) was RM729 per ton. The average price of FFB yield in January 2005 was about RM 288.00 per ton. Assuming that other costs such as management cost are constant. A very simple calculation was conducted to obtain the optimum profit from several radius levels. The calculation of total profit for stations S2 and S3 at certain radius was given in Table 4.

The results suggested that, given an annual application of 5.148 kg of N, 2.7054 kg of P, 3.2196 kg of K and 2.8126 kg of Mg fertilizer, palm oil grown in Bungor soil series were capable of producing an average FFB yield of 30.1826 tonnes per hectare per year and of making total profit of RM7254.73. In station S2, the total profit was increased as the estimated FFB yield increased. At radius 0.0 the total profit was RM 6959.64, and the highest total profit was recorded as RM7281.33. The results suggested that given an annual application of 5.6169 kg for N, 1.9442 kg for P, 4.5206 kg for K and 3.2781 kg for Mg fertilizer, oil palm grown in Bungor soil series were capable of producing an average FFB yield 31.0969 tonnes per hectare and of making a total profit of RM7281.33. With the combination of 6.4966 kg of N, 2.3188 kg of P, 3.5574 of K and 2.3068 kg of Mg fertilizer at Durian soil series (station S3), 27.5043 tonnes per hectare of FFB yield could be produced with a total profit of RM6373.06. A combination of the fertilizers suggested at 5.6169 of N fertilizer, 1.9442 of P fertilizer, 4.5206 of K and 3.2781 kg of Mg fertilizer, managed to produce an average FFB yield of 31.0969 tonnes per year at station S2 (Briah soil series).

After determining the optimum level of fertilizers and the maximum profit for each station, a comparative study of the optimum fertilizer needed for each station was performed. Table 5 provides the summary of the fertilizers required by oil palms. It is obvious that the predominantly required fertilizers for oil palm are the N and K fertilizers. The recordings of the S1 (Bungor soil series), S2 (Briah soil series) and S3 (Durian soil series) stations disclosed a need for the N fertilizer

Table 4. The fertilizer level, average estimated FFB yield and total profit for stations S2 and S3.

Station	Radius	Estimated FFB yield (ton/hec/yr)	Fertilizer cost (RM)	Total income (RM)	Total profit (RM)
S2	0.0	28.2359	1172.304	8131.939	6959.635
	0.1	28.6748	1228.148	8258.342	7030.195
	0.2	29.0506	1265.799	8366.573	7100.774
	0.3	29.3638	1307.291	8456.774	7149.483
	0.4	29.6169	1358.573	8529.667	7171.094
	0.5	29.8253	1428.479	8589.686	7161.207
	0.6	30.0304	1492.237	8648.755	7156.518
	0.7	30.2560	1544.327	8713.728	7169.401
	0.8	30.5079	1590.567	8786.275	7195.708
	0.9	30.7880	1633.566	8866.944	7233.378
	1.0	31.0969	1674.578	8955.907	7281.329
S3	0.0	24.4803	1257.452	7050.326	5792.874
	0.1	24.7287	1280.222	7121.866	5841.644
	0.2	24.9778	1308.184	7193.606	5885.423
	0.3	25.2337	1338.455	7267.306	5928.851
	0.4	25.5011	1369.646	7344.317	5974.670
	0.5	25.7835	1400.894	7425.648	6024.754
	0.6	26.0841	1431.708	7512.221	6080.512
	0.7	26.4050	1461.843	7604.640	6142.797
	0.8	26.7479	1491.270	7703.395	6212.125
	0.9	27.1141	1520.029	7808.861	6288.832
	1.0	27.5043	1548.178	7921.238	6373.060

Table 5. The optimum level of fertilizer need by the palm.

Station/soil series	Fertilizer level (kg/palm/year)				Profit (RM)
	N	P	K	Mg	
S1/Bungor	5.1480	2.7050	3.2190	2.8126	7254.73
S2/Briah	5.6169	1.9442	4.5206	3.2781	7281.32
S3/Durian	6.4966	2.3188	3.5574	2.3068	6373.06

to be higher than the K fertilizer. In general, the need for P and Mg fertilizers is less than that for the N and K fertilizers.

The foliar nutrient composition levels and the average estimate for the FFB yield giving the maximum profit are shown in Table 6. The findings show that the combination of foliar nutrient composition 2.5303% of N, 0.1698% of P, 1.0855% of K, 0.5757% of Ca and 0.3562% of Mg were capable of producing an average FFB yield in the S1 station (Bungor soil series) of 30.1826 tonnes per hectare. The results suggested that the Briah soil series produced an estimated FFB yield of 31.0969 tonnes per hectare when the composition of the combination of N, P, K, Ca and Mg nutrient composition are 2.5646, 0.1618, 0.6196, 0.4863 and 0.4425% respectively. On the Durian soil series, the combination of foliar nutrient composition of

Table 6: The estimated FFB yield and the foliar nutrient composition level (%)

Station	Estimated FFB yield (tonnes/hectare/year)	Foliar nutrient composition (%)				
		N	P	K	Ca	Mg
S1	30.1826	2.5303	0.1698	1.0855	0.5757	0.3562
S2	31.0969	2.5646	0.1618	0.6196	0.4863	0.4425
S3	27.5043	2.6264	0.1517	0.9626	0.5795	0.5365

N, P, K, Ca and Mg are 2.6264%, 0.1517%, 0.962%, 0.5795% and 0.5365%, respectively produced about 27.5043 tonnes of FFB yield.

The N concentration was consistently the highest composition followed by the K concentration among the experimental stations. The result is consistent with the findings of the fertilizer level needed by the oil palm. The P concentration recorded the lowest level in the foliar analysis when compared to the other nutrients. The sequence (ascending order) of the foliar nutrient composition needed by the oil palm is the P, Mg, Ca, K and the N concentration.

Conclusion

The results discussed on the previous section clearly indicated that the R^2 value for fertilizer treatments are comparable to the study done by Mohammed, Foster, Zakaria and Chow (1986), Mohammed, Zakaria, Dolmat *et al*. (1991), Foster (1995), Mohammed, Abu Bakar, Dolmat and Chan (1999) and Green (1976). The canonical analysis for the fertilizer levels identified station S1 as at the maximum point and station S2 and S3 are marked at the saddle points. The ridge analysis disclosed the optimum level of the estimated FFB yield. The oil palms are expected to produce around 27 to 31 tonnes of FFB yield per hectare per year with the suggested levels of the fertilizers. The total profit can be obtained from the optimal level of the fertilizers. The foliar nutrient combinations found in this study are in the range of the optimal levels suggested by Foster and Chang (1977a). The fertilizer levels needed by the oil palm are different between the experimental stations. The soil nutrients and the climate appeared to be other factors which affected the production of FFB yield (Foster, Chang, Dolmat, Mohammed and Zakaria (1987a); Foster, Mohammed and Zakaria (1987a); Foster, Dolmat and Gurmit (1987b), Soon and Hong (2001) and Foster (2003).

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Appendix A

Consider the **B** matrix discussed (4). Consider also the **P** matrix (orthogonal), which diagonalizes **B**. That is,

$$\mathbf{P'BP} = \mathbf{\Lambda}$$

$$= \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \cdots & \\ 0 & & & \lambda_m \end{bmatrix}, \quad (12)$$

where the λ_i are the eigenvalues of **B**. The solution **x** that produces locations where $\frac{\partial L}{\partial \mathbf{x}} = \mathbf{0}$ is given by

$$(\mathbf{B} - \kappa\mathbf{I})\mathbf{x} = -\frac{1}{2}\mathbf{b}, \quad (13)$$

If. (13) pre-multiply (**B** - κ**I**) by $\mathbf{P}^T$ and post-multiple by **P** we obtain

$$\mathbf{P'}(\mathbf{B} - \kappa\mathbf{I})\mathbf{P} = \mathbf{\Lambda} - \kappa\mathbf{I},$$

because $\mathbf{P}^T\mathbf{P} = \mathbf{I}_m$. If (**B** - κ**I**) is negative definite, the resulting solution **x** is at least a local maximum on the radius $\mathbf{H} = (x'x)^{1/2}$. On the other hand if (**B** - κ**I**) is positive definite, the result is a local minimum. Because

$$(\mathbf{B} - \kappa\mathbf{I}) = \mathbf{\Lambda} - \kappa\mathbf{I}$$

$$= \begin{bmatrix} \lambda_1 - \kappa & & 0 \\ & \lambda_2 - \kappa & \\ 0 & & \lambda_m - \kappa \end{bmatrix},$$

then if $\kappa > \lambda_{max}$, (**B** - κ**I**) is negative definite and if $\kappa < \lambda_{min}$, (**B** - κ**I**) is positive definite.