REVIEW ARTICLE

State of the art in quality assurance issues of structures with particular emphasis on strength degradation

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Abstract

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With the increase of the economic competition in the industrial world, much attention is being paid to the deterioration of the design structures before and after their first use. Even with the highest quality of materials and workmanship, the occurrence of some form of imperfections during manufacture is inevitable and there will be a typical distribution of imperfection sizes associated with a particular manufacturing process and quality. The origin of defects in a material can take place during manufacturing stage, or during assembly, installation, commissioning or during in service. The defects will be generated due to deterioration of the component/structure which, in turn, results in deterioration of mechanical properties, crack initiation and propagation, leaks in pressurized components and catastrophic failures. Hence, Non Destructive Inspection (NDI) is required at regular intervals and the results can be used for maintenance to mitigate fatigue risk. However, no in-service inspection is perfect. NDI outputs normally depend on many uncertain factors such as the condition of the inspected structure and its service environment etc. In order to take into

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account those uncertainties, the probabilistic approach is capable of identifying variables affecting the design life of the components. It has also been proved that the probabilistic method can be extended to provide very useful information to help managers in making decisions regarding the operation and inspection time of the structures in order to maintain their reliability.

In the present paper, a literature review of the current approaches and methodologies that has been utilized in the area of structural risk and reliability analysis for structures is presented. The parameters used to quantify the uncertainty and reliability of NDI technique is explored. Several probabilistic models regarding the updating flaw information and inspection schedule using different approaches are discussed. Finally, examples of such application on engineering structures are presented.

Key words : quality assurance, nondestructive inspection (NDI), uncertainties, structural reliability

บทคัดย่อ

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การประเมินคุณภาพของโครงสร้างโดยเน้นที่การถดถอยของกำลังรับน้ำหนัก

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สืบเนื่องจากการแข่งขันในโลกอุตสาหกรรม การตรวจสอบอายุการใช้งานของผลิตภัณฑ์ ชิ้นส่วนต่าง ๆ ของโครงสร้างหรือเครื่องจักรกลก่อนและระหว่างการใช้งานจึงมีความสำคัญมาก โดยถึงแม้ว่าในทุกขั้นตอนของการผลิตได้เลือกใช้วัสดุที่มีคุณภาพดีและมีการตรวจสอบคุณภาพทุกขั้นตอน ความไม่สมบูรณ์ของชิ้นส่วนก็ยังสามารถเกิดขึ้นได้ เช่น ความผิดพลาดของขนาด เป็นต้น สำหรับตำหนิที่เกิดขึ้นในวัสดุสามารถเกิดขึ้นได้ในขบวนการผลิต การขึ้นรูปของชิ้นส่วน การติดตั้ง หรือแม้กระทั่งการขนส่ง รอยตำหนิเพียงเล็กน้อยนี้ในทางวิศวกรรมสามารถขยายขนาดหรือที่เรียกว่าการขยายของรอยแตก (crack propagation) และส่งผลไปสู่ความสามารถในการรับน้ำหนักโดยรวมของโครงสร้างได้ ดังนั้นการตรวจสอบโครงสร้างแบบไม่ทำลายจึงถูกแนะนำให้ปฏิบัติอย่างสม่ำเสมอเพื่อเป็นการบำรุงรักษาโครงสร้างและลดความเสี่ยงในการพังทลายของโครงสร้างที่ต้องการความปลอดภัยสูง เช่น เตาปฏิกรณ์นิวเคลียร์ แท่นขุดเจาะน้ำมัน และเครื่องบิน เป็นต้น อย่างไรก็ตาม จากการศึกษาค้นคว้าพบว่าไม่มีการตรวจสอบโครงสร้างแบบไม่ทำลายชนิดใดเลยที่ให้ความถูกต้อง 100 เปอร์เซ็นต์ ซึ่งความไม่แน่นอนดังกล่าวอาจเกิดขึ้นจากสภาพของชิ้นส่วนที่จะตรวจสอบหรือสภาพแวดล้อมที่โครงสร้างตั้งอยู่ ดังนั้นการที่จะลดความไม่แน่นอนดังกล่าวสามารถที่จะทำได้โดยการประยุกต์ใช้ศาสตร์ทางด้านสถิติช่วยในการตัดสินใจของผู้ที่จะดำเนินการวางแผนการปฏิบัติงานและช่วงเวลาการตรวจสอบโครงสร้างเพื่อยืดอายุการใช้งานของโครงสร้าง

บทความนี้จึงได้รวบรวมเทคนิคและวิธีการที่นิยมใช้กันอย่างแพร่หลายในการตรวจสอบความน่าเชื่อถือของโครงสร้าง ปัจจัยที่มีผลต่อการประเมินค่าความไม่แน่นอนของโครงสร้าง แบบจำลองทางสถิติเกี่ยวกับการตรวจสอบจำนวนของรอยแตกในโครงสร้างและการวางแผนการตรวจสอบก็ได้นำเสนอและเปรียบเทียบในบทความนี้ สุดท้ายตัวอย่างของการประยุกต์ใช้แบบจำลองทางสถิติกับโครงสร้างจริงก็ได้ถูกแสดงในส่วนสุดท้ายของบทความ

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With the increase of the economic competition in the industrial world, much attention is being paid to the deterioration of the design structures before and after their first use. Even with the highest quality of materials and workmanship, the occurrence of some form of imperfections during manufacture is inevitable and there will be a typical distribution of imperfection sizes associated with a particular manufacturing process and quality. The origin of defects in a material can take place during manufacturing stage, or during assembly, installation, commissioning or during in-service. We can broadly categorize these steps into two stages i.e. pre-service and in-service. In the pre-service scenario, the defects may be present in the raw material stage or may be introduced during machining, fabrication, heat treatment, assembling. The pre-service quality can be achieved essentially by good engineering practice i.e. by way of selecting suitable quality raw materials and by ensuring that harmful defects are not produced during the subsequent stages of fabrication and assembly, prior to putting the part/component into service.

In-service, one of the important mechanisms of the deterioration is the fatigue effect on mechanical components subjected to repeated or cyclic load pattern. The defects will be generated due to deterioration of the component/structure as a result of one or combination of the operating conditions like elevated temperature, pressure, stress, hostile chemical environment and irradiation leading to creep, fatigue, stress corrosion, embattlement, residual stresses, micro-structural degradation etc. which, in turn, result in deterioration of mechanical properties, crack initiation and propagation, leaks in pressurized components and catastrophic failures. Hence, Non Destructive Inspection (NDI) are required at regular intervals and the results can be used for maintenance to mitigate fatigue risk (Schuëller, 1990). However, no in-service inspection is perfect. NDI outputs normally depend on many uncertain factors such as the condition of the inspected structure and its service environment etc. In order to take into account those uncertainties, the probabilistic approach is capable of identifying variables affecting the design life of the components. It has also been proved that the probabilistic method can be extended to provide very useful information to help managers in making decisions regarding the operation and inspection time of the structures in order to maintain their reliability.

In the present paper, a literature review of the current approaches and methodologies that has been utilized in the area of structural risk and reliability analysis for structures is presented. The parameters used to quantify the uncertainty and reliability of NDI technique is explored. Several probabilistic models regarding the updating flaw information and inspection schedule using different approaches are discussed. Finally, examples of such application on engineering structures are presented.

Nondestructive Inspection (NDI)

State-of-the-art nondestructive inspection (NDI) techniques provide an opportunity to obtain data on fatigue crack growth in service without damaging the structure. NDI plays an essential role in in-service condition assessment and repair decisions, especially when combined with methods of failure analysis derived from fracture mechanics. The most common NDI techniques for metallic structures are Visual Inspection (VI), Penetrant Inspection (PI), Magnetic Particle Inspection (MI), Eddy Current (EC), Radiographic Inspection (RI), Ultrasonic Inspection (UI) and Acoustic Emission (AE) (ASM, 1989).

Each of the methods has its own advantage and disadvantage. For example, VI is the simplest and least expensive. However, the reliability of results depends strongly on the skill of the inspector. Inspection conditions such as illumination and surface condition are the other factor that should be taken into account on the reliability of results. Recently, the new NDI technique, namely AE, was introduced. However, there are still some disadvantages of using the AE method, i.e. interpreting error. Therefore one can say that no in-service inspection is perfect. NDI outputs normally depend on many uncertain factors such as the condition of the inspected structure and its service environment, the sensitivity of inspection equipment, material imperfections and operator training and skills. Neglecting these uncertainties not only may result in misinformed decision-making, but also may lead to unnecessary repair or damage, which later must be repaired. A rational approach to evaluating the role of these sources of uncertainty in structural condition assessment is needed. Probabilistic methods, with their framework for the rational analysis of uncertainty, provide such an approach.

Reliability of NDI technique

General Remarks

Although NDI becomes an essential tool for the assessment of an aging structure, it introduces additional uncertainty in the reliability analysis i.e. due to the uncertainties in the inspection technique itself. Several measures have been introduced including probability of detection (POD), flaw size measurement accuracy and so called "false call" probability (FCP). Those measures will be used to quantify the uncertainty and reliability of NDI technique as shown in the following sections:

Probability Of Detection (POD)

The Probability of Detection (POD) generally gives the probability of detecting cracks of various lengths and depths under various inspection conditions. In order to obtain the POD curve for a particular NDI method, flaws with various sizes are introduced into a test specimen and the ratio of number of flaws detected to the number of flaw existing are calculated. Figure 1 shows an example of the detection probability of crack detection in Titanium alloy plate taken from aircraft body. It can be seen that POD increases with flaw size and eventually attains a maximum value at which non-detection is governed by other complicating factors, e.g. human errors, that may dominate the detection process (Simonen, 1995).

From Figure 1, it can be seen that the POD curves of X-ray and Eddy current incorporate the possibility that a very large flaw may not be detected, and the POD curve of X-ray also gives false call probability, i.e. a nonzero probability of detection when there is actually no flaw. The POD curve of Ultrasonic immersion, which has been used in some studies, ignores the probability of false calls and non-detection of large flaws. It can be seen that the shape of the distribution is log-normal shape. Therefore, the "Log-odds" function has been developed and it widely accepted for approximating POD function. Hence, Berens (1989) suggested a log-odds function as follows;

$$POD(a) = \left(1 + \exp\left(-\left(\frac{\pi}{3}\left(\frac{\ln a - \mu}{\sigma}\right)\right)\right)\right)^{-1} ;a > 0 \tag{1}$$

Parameter $\mu = \ln a_{0.5}$, where $a_{0.5}$ is the median flaw size satisfying POD($a_{0.5}$); σ is related to the steepness of the POD(a) curve, a smaller value of σ being associated with a steeper POD(a) curve.

It is also suggested that the POD function can be formulated by the exponential distribution;

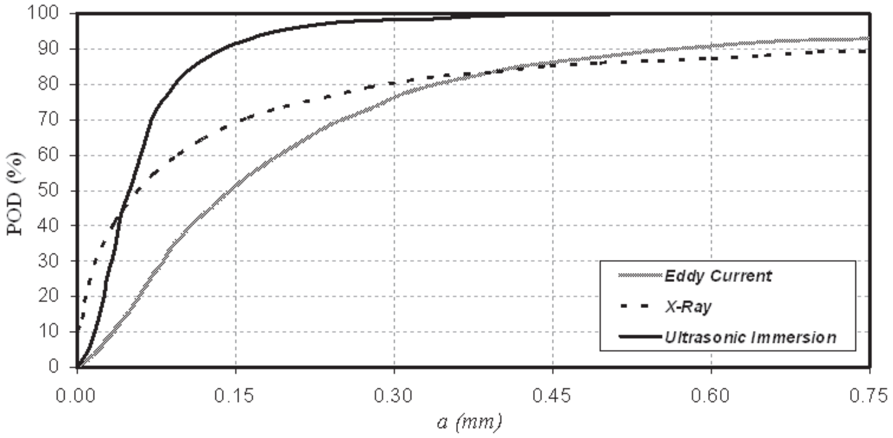

Figure 1. Comparison of POD Curves between three NDI Methods (NTIAC, 1997)

$$POD(a) = \begin{cases} 1 - \exp(-c(a - a_{th})); & a \geq a_{th} \\ 0 & ; \quad a \leq a_{th} \end{cases} \tag{2}$$

in which a_{th} is the minimum detectable flaw and c is a constant, both of which depend on the NDI device and its resolution (see e.g. Tsai and Wu (1993)).

In some situations, however NDI cannot detect even the large flaws. Therefore, an alternative expression of exponential POD can be shown as follows (Staat, 1993):

$$POD(a) = (1 - p)(1 - \exp(-ca)); \; a \geq 0 \tag{3}$$

where c is a parameter derived from experimental data. This POD(a) is asymptotically equal to 1-p for large values of a. Typically, p would be of the order of 0.01-0.05 for flaw sizes of practical interest. Figure 2 shows the comparison of POD curves.

However, the best of the statistical procedures for data analysis today are the log-odds model (Singh, 2000).

Chang *et al.* (1994) have derived the POD curve for X-ray inspection of welded joints of SAE4130 steel used in aircraft frames. They pointed out that uncertainties at each inspection are due to material properties, defect location, geometry and orientation. However for each inspection there is also the uncertainty due to different inspectors and the state of the equipment used. In their study they varied the defects, inspectors and equipment and adopted a log-normal distribution to describe the results.

Baker and Deschamps (1999) describe sub-sea inspections methods used and the locations inspected in fixed offshore structures. The most important method of nondestructive underwater inspection is direct visual inspection either using divers or remotely operated vehicles. Methods of detecting the occurrence of surface breaking defects of cracking include magnetic particle inspection, eddy current inspection and alternating current field measurement. Methods for measuring the depth of the defects include A-scan ultrasonic testing, alternating current potential drop and alternating current field measurement. A different and much less costly technique is the flooded member detection, which aims to detect water ingress due to through thickness crack. Detection is carried out by ultrasonic equipment or by Gamma-ray source and detector, using a ROV. However this detection is at a late stage of development of the crack. For early detection it is necessary to have divers with magnetic particle inspection or alternating current field measurement equipment which is much more costly.

Flaw size measurement error (FSME)

After NDI inspection has been performed on a structural component, the first step is to calibrate the measured flaw size with the actual flaw size in the structural component. An error in flaw sizing measures the difference between the true size of

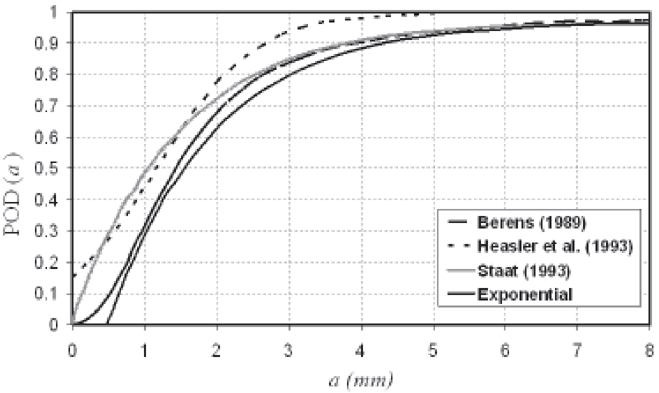

Figure 2. Schematic of POD model (Zheng and Ellingwood, 1998)

the flaw and its estimated size as measured by the NDI technique. By performing regression analysis, the relationship between the actual and measured flaw size can be obtained (Tang, 1973; Jiao, 1989; Rummel *et al.*, 1989) as shown in Equation (4)

$$C_a = \alpha + \beta C_m + \varepsilon \qquad (4)$$

where C_a, C_m are the actual and measured flaw sizes respectively (the size could be length, width, depth, etc.); α, β are coefficients determined from linear regression analysis on a particular set of data, and ε represents the calibration error which is a normal random variable with zero mean and standard deviation σ. In general, the parameters α, β and ε in the above calibration curve would be established for each NDI device, method of operation and pulse amplitude calibration, size characteristic (for example, length or depth, etc.), flaw characteristic (for example, surface cracks, inclusions, etc.), and material to be inspected.

However the actual size C_a is given while C_m is regressed on C_a in many practical laboratory test programs (Heasler *et al.*, 1993) as shown below;

$$C_m = \alpha + \beta C_a + \varepsilon \qquad (5)$$

False Call Probability (FCP)

False Call Probability (FCP) is normally defined as the fraction of times that an unflawed component or structural element will be incorrectly classified as being flawed. Hence, repair or replacement due to false call can lead to unnecessary economic cost and may result in potential damage. In order to obtain FCP, one can perform NDI on unflawed components. From Equation (2) and Equation (3), it can be seen that none of them are taking into account of false call probability. Thus the updating form of Equation (1) is introduced by Heasler *et al.* (1993);

$$POD(a) = (1 + \exp(-(\alpha + \beta a)))^{-1} \qquad (6)$$

By comparing the definition of POD and FCP, it can be seen that FCP is the value of POD at flaw size $a = 0$. Hence, FCP can be obtained from Equation (7).

$$FCP = POD(0) = (1 + \exp(-\alpha))^{-1} \qquad (7)$$

where α and β are parameters regressed from experimental data.

Relationship between POD, FCP and FSME

From the previous section, it can be seen that POD, FSME and FCP are used to describe different aspects of NDI performance. However they are not independent measures (Zhang and Mahadevan, 2001). The probability of detecting a crack size a can be expressed as a conditional probability;

$$POD(a) = P(C_m > 0 \mid C_a = a) \qquad (8)$$

where C_a is actual flaw size, C_m is measured flaw size.

Substituting C_m in Equation (8) with C_m in Equation (5) gives;

$$POD(a) = P(\alpha + \beta C_a + \varepsilon > 0) = P(\varepsilon < \alpha + \beta C_a) \qquad (9)$$

For the case of false call probability (FCP), it can be expressed as;

$$\begin{aligned} FCP &= P(C_m > 0 \mid C_a = 0) = P(\alpha + \varepsilon > 0) \\ &= P(\varepsilon < \alpha) = POD(0) \end{aligned} \qquad (10)$$

The random error, denoted by (ε), is assumed to be a random variable with density function following the normal distribution $\varepsilon \approx N(0, \sigma_\varepsilon)$. Therefore Equation (9) and Equation (10) become;

$$POD(a) = \Phi\left(\frac{\alpha + \beta C_a}{\sigma_\varepsilon}\right) \qquad (11)$$

and

$$FCP = \Phi\left(\frac{\alpha}{\sigma_\varepsilon}\right) \qquad (12)$$

where $\Phi(\cdot)$ is the standard normal distribution function.

Parameters α and σ_ε in Equation (11) and Equation (12) affect the POD and FCP as;

- σ_ε relates to the steepness of the POD curve in the way that a smaller value of σ_ε (a small variance of measurement accuracy) gives a steeper POD curve.
- A negative α reduces the probability of false calls while decreasing the probability of detecting when $a > 0$, and a positive α has the opposite effect.

Probabilistic model on reliability-based inspection

General Remarks

In the 1970's, the need for quality assurance of nondestructive inspection (NDI) methods was recognized and first attempts at assessing and modeling the inspection performance were made. In the same period it was first described how such models can be used to update probabilistic models of flaws (Tang, 1973). Since then quantitative models for the representation of the quality of NDI were developed mainly within the aerospace, nuclear and offshore industries for techniques aimed at the detection of flaws and cracks (Yang, 1994). Several studies have been reported in the literature to incorporate the information from inspection to update flaw information as summarized in the following sections.

Bayesian approach

The *Bayesian approach* has special significance to engineering design where available information is invariably limited and subjective judgment is often necessary. In the case of parameter estimation, the engineer often has some knowledge of the possible values, range of the values, of a parameter; moreover, he may also have some intuitive judgment on the values that are more likely to occur that others. Then if additional information becomes available, i.e. inspection results, the prior assumptions may be modified formally through Bayes' theorem. In this section a literature review on probabilistic model of reliability-based inspection using Bayesian approach is explored.

A framework for updating both size and density of flaws based on NDI data was first proposed by Tang (1973). The main concept is to pass a distribution of flaw size through a filter, which is defined by the detectability function that governs the reliability of each NDI device. The updated distribution for various repair levels on flaws detected is performed by using Bayes' Theorem. Based on the proposed formulation, an up-to-date description of flaw size and density can be shown in terms of probability distribution.

Since then the idea of updating failure probability using the information from non-destructive inspection (NDI) with the Bayesian approach was adopted. If Bayes' rule is applied to update a Probability Density Function (PDF) based on the observation of an event E_1, the posterior PDF of x can be written as (Madsen, 1987, 1997);

$$f''_X(x|E_1) = L(E_1|x).f'_x(x).const \tag{13}$$

$f''_X(x)$ is known as the posterior PDF of x, $f'_x(x)$ as the prior PDF. The constant in Equation (13) is determined by the condition that the integration of $f''_X(x)$ over the total domain of X must results in unity. Figure 3 shows the updating of a probability density function.

Itagaki *et al.* (1989); Itagaki and Yamamoto (1985); Fugimoto *et al.* (1989) adopted the Bayesian method in structural reliability analysis in order to determine appropriate inspections which are significant for continuing structural integrity. Due to the difficulty in determining the PDF governing the life a priori, the fatigue life of a member is defined as the duration between the beginning of service and the time when a fatigue crack is propagated to the minimum length detectable by visual inspection. Therefore, an estimation of an uncertain scale parameter β in a two-parameter Weibull distribution is assumed to be;

$$F_T(t|\beta) = 1 - \exp\left\{-\left(\frac{t}{\beta}\right)^{\alpha}\right\} \tag{14}$$

where t is time to crack initiation.

In addition, the length of a fatigue crack and probability of detecting a crack of length x are

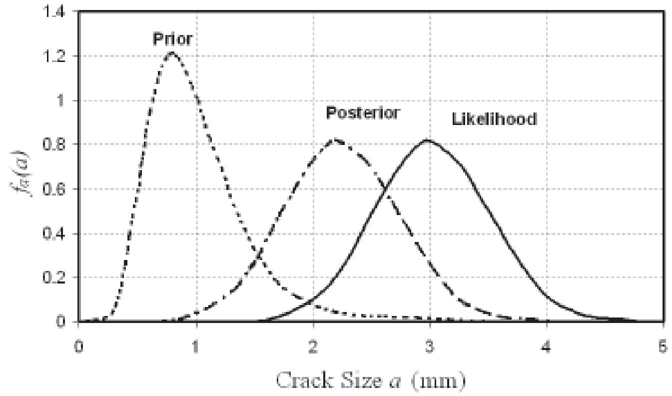

Figure 3. Illustration of the updating of a probability density function (Madsen, 1987, 1997)

assumed to be an exponential function as shown in Equations (15) and (16) respectively.

$$F_X(x|\lambda) = \left[1 - \exp\left\{-\lambda(\tau).(x-\mu)\right\}\right] \quad x \geq \mu \quad (15)$$

where μ is the minimum crack length. $\lambda(\tau)$ is a constant parameter at a certain time τ.

$$D(x|a) = 1 - \exp\{-a(x-b)\} \quad x \geq b \quad (16)$$

where b is the minimum crack length detectable by visual inspection.

From Equations (15) and (16) an average detectability can be derived as shown in Equation (17).

$$\overline{D}(\tau|a,\lambda) = \int_0^\infty D(x|a)\left\{dF_X(x|\lambda)/dx\right\}dx \quad (17)$$

Based on Equations (15)-(17), the three possible events at time of inspection are defined including, 1) a certain crack length of crack detected, 2) an existing crack but not detected and 3) no crack existing and no crack detected. After that Bayesian analysis is adopted to obtain the posterior density after inspection. Finally an expected reliability with respect to β and $\overline{D}$ is given;

$$\overline{R}_M(t) = \int_{\beta_1}^{\beta_2}\int_{\overline{D}_1}^{\overline{D}_2} R_M(t|\beta,\overline{D}).p^k(\beta,\overline{D})d\overline{D}d\beta \quad (18)$$

A stochastic model for fatigue crack growth, which accounts for uncertainties in loading, initial and critical crack sizes, material parameters, and the uncertainty related to computation of the stress intensity factor was studied by Skjong and Torhaug (1991). Failure probabilities were computed by first and second order reliability methods. Model updating based on in-service inspection results were formulated within the first order reliability method.

Deodatis *et al.* (1996, 1992); Ito *et al.* (1992) extended the Itagaki *et al.* (1989); Itagaki and Yamamoto (1985); Fugimoto *et al.* (1989) approach by adding the uncertainty in crack propagation into the model. The main purpose of their study is to determine appropriate non-periodic inspection intervals so that the reliability of fatigue-sensitive structures remains above a pre-specified minimum level throughout their service life. At an inspection the parameters of the models are updated due to the output of the inspection as well as a result of the repair actions. The updated model is used to decide the time of the following inspection. The main conclusion is that the first inspection can be delayed but the following ones should be done at shorter time intervals.

In contrast with the fatigue reliability analysis based on the knowledge of probability distribution of overall pre-service crack population at a weld joint, Pandey (1998) proposed probabilistic models for condition assessment of oil and gas pipelines since once the pipeline is buried under ground, information about the overall defect population becomes difficult to obtain. Therefore the only in-line inspection data represent the censored defect population as a result of imperfect inspection tool. In order to obtain the inspection time, the repair criterion is defined in terms of the Maximum Allowable Operating Pressure (MAOP) of the pipeline. After that condition assessment and maintenance planning is defined using Bayesian updating technique.

Rocha (1994); Rocha and Schuëll*er* (2005) used the Bayesian technique to update the detection probability. In their study, is assumed to be size dependent in which subsequent uncertainties in this dependence can be accounted for by considering one or more parameters in detection function to be random variables. They divided the event into two subevents;

1. A crack of size a is detected:

$$f''_A(\alpha_1) = \frac{P_D(\alpha_1|a)}{E[P_D(\alpha_1|a)]} f'_A(\alpha_1) \tag{19}$$

and

2. A crack of size a is not detected (but its existence is known):

$$f''_A(\alpha_1) = \frac{1 - P_D(\alpha_1|a)}{1 - E[P_D(\alpha_1|a)]} f'_A(\alpha_1) \tag{20}$$

where

$$E[P_D(\alpha_1|\alpha)] = \int_0^\infty P_D(\alpha_1|\alpha) f'_A(\alpha_1) d\alpha_1 \tag{21}$$

Another suggestion is to consider the entire possible event, i.e. undetected large flaw and false call probability. Many researchers (Byers *et al.*, 1997; Tang, 1973; Zhang and Mahadevan, 2001; Zheng and Ellingwood, 1998) suggest that the POD function should be incorporated directly into the failure probability updating to account for the uncertainties in the NDI performance more comprehensively including false call probability. Zhang and Mahadevan (2001) used the Bayes theorem to update failure probability as shown in Equation (22)

$$\begin{aligned} P_{f,up} &= P(a_c - a_N \le 0|\overline{D}) \\ &= \frac{P(\overline{D}|a_c - a_N \le 0)P(a_c - a_N \le 0)}{P(\overline{D})} \end{aligned} \tag{22}$$

$$P(\overline{D}) = \int_0^\infty (1 - POD(a)) f_{a_{N_0}}(a) \mathrm{d}a \tag{23}$$

where a_c is a critical crack size. a_N is in-service crack size corresponding to N stress cycles. $f_{a_{N_0}}$(a) is the Probability Density Function (PDF) of the crack size at the time of inspection.

Limit State Function approach

The classical Limit State Function (LSF) for the description of the event of detection, see e.g. Madsen (1987, 1997), is given in Equation (24). It is defined corresponding to different inspection results, and the failure probability, g_D, is updated directly based on inspection results.

$$g_D = s_D - s \tag{24}$$

where s_D and *s* are the detectable defect size and true defect size respectively.

Because the POD is a monotonically increasing function, the probability of detecting a crack smaller than or equal to *s* is POD(*s*). Therefore, if the POD asymptotically becomes 1 for very large crack sizes, s_D can be related to the POD by;

$$\begin{aligned} F_{s_D}(s) &= POD(s) \\ F_{s_D}(s) &= \frac{\mathrm{d}\, POD(s)}{\mathrm{d}s} \end{aligned} \tag{25}$$

The updated failure probability can be shown in Equation (26);

$$\begin{aligned} P_{f,up} &= P[g(D) \le 0|s_D - s = 0] \\ &= \frac{\frac{\partial}{\partial x} P[s_c - s \le 0 \le 0 \cap s_D - s \le x]_{x=0}}{\frac{\partial}{\partial x} P[s_c - s \le 0 \le x]_{x=0}} \end{aligned} \tag{26}$$

where s_c is a critical crack size and the derivatives are computed for x = 0.

It can be seen that the possible outcomes of inspection is not only the event of crack detected with size measurement (defined through the limit state function (Madsen, 1987; 1997)) but there are two more possible outcomes of inspection, i.e. no

crack detected and crack detected without size measurement. Madsen (1987; 1997) and Zhao *et al.* (1994) proposed the limit state for those remaining cases as follow;
For D: no crack detection:

$$s_D - s_d < 0 \tag{27}$$

For D: crack detection:

$$s_D - s_d > 0 \tag{28}$$

where s_d is the detectability of the particular NDI technique used.

However, there is the possibility that a very large flaw may not be detected and also gives false call probability. It can be seen that LSF introduced by Madsen (1987) does not cover the entire possible event. This led to the introduction of another formulation of LSF using Probability of an Indication, $P(I)$ (Hong, 1997). For a given multidimensional crack size distribution $f_{\underline{s}}(\underline{s})$, the probability of an indication can be written as;

$$P(I) = \int_{\underline{s}} POI(\underline{s}) f_{\underline{s}} \mathrm{d}s \tag{29}$$

Equation (29) can be rewritten as shown below;

$$P(I) = \int_{\underline{s}} \left[\int_0^{POI(\underline{s})} f_u(u) du \right] f_{\underline{s}}(\underline{s}) \mathrm{d}\underline{s} = \int_{g_1<0} f_u(u) f_{\underline{s}}(\underline{s}) \mathrm{d}u \mathrm{d}\underline{s} \tag{30}$$

where u is a uniformly distributed random variable with range from 0 to 1. $POI(\underline{s})$ is Probability Of Inspection (POI) of crack size $\underline{s}$. g_1 is a limit state function as shown in Equation (31).

$$g_1 = u - POI(\underline{s}) \tag{31}$$

Equation (31) can be rewritten as (Hong, 1997);

$$g_1 = z - \Phi^{-1}(POI(\underline{s})) \tag{32}$$

where z is a standard normal distributed random variable and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal distribution function.

Markov Chain approach

The deterioration processes, e.g. the fatigue process described by a crack growth is often modeled by a Markov process. The crack size as well as the time are utilized in a discretized form in order to apply a Markov Chain model. Kozin and Bogdanoff (1981, 1983) considered the process of fatigue crack propagation as a kind of Markov process. They gave the initial statistical distribution of crack length and calculated the mean, variance and distribution of crack length at an arbitrary time, as well as the number of cycles to reach the specified length of the Markov Chain. Their model can be directly fitted to experimental data. However, this makes predictions for other load conditions or geometrical configurations a difficult task. This problem can be circumvented by using a stochastic crack growth model for the determination of the transformation probabilities (Oswald and Schuëller, 1983, 1984a,b; Schuëller and Oswald, 1984). The method starts from dividing the range of the crack length into b interval of the length $\Delta a = \frac{W}{b}$ (with W as the component width), which is considered as "stage" of the crack length. After each load event the crack length stage will be equal or higher than before. Hence, the transition probability matrix P can be shown as follows

$$P = \begin{bmatrix} p_{0,0} & p_{0,1} & \cdots & \cdots & \cdots & p_{0,b} \\ 0 & p_{1,1} & \cdots & \cdots & \cdots & p_{1,b} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ 0 & 0 & \cdots & \cdots & \cdots & p_{b-1,b} \\ 0 & 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix} \tag{33}$$

where the transition probabilities $p_{i,k}$ are defined as

$$p_{i,k} = \int_{(k-i-\frac{1}{2})\Delta a}^{(k-i+\frac{1}{2})\Delta a} f(z) \mathrm{d}z \tag{34}$$

where z is the crack increment due to load event which is determined by using a crack propagation law (e.g. Paris-Erdogan law) based on random parameters.

Shimada *et al.* (1989) have extended Kozin and Bogdanoff's approach by indicating that fatigue crack propagation is a process having the constant ratio of r = p/q (where p and q are the elements in the stochastic transition matrix of the Markov Chain) and verifying the possibilities of applying the Markov Chain to fatigue crack propagation. By assuming the crack length or its state as the random variable, the study has been performed for the case of nondestructive inspection (NDI) to investigate the probability of a fracture vs. time relationship and the effect of the number of inspections on the relationship. The probability of detecting cracks can be defined as;

$$D_j = \text{Prob [detecting crack } | \text{ crack length } a_j] \quad (35)$$

If cracks are not detected until they reach a crack length a_{i-1}, and can first be detected when reaching a_j, D_j can be expressed as Equation (36)

$$\begin{aligned} &D_j = 0 \qquad (j = 1,2,\ldots,i-1) \\ &0 \le D_j \le 1 \quad (j = i,i+1,\ldots,b-1) \end{aligned} \qquad (36)$$

where b represents the critical crack state. This case will be developed by considering the two cases of the '*repair model*' and the '*replacement model*'.

In order to take into account the uncertainties related to the efficiency of inspection and repair, Rocha (1994); Rocha and Schuëller (2005) performed the analysis using Markov chain technique. The approach is based on the assumption that one-dimensional crack growth phenomena can be modeled by an equivalent discrete Markov process. They concluded that with respect to the computational aspects, the Markov Chain technique requires a convenient definition of representative discrete crack sizes, denoting the systems states, and the estimation of transition probabilities comprising the Transition Probability Matrix (TPM). For updating crack size, only simple vector-matrix multiplications have to be performed. A similar approach was adopted by Lassen (1991); Lassen and Sørensen (2002) to develop a stochastic model for a reliability analysis of the fatigue fracture of welded steel plate joints. After that the influence of scheduled inspection and repair is incorporated to the model.

Risk-Based Inspection Planning

Risk-based inspection involves the planning of an inspection on the basis of the information obtained from a risk analysis of the components. The purpose of the risk analysis is to identify the potential degradation mechanisms and threats to the integrity of the equipment and to assess the consequences and risks of failure. The inspection plan can then target the high-risk equipment and be designed to detect potential degradation before fitness-for-service could be threatened.

In general, inspection strategies have either been based in risk or in cost formulations. Also, in some industries there are fixed intervals between inspections, while others have variable intervals. In these cases the probabilistic models are updated with the results of each inspection and future planning uses the updated models. Application of probabilistic models for inspection in the offshore and marine industries was assessed e.g. by Skjong and Torhaug (1991) deriving optimum inspection plans considering the cost of design, inspection, repair and consequences cost of failure. They demonstrated that reliability based optimization programs based on First Order Reliability Method (FORM) results can be used for design optimization.

As stated above, a probabilistic cumulative fatigue damage model based on a simple Markov Chain approach was proposed by Lassen (1991). He developed a tool for a reliability assessment and a strategy for periodic inspection of welded joints in marine steel structures. The probabilities of taking into account crack initiation and crack threshold are discussed.

By considering maintenance schedule as a series of decision problems, a risk- based framework for structural maintenance planning can be obtained (Faber, 1997). It considers the reassessment of the status of a structure with fatigue cracks and the decisions about the rehabilitation, which need to be made along the life of the structure. The decisions are made on the basis of economic criteria and the inspection plans are updated with the results of inspections.

Berens and Burns (1994) described the risk-based approach to the maintenance planning for a fleet of aircraft. Cracks are considered at every frame and failure is the fracture resulting from the excess of the stress intensity factor at a detail, which is a realistic limit state condition for structures made of aluminum. They consider that at maintenance action the population of details is inspected and all detected cracks are repaired. Detection is according to a POD curve and the repair quality is expressed by an equivalent crack size distribution. The costs associated with different options of inspection intervals, inspection method and repair quality can be used to choose the least expensive solution.

A procedure developed and applied for optimization of quality assurance parameters, which determine the maintenance efforts required for welded structures and components, was proposed by Gasser and Schuëller (1999). The intention is to evaluate the quality of inspection of welded components such that expected cost is minimized. Maintenance costs, which depend on the inspection quality, and expected costs are also taken into account.

Cost-Based Inspection Planning

It is undoubtedly important in engineering to construct a more effective inspection strategy, which is obtained by paying attention to a physical feature of the structures. If reliability is the only criterion based on which the effect of inspections is quantitatively assessed, we take only a positive effect of inspections into consideration and inevitably arrive at an unrealistic conclusion that it is the best way to make as many inspections as possible. However, by introducing a cost-based criterion, we can make a quantitative assessment in consideration of both positive and negative effects brought out through inspections, e.g. the deterioration of the system availability and the increase of cost as a result of inspections and replacements. Such an assessment leads to a more effective and realistic inspection strategy.

Toyoda-Makino and Tanaka (1998); Toyoda-Makino (1999) proposed an optimal inspection strategy for random fatigue crack growth based upon cost-minimization, by the use of a diffusive crack growth model, where the randomness associated with the material inhomogeneity as well as the initial crack length is taken into analysis. They consider that a criterion based only on reliability is against the engineering reality, since it causes not only the deterioration of the system availability but also the increase of cost. It is more effective to determine the inspection strategy based upon a cost criterion. They also consider that periodical inspections are not always effective for fatigue failure, since a fatigue crack growth rate is gradually accelerated as the fatigue damage grows. Thus they propose an optimal inspection schedule against fatigue failure based upon cost-minimization.

Delmar and Sørensen (1992) presented a technique to provide information for decision-making considering uncertainty in a quantified way. The technique can be used to select the most important parameters so that only the necessary information is presented for decision-makers. It was shown how decision theory in combination with reliability theory and optimization theory could be used as a tool to consider the uncertainty.

In the method presented by Madsen *et al.* (1989) the safety against fatigue failure is achieved through design of individual elements, introduction of structural redundancy and inspection for fatigue cracks with subsequent repair of detected cracks. Different repair strategies were compared and the total expected cost of design, inspection repair, and failure was minimized. The optimization parameters were member thickness inspection times and inspection quantities.

Fujimoto and Mizutani (1994) presented a method for sequential cost minimization for the inspection-planning problem of fatigue deteriorating structures. The method aims at finding an optimal inspection strategy so that the total cost expected in the period between subsequent inspections is minimal.

Jiang *et al.* (1998) have proposed a mathematical framework of partially observable Markov decision processes to optimize the inspection and repair policies of structures. They account for costs and information content of various inspection strategies.

Application of NDI reliability on engineering structures

Several researchers have studied the reliability of engineering structures subjected to fatigue loading. This section will focus on the application of reliability-based inspection on planning the maintenance strategies.

Aircraft structures

Based on the application of random vibration theory, Yang and Trapp (1974) proposed a method for calculating the reliability of aircraft structures. Operational service loads were considered random. The fatigue process was described as crack initiation, crack propagation, and strength degradation. The outcome of the detection of an existing fatigue crack during inspection was also considered a random variable.

Several authors have dealt with aircraft structures under periodic inspection, such as Itagaki and Ito (1998); Asada *et al.* (1985); Fugimoto *et al.* (1989); Deodatis *et al.* (1992); Itagaki *et al.* (1998). The main features of the approaches are to predict the reliability in the period up to the following inspection, which should be performed whenever the reliability reaches a threshold value. At an inspection the parameters of the models are updated due to the output of the inspection as well as a result of the repair actions. The updated model is used to decide the time of the following inspection. The main conclusions are that the first inspection can be delayed but the following ones should be done at shorter time intervals.

Offshore structures

The offshore industry currently requires that the structural integrity of fixed offshore platforms be ensured by inspecting them periodically. For a long time, decision on inspection, repair, and maintenance has been made by experienced engineers applying their judgment together with the appropriate deterministic analyses. However, it is now expected that, employing recently developed techniques based on structural reliability method considering the effect of uncertainties, inspection and maintenance scheduling can be made based on more quantified information.

Wirsching *et al.* (1990) formulated fatigue reliability analysis on the integrity of structural systems in offshore platforms subjected to variable amplitude loading and fracture under extreme loading, which includes a design, inspection and repair process to minimize life-cycle cost. A Monte Carlo simulation was employed for performing reliability analysis, given a program of periodic inspection and repair. A fracture mechanics model described fatigue crack growth. Model parameters and other design factors were considered as random variables. Probabilities of failure estimates are used for an economic value analysis to establish optimal strategies for design and for a maintenance schedule.

The paper by Baker and Deschamps (1999) deals with sub-sea inspections strategies, the inspection methods used, the locations inspected and the probability of inspection in fixed offshore structures.

Inspection and maintenance planning of pipeline under external corrosion considering generation of new defects is studied by Hong (1998) adopting a Markov process model. The matrix of probability transition, the probability of defect size detection and the probability distribution of sizing a detected defect is incorporated in estimating the probability of failure. The generation of new corrosion defects is modeled by a Poisson process.

Moan *et al.* (2000) presented a probabilistic inspection tool, which is used to provide information about where and when to perform inspections to detect fatigue crack growth. It is based on probabilistic fatigue crack growth model and has been calibrated with an extensive number of inspections in jacket platforms.

Conclusions

This paper presents state of the art in quality assurance issues of structures with particular emphasis on strength degradation. A literature review of the current approaches and methodologies that has been utilized in the area of structural risk and reliability analysis for structures is presented. The parameters used to quantify the uncertainty and reliability of NDI technique is explored. Several probabilistic models regarding to the updating flaw information and inspection schedule using different approaches are discussed. Finally, the examples of such application on engineering structures are illustrated.

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