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Original Article

Asymptotic stability of time-varying delay-difference system of cellular neural networks via matrix inequalities and application

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Abstract

In this paper, we obtain some criteria for determining the asymptotic stability of the zero solution of time-varying delay-difference system of cellular neural networks in terms of certain matrix inequalities by using a discrete version of the Lyapunov second method. The result is applied to obtain new stability conditions for some classes of time-varying delay-difference equations such as time-varying delay-difference system of cellular neural networks with multiple delays in terms of certain matrix inequalities. Our results can be well suited for computational purposes.

Keywords: asymptotic stability, cellular neural networks, Lyapunov function, time- varying delay-difference system, matrix inequalities

1. Introduction

In recent decades, cellular neural networks have been extensively studied in many aspects and successfully applied to many fields such as pattern identifying, voice recognizing, system controlling, signal processing systems, static image treatment, and solving nonlinear algebraic equations, etc. Such applications are based on the existence of equilibrium points, and qualitative properties of systems. In electronic implementation, time delays occur due to various reasons such as circuit integration, switching delays of the amplifiers and communication delays, etc. Therefore, the study of the asymptotic stability of cellular neural networks with delays is of particular importance to manufacturing high quality microelectronic cellular neural networks.

While stability analysis of continuous-time neural networks can employ the stability theory of differential equations by Wei et al. (2005), it is much harder to study the stability of discrete-time neural networks by Gubta and Jin (1996) with time delays by Arik (2005) or impulses by Liu *et al.* (2003). The techniques currently available in the literature for discrete-time systems are mostly based on the construction Lyapunov second method by Infante (1978). For Lyapunov second method, it is well known that no general rule exists to guide the construction of a proper Lyapunov function for a given system. In fact, the construction of the Lyapunov function becomes a very difficult task.

In this paper, we consider time varying delay-difference system of cellular neural networks of the form

$$u(k+1) = -C(k)u(k) + A(k)S(u(k)) + B(k)S(u(k-h)) + f, \quad (1)$$

where $u(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $h \geq 0$, $C(k) = diag\{c_1(k), \ldots, c_n(k)\}$, $c_i(k) \geq 0$, $i = 1, 2, ..., n$ is the $n \times n$ relaxation matrix function, $A(k) = (a_{ij}(k))_{n\times n}$ and $B(k) = (b_{ij}(k))_{n\times n}$ are the $n \times n$ weight matrix functions, $f = (f_1, \ldots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1,1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, ..., n$.

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The asymptotic stability of the zero solution of the delay-differential system of cellular neural networks has been developed during the past several years. We refer to monographs by Arik (2002) and Chua and Yang (1988) and the references cited therein. Much less is known regarding the asymptotic stability of the zero solution of the time- varying delay-difference system of cellular neural networks. Therefore, the purpose of this paper is to establish sufficient conditions for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

2. Preliminaries

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all non-negative real numbers; $\mathbb{Z}^+$ denotes the set of all non-negative integers; $\mathbb{R}^n$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|\cdot\|$ and the scalar product between x and y is defined by $x^T y$; $M^{n\times m}$ denotes the space of all $(n\times m)$ -matrices; and A^T denotes the transpose of the matrix A; is the symmetric matrix if .

We assume that the neuron activation functions are bounded and satisfy the following hypotheses, respectively:

$$0 \le \frac{s_i(r_1)-s_i(r_2)}{r_1-r_2} \le l_i, \ \forall r_1, r_2 \in \mathbb{R}, \text{ and } r_1 \ne r_2, \qquad (2)$$

where $l_i > 0$ are constants for $i = 1,2,...,n$.

By assumption (2) we know that the functions $s_i(\cdot)$ satisfy

$$|s_i(x_i)| \le l_i |x_i|, \quad i = 1,2,...,n,$$

and

$$s^2_i(x_i) \le l_i x_i s_i(x_i), \qquad (3)$$

Matrix $Q \in \mathbb{R}^{n\times n}$ is positive semidefinite $(Q \ge 0)$ if $x^T Q x \ge 0$, for all $x \in \mathbb{R}^n$. If $x^T Q x > 0 (x^T Q x < 0$, resp.) for any $x \ne 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0, (Q < 0,$ resp.). It is easy to verify that $Q > 0$, $(Q < 0$, resp.) Iff

$$\exists \beta > 0: \ x^T Q x \ge \beta \|x\|^2, \forall x \in \mathbb{R}^n,$$

$$(\exists \beta > 0: \ x^T Q x \le -\beta \|x\|^2, \forall x \in \mathbb{R}^n, \text{ resp.});$$

Matrix function $Q(t) \in M^{n\times n}$ is positive definite if

$$\exists \beta > 0: \ x^T Q(t) x \ge \beta \|x\|^2, \ \ \forall t \in \mathbb{R}^+, \ \ x \in \mathbb{R}^n.$$

Fact 2.1 For any positive scalar ε and vectors x and y, the following inequality holds:

$$x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y.$$

Let us denote $V_\delta = \{x \in \mathbb{R}^n : \ \|x\| < \delta\}$.

Lemma 2.1 (Hale (1977)) The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(k, x(k)) : \mathbb{R}^n \to \mathbb{R}^+$ such that

$$\exists \beta > 0 : \Delta V(k, x(k)) = V(k, x(k+1)) - V(k, x(k)) \le -\beta \|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is locally asymptotically stable.

We present the following technical lemmas, which will be used in the proof of our main result.

Lemma 2.2 (Chua and Yang (1988)) For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M = M^T > 0$, scalar $s \in \mathbb{Z}^+ / \{0\}$, vector function $W : [0, s] \to \mathbb{R}^n$, we have

$$s \sum_{i=0}^{s-1} (w^T(i) M w(i)) \ge \left(\sum_{i=0}^{s-1} w(i) \right)^T M \left(\sum_{i=0}^{s-1} w(i) \right).$$

3. Main results

In this section, we consider the asymptotic stability of the zero solution u^* of (1) in terms of certain matrix inequalities. With out loss of generality, we can assume that $u^* = 0, S(0) = 0$ and $f = 0$ (for otherwise, we let $x = u - u^*$ and define $S(x) = S(x+u^*) - S(u^*)$).

The new form of (1) is now given by

$$x(k+1) = -Cx(k) + AS(x(k)) + BS(x(k-h)). \quad (4)$$

Theorem 3.1 The zero solution of the time-varying delay-difference system (4) is asymptotically stable if there exist symmetric positive definite matrices $P(k), G(k), W(k)$, and $L = diag[l_1, \dots, l_n] > 0$ satisfying the following matrix inequalities:

$$\psi = \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} < 0, \qquad (5)$$

where

$$\begin{aligned}
(1,1) \ = \ & C^T(k)P(k+1)C(k) - P(k) + hG(k) + W(k) \\
& + \varepsilon A^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k) \\
& + \varepsilon_1 C^T(k)P(k)B(k)B^T(k)P(k)C(k) \\
& + \varepsilon_2 LA^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)L \\
& + LA^T(k)P(k+1)A(k)L + \varepsilon^{-1}LL, \\
(2,2) \ = \ & LB^T(k)P(k+1)B(k)L + \varepsilon_1^{-1}LL \\
& + \varepsilon_2^{-1}LL - W(k), \text{ and} \\
(3,3) \ = \ & -hG(k).
\end{aligned}$$

Proof Consider the Lyapunov function $V(k, y(k)) = V_1(k, y(k)) + V_2(k, y(k)) + V_3(k, y(k))$ where

$$V_1(k, y(k)) = x^T(k)P(k)x(k),$$

$$V_2(k, y(k)) = \sum_{i=k-h+1}^{k} (h-k+i)x^T(i)G(k)x(i),$$

$$V_3(k, y(k)) = \sum_{i=k-h+1}^{k} x^T(i)W(k)x(i),$$

$P(k), G(k)$, and $W(k)$ being symmetric positive definite solutions of (5) and $y(k) = [x(k), x(k-h)]$.

Then difference of $V(k, y(k))$ along trajectory of solution of (4) is given by $\Delta V(k, y(k)) = \Delta V_1(k, y(k)) + \Delta V_2(k, y(k)) + \Delta V_3(k, y(k))$, where

$$\begin{aligned}
\Delta V_1(k, y(k)) &= V_1(k, x(k+1)) - V_1(k, x(k)) \\
&= [-C(k)x(k) + A(k)S(x(k)) \\
&\quad + B(k)S(x(k-h))]^T P(k+1) \\
&\quad \times[-C(k)x(k) + A(k)S(x(k)) \\
&\quad + B(k)S(x(k-h))] - x^T(k)P(k)x(k) \\
&= x^T(k)[C^T(k)P(k+1)C(k) - P(k)]x(k) \\
&\quad -x^T(k)C^T(k)P(k+1)A(k)S(x(k)) \\
&\quad - S^T(x(k))A^T(k)P(k+1)C(k)x(k) \\
&\quad -x^T(k)C^T(k)P(k+1)B(k)S(x(k-h)) \\
&\quad - S^T(x(k-h))B^T(k)P(k+1)C(k)x(k) \\
&\quad +S^T(x(k))A^T(k)P(k+1)B(k)S(x(k-h)) \\
&\quad +S^T(x(k-h))B^T(k)P(k+1)A(k)S(x(k)) \\
&\quad +S^T(x(k))A^T(k)P(k+1)A(k)S(x(k)) \\
&\quad +S^T(x(k-h))B^T(k)P(k+1)B(k)S(x(k-h)),
\end{aligned}$$

$$\Delta V_2(k, y(k)) = \Delta\left(\sum_{i=k-h+1}^{k} (h-k+i)x^T(i)G(k)x(i)\right) =$$

$$hx^T(k)G(k)x(k) - \sum_{i=k-h+1}^{k} x^T(i)G(k)x(i),$$

and

$$\Delta V_3(k, y(k)) = \Delta\left(\sum_{i=k-h+1}^{k} x^T(i)W(k)x(i)\right) = x^T(k)W(k)x(k)$$

$$- x^T(k-h)W(k)x(k-h), \qquad (6)$$

where (3) and Fact 2.1 are utilized in (6), respectively.

Note that

$$-x^T(k)C^T(k)P(k+1)A(k)S(x(k)) - S^T(x(k))A^T PCx(k) \le$$
$$\varepsilon x^T(k)C^T(k)P(k+1)A(k)A^T(k)P(k+1)C(k)x(k)$$
$$+\varepsilon^{-1}S^T(x(k))S(x(k)),$$

$$-x^T(k)C^T(k)P(k+1)B(k)S(x(k-h))$$
$$- S^T(x(k-h))B^T(k)P(k+1)C(k)x(k) \le$$
$$\varepsilon_1 x^T(k)C^T(k)P(k+1)B(k)B^T(k)P(k+1)C(k)x(k)$$
$$+\varepsilon_1^{-1}S^T(x(k-h))S(x(k-h)),$$

$$S^T(x(k))A^T(k)P(k)B(k)S(x(k-h))$$
$$+ S^T(x(k-h))B^T(k)P(k)A(k)S(x(k)) \le$$
$$\varepsilon_2 S^T(k)A^T(k)P(k)B(k)B^T(k)P(k)A(k)S(k)$$
$$+\varepsilon_2^{-1}S^T(x(k-h))S(x(k-h)),$$

$$S^T(x(k-h))B^T(k)P(k+1)B(k)S(x(k-h)) \le$$
$$x^T(k-h)LB^T(k)P(k+1)B(k)Lx(k-h),$$

$$S^T(x(k))A^T(k)P(k+1)A(k)S(x(k)) \le$$
$$x^T(k)LA^T(k)P(k+1)A(k)Lx(k),$$

$$\varepsilon_2 S^T(k)A^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)S(k)$$
$$\le \varepsilon_2 x^T(k)LA^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)Lx(k),$$

$$\varepsilon_1^{-1}S^T(x(k-h))S(x(k-h)) \le \varepsilon_1^{-1}x^T(k-h)LLx(k-h)$$

$$\varepsilon_2^{-1}S^T(x(k-h))S(x(k-h)) \le \varepsilon_2^{-1}x^T(k-h)LLx(k-h),$$

and

$$\varepsilon^{-1}S^T(x(k))S(x(k)) \le \varepsilon^{-1}x^T(k)LLx(k),$$

hence

$$\Delta V_1(k, y(k)) \le x^T(k)[C^T(k)P(k+1)C(k) - P(k)]x(k)$$
$$+\varepsilon x^T(k)A^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)x(k)$$
$$+\varepsilon_1 x^T(k)C^T(k)P(k+1)B(k)B^T(k)P(k+1)C(k)x(k)$$
$$+x^T(k-h)LB^T(k)P(k+1)B(k)Lx(k-h)$$
$$+x^T(k)LA^T(k)P(k+1)A(k)Lx(k)$$
$$+\varepsilon_2 x^T(k)LA^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)Lx(k)$$
$$+\varepsilon_1^{-1}x^T(k-h)LLx(k-h) + \varepsilon_2^{-1}x^T(k-h)LLx(k-h)$$
$$+\varepsilon^{-1}x^T(k)LLx(k).$$

Then we have

$$\Delta V(k, y(k)) \le x^T(k)[C^T(k)P(k+1)C(k) - P(k) + hG(k)$$
$$+W(k) + \varepsilon A^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)$$

$$+\varepsilon_1 C^T(k)P(k+1)B(k)B^T(k)P(k+1)C(k)$$
$$+\varepsilon_2 LA^T(k)P(k+1)B(k)B^T(k)P(k)A(k)L$$
$$+LA^T(k)P(k+1)A(k)L+\varepsilon^{-1}LL]x(k)$$
$$+x^T(k-h)[LB^T(k)P(k+1)B(k)L+\varepsilon_1^{-1}LL+\varepsilon_2^{-1}LL$$
$$-W(k)]x(k-h)-\sum_{i=k-h+1}^{k} x^T(i)G(k)x(i).$$

Using **Lemma 2.2**, we obtain

$$\sum_{i=k-h+1}^{k} x^T(i)G(k)x(i) \geq \left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG(k))\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right).$$

From the above inequality it follows that:

$$\Delta V(k,y(k)) \leq x^T(k)[C^T(k)P(k+1)C(k)-P(k)+hG(k)$$
$$+W(k)+\varepsilon A^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)$$
$$+\varepsilon_1 C^T(k)P(k+1)B(k)B^T(k)P(k+1)C(k)$$
$$+\varepsilon_2 LA^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)L$$
$$+LA^T(k)P(k+1)A(k)L+\varepsilon^{-1}LL]x(k)$$
$$+x^T(k-h)[LB^T(k)P(k+1)B(k)L+\varepsilon_1^{-1}LL+\varepsilon_2^{-1}LL$$
$$-W(k)]x(k-h)-\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)^T (hG(k))\left(\frac{1}{h}\sum_{i=k-h+1}^{k} x(i)\right)$$

$$=\left(x^T(k), x^T(k-h), (\frac{1}{h}\sum_{i=k-h}^{k-1} x(i))^T\right)\begin{pmatrix}(1,1) & 0 & 0\\ 0 & (2,2) & 0\\ 0 & 0 & (3,3)\end{pmatrix}\begin{pmatrix}x(k)\\ x(k-h)\\ (\frac{1}{h}\sum_{i=k-h}^{k-1} x(i))\end{pmatrix}$$

$$= y^T(k)\begin{pmatrix}(1,1) & 0 & 0\\ 0 & (2,2) & 0\\ 0 & 0 & (3,3)\end{pmatrix}y(k)$$

$$= y^T(k)\psi y(k),$$

where

$$(1,1) = C^T(k)P(k+1)C(k)-P(k)+hG(k)+W(k)$$
$$+\varepsilon A^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)$$
$$+\varepsilon_1 C^T(k)P(k)B(k)B^T(k)P(k)C(k)$$
$$+\varepsilon_2 LA^T(k)P(k+1)B(k)B^T(k)P(k+1)A(k)L$$
$$+LA^T(k)P(k+1)A(k)L+\varepsilon^{-1}LL,$$

$$(2,2) = LB^T(k)P(k+1)B(k)L+\varepsilon_1^{-1}LL+\varepsilon_2^{-1}LL-W(k),$$

$$(3,3) = -hG(k),$$

and $y(k)=\begin{pmatrix}x(k)\\ x(k-h)\\ \left(\frac{1}{h}\sum_{i=k-h}^{k-1} x(i)\right)\end{pmatrix}$.

By the condition (5), ΔV is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq -\beta\|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 2.1**. This completes the proof.

Example 3.1 Let us consider the time-varying delay-difference system (4), given by the system

$$x(k+1) = -C(k)x(k)+A(k)S(x(k))+B(k)S(x(k-h)),$$

where the matrices are

$$C(k)=\begin{pmatrix}e^{-2} & 0\\ 0 & e^{-2-0.5t}\end{pmatrix},\ A(k)=\begin{pmatrix}1.9-0.5e^{-5.8t}-e^{-5.8t} & 1\\ -e^{-5.8t} & 0.5e^{-t}-2\end{pmatrix},$$

$$B(k)=\begin{pmatrix}3-e^{6t} & -1\\ 0 & -0.5e^{-2t}\end{pmatrix},\ s_i(x_i)=\frac{2}{\pi}\tan^{-1}(x_i), i=1,2,$$

$\varepsilon = 0.5$ and $h=1$.

Using the LMI Toolbox in MATLAB, we found that the LMIs in **Theorem 3.1** are feasible and

$$P(k)=\begin{pmatrix}e^{-5.8t} & 0\\ 0 & 1\end{pmatrix},\ G(k)=\begin{pmatrix}1 & 0\\ 0 & e^{-0.5t}\end{pmatrix},\ W(k)=\begin{pmatrix}e^{-t} & 0\\ 0 & 1\end{pmatrix},$$

and $L=\begin{pmatrix}2.7293 & 0\\ 0 & 2.8806\end{pmatrix}$ are set of solutions to the LMIs (5).

Therefore, the system is asymptotically stable.

For a given initial condition $x(\theta)=[-0.5,-2]^T$, convergence behavior of is shown in Figure 1. As we can see

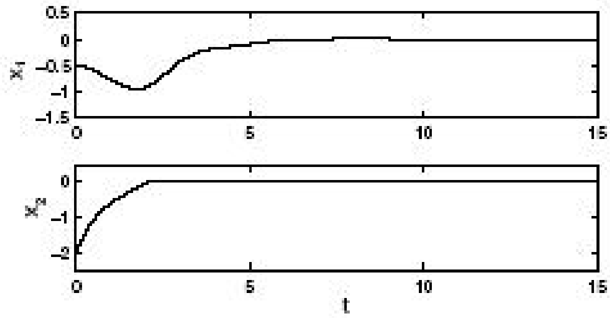

Figure 1. The convergence dynamics of the system in example 3.1.

from this figure, the steady state of cellular neural networks is indeed asymptotically stable.

4. Application

In this section, we apply the main result to obtain stability conditions for some specific classes of time-varying delay-difference equation. We consider time-varying delay-difference system of cellular neural networks with multiple delays in terms of certain matrix inequalities of the form

$$u(k+1) = -Cu(k) + AS(u(k)) + \sum_{i=1}^{m} B_i S(u(k-h_i)) + f, \quad (7)$$

where $u(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $0 \le h_1 \le \ldots \le h_m$, $C(k) = diag\{c_1(k), \ldots, c_n(k)\}$, $c_i(k) \ge 0$, i = 1,2, ...,n is the $n \times n$ relaxation matrix function, $A_i(k)$ and $B_i(k)$, $i = 1,2,...,m$ are the $n \times n$ weight matrices function, $f = (f_1, \ldots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1,1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1,2,...,n$.

We consider the asymptotic stability of the zero solution u^* of (7) in the terms of certa
in matrix inequalities. With out loss of generality, we can assume that $u^* = 0, S(0) = 0$ and f = 0 (for otherwise, we let $x = u - u^*$ and define $S(x) = S(x+u^*) - S(u^*))$.

The new form of (7) is now given by

$$x(k+1) = -A(k)x(k) + \sum_{i=1}^{m} B_i(k) S(x(k-h_i)). \quad (8)$$

Theorem 4.1 The zero solution of the time-varying delay-difference system (8) is asymptotically stable if there exist symmetric positive definite matrices $P(k), G_i(k)$, and $W_i(k)$, $i = 1,2,...,m$ and $L = diag[l_1, \ldots, l_n] > 0$ satisfying the following matrix inequalities:

$$\psi = \begin{bmatrix} (0,0) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & (1,1) & (1,2) & \cdots & (1,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & (2,1) & (2,2) & \cdots & (2,m) & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (m,1) & (m,2) & \cdots & (m,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & (m+1,m+1) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (m+2,m+2) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & (2m,2m) \end{bmatrix} < 0, \quad (9)$$

where

$$(0,0) = C^T(k)P(k+1)C(k) - P(k) + \sum_{i=1}^{m} h_i G_i(k) + W_i(k)$$
$$+\varepsilon \sum_{i=1}^{m}\sum_{j=1}^{m} A^T(k)P(k+1)B_i(k)B_j^T(k)P(k+1)A(k)$$
$$+\varepsilon_1 \sum_{i=1}^{m}\sum_{j=1}^{m} C^T(k)P(k+1)B_i(k)B_j^T(k)P(k+1)C(k)$$
$$+\varepsilon_2 \sum_{i=1}^{m}\sum_{j=1}^{m} LA^T(k)P(k+1)B_i(k)B_j^T(k)P(k+1)A(k)L$$
$$+LA^T(k)P(k+1)A(k)L + \varepsilon^{-1}LL,$$

$(1,1) = LB_1^T(k)P(k+1)B_1(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL - W_1(k)$,

$(1,2) = LB_1^T(k)P(k+1)B_2(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL$,

$(1,m) = LB_1^T(k)P(k+1)B_m(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL$,

$(2,1) = LB_2^T(k)P(k+1)B_1(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL$,

$(2,2) = LB_2^T(k)P(k+1)B_2(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL - W_2(k)$,

$(2,m) = LB_2^T(k)P(k+1)B_m(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL$,

$(m,1) = LB_m^T(k)P(k+1)B_1(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL$,

$(m,2) = LB_m^T(k)P(k+1)B_2(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL$,

$(m,m) = LB_m^T(k)P(k+1)B_m(k)L + \varepsilon_1^{-1}LL + \varepsilon_2^{-1}LL - W_m(k)$,

$(m+1,m+1) = -h_1 G_1(k)$,

$(m+2,m+2) = -h_2 G_2(k)$, and

$(2m,2m) = -h_m G_m(k)$.

Proof Consider the Lyapunov function $V(k,y(k)) = V_1(k,y(k)) + V_2(k,y(k)) + V_3(k,y(k))$, where

$V_1(k,y(k)) = x^T(k)P(k)x(k)$,

$$V_2(k,y(k)) = \sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} (h-k+i)x^T(j)G_i(k)x(j),$$

$$V_3(k,y(k)) = \sum_{i=1}^{m} \sum_{j=k-h_i+1}^{k} x^T(j)W_i(k)x(j),$$

$P(k), G_i(k)$, and $W_i(k)$, $i = 1,2,...,m$ being symmetric positive definite solutions of (9) and $y(k) = [x(k), x(k-h_1), \ldots, x(k-h_m)]$.

Then difference of $V(k,y(k))$ along trajectory of solution of (8) is given by $\Delta V(k,y(k)) = \Delta V_1(k,y(k)) + \Delta V_2(yk,(k)) + \Delta V_3(k,y(k))$, where

$$\Delta V_1(k,y(k)) = V_1(k,x(k+1)) - V_1(k,x(k))$$
$$= [-C(k)x(k) + A(k)S(x(k))x(k)$$
$$+ \sum_{i=1}^{m} B_i(k)S(x(k-h_i))]^T P(k+1)$$
$$\times[-C(k)x(k) + A(k)S(x(k))x(k)$$

$$+\sum_{i=1}^{m} B_i(k)S(x(k-h_i))] - x^T(k)P(k)x(k)$$

$$= x^T(k)[C^T(k)P(k+1)C(k) - P(k)]x(k)$$

$$-x^T(k)C^T(k)P(k+1)A(k)S(x(k))$$

$$-S^T(x(k))A^T(k)P(k+1)C(k)x(k)$$

$$-\sum_{i=1}^{m} x^T(k)C^T(k)P(k+1)B_i(k)S(x(k-h_i))$$

$$-\sum_{i=1}^{m} S^T(x(k-h_i))B_i^T(k)P(k+1)C(k)x(k)$$

$$+\sum_{i=1}^{m} S^T(x(k))A^T(k)P(k+1)B_i(k)S(x(k-h_i))$$

$$+\sum_{i=1}^{m} S^T(x(k-h_i))B_i^T(k)P(k+1)A(k)S(x(k))$$

$$+\sum_{i=1}^{m} S^T(x(k))A^T(k)P(k+1)B_i(k)S(x(k-h_i))$$

$$+\sum_{i=1}^{m} S^T(x(k-h_i))B_i^T(k)P(k+1)A(k)S(x(k))$$

$$+S^T(x(k))A^T(k)P(k+1)A(k)S(x(k))$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{m} S^T(x(k-h_i))B_i^T(k)P(k+1)B_j(k)S(x(k-h_j)),$$

$$\Delta V_2(k, y(k)) = \Delta\left(\sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} (h_i - k + j)x^T(j)G_i(k)x(j)\right)$$

$$= \sum_{i=1}^{m} h_i x^T(k)G_i(k)x(k) - \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)G_i(k)x(j),$$

$$\Delta V_3(k, y(k)) = \Delta\left(\sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k} x^T(j)W_i(k)x(j)\right)$$

$$= \sum_{i=1}^{m} x^T(k)W_i(k)x(k) - \sum_{i=1}^{m} x^T(k-h_i)W_i(k)x(k-h_i).$$

The rest of the proof is similar to that of Theorem 3.1. need hold.

5. Conclusions

In this paper, based on a discrete analog of the Lyapunov second method, we have established a sufficient condition for the asymptotic stability of time-varying delay-difference system of cellular neural networks in terms of certain matrix inequalities. The result has been applied to obtain new stability conditions for some classes of time-varying delay-difference equation such as delay-difference system of cellular neural networks with multiple delays in the terms of certain matrix inequalities.

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