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Original Article

Application of micro truss and strut and tie model for analysis and design of reinforced concrete structural elements

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Abstract

Generally structural members can be broadly divided into two regions, namely B or Bernoulli regions where the strain distributions are linear, and D or disturbed regions where the strain distributions are nonlinear. Even though the strut and tie model (STM) is an effective tool for the design of B and D regions, the designer should have a certain level of experience to develop the model. In this paper, the micro truss model, which can be considered as a generalization of STM, is used to develop STM. The micro truss models can be used to predict the nonlinear response of reinforced concrete structures, and this is illustrated by considering a simply supported deep beam. Micro truss models were used to develop the STM, and the beams were designed using these STM. The nonlinear response of the beams was simulated using the micro truss model, and it was found that the beams behaved satisfactorily until the service stage, and the ultimate load was greater than the design capacity of the beams.

Keywords: micro truss, nonlinear analysis, reinforced concrete, framework method, strut and tie model

1. Introduction

Those regions of a structural member in which the Bernoulli hypothesis of plane distribution of strain is valid are known as B regions, where B stands for beam or Bernoulli. The other regions where the strain distributions are nonlinear are known as D regions, where D stands for discontinuity or disturbance. While well defined theories are available for designing B regions, thumb rules or empirical equations are still being used to design D regions, though B and D regions are equally important. A strut and tie model (STM) offers an alternative to such empirical methods and it provides design engineers with a more flexible and intuitive option for designing structural elements. Since STM is a realistic approach, this has found place in many design standards like the American code (ACI 318-08), Australian code (AS 3600-2001), Canadian code (A23.3-04), Eurocode (BS EN-1992-1-1: 2004), Model code (CEB-FIP Model Code 1990), New Zealand code (NZS 3101: 2006), and others.

Though STM can be used to design reinforced concrete structures, it is essential that the designer should have a minimum level of experience to develop the appropriate truss. Further, STM can only be used to determine the limiting capacity and it cannot be used to predict the response of the structure. Hence it is not possible to check whether the structure designed using STM performs satisfactorily in the service stage.

In the present study, the reinforced concrete structural element is replaced by a simpler model known as micro truss model (Salem, 2004). In this study, micro truss model is based on the framework method (Hrennikoff, 1941) and uses

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a lattice type mesh. Micro truss can be considered as a generalization of STM and can be used to develop STM. Further, it can be used to predict the nonlinear response of reinforced concrete structural elements. For illustrating the use of micro truss model, a simply supported deep beam subject to different types of loading like central point load, two point load and uniformly distributed load (UDL) is considered. The STM for these cases are developed using micro truss model and are designed using STM. The nonlinear response of these beams is also predicted by using the micro truss model.

2. A brief review of STM

In STM, a reinforced concrete member is idealized as an equivalent truss, and analysed for applied loads. The compression and tension zones are converted into equivalent struts and ties, respectively, which are in turn connected at the nodes to form a statically admissible truss. It is assumed that the ties will yield before the strut fails. The various components in a STM are struts, ties, and nodes. Struts are compression members and the different types of struts are shown in Figure 1. Ties are the tension members and they represent reinforcing steel. Nodes form at points where struts and/or ties intersect. Nodes are described by the type of the members that intersect at the nodes. For example, a CCT node is one, which is bounded by two struts (C) and one tie (T). Using this nomenclature nodes are classified as CCC, CCT, CTT or TTT (Figure 2). For more details regarding STM, Schlaich *et al.* (1987) and SP-208 (2003) can be referred to.

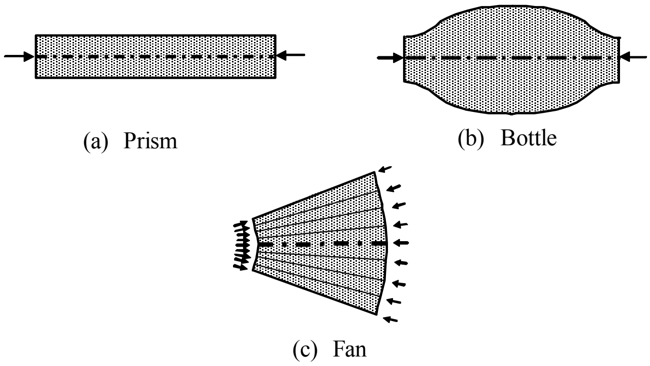


Figure 1. Different types of struts

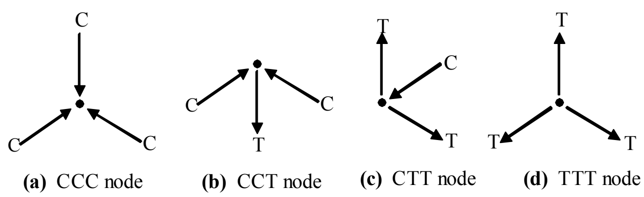


Figure 2. Different types of nodes

In the case of real truss, the identification of member areas, joint details and their design is fairly straight forward. However, in the case of an implicit truss embedded in concrete, the determination of appropriate member cross sectional areas and node dimensions is not so simple, especially for the determination of concrete strut and node dimensions. Hence, in the present study guidelines given in ACI 318-08 for the determination of dimensions and permissible stresses of struts and nodes are used. The safety factors recommended in IS 456-2000 are used for the design of deep beams.

3. Introduction to micro truss model

3.1 Background

As mentioned earlier, the micro truss is based on the framework method proposed by Hrennikoff (1941), in which the structure is replaced by an equivalent pattern of truss elements. One of the patterns proposed by Hrennikoff for modelling plane stress problems is shown in Figure 3.

In Figure 3, A_h, A_d and A_v are the cross sectional areas of horizontal, diagonal, and vertical truss members, respectively, and they are evaluated as:

$$A_h = \frac{3}{8}(3 - k^2)at \tag{1}$$

$$A_d = \frac{3}{16}\frac{(1+k^2)^{3/2}}{k}at \tag{2}$$

$$A_v = \frac{3}{8}\frac{(3k^2 - 1)}{k}at \tag{3}$$

In these expressions, t is the thickness of the plate. This pattern is accurate for materials having a Poisson's ratio equal to 1/3. This choice of pattern for reinforced concrete is justified by the fact that the Poisson's ratio for concrete changes from 0.2 to 0.4 from the elastic state to the failure stage (Kotronis *et al.*, 2003). The pattern of truss elements shown in Figure 3 is used to develop a micro truss model. Figure 4 shows the micro truss model for a reinforced concrete beam.

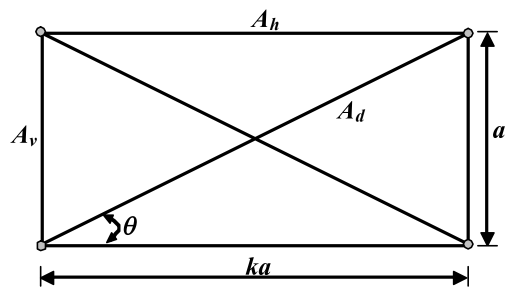


Figure 3. Pattern of truss elements for plane stress problems

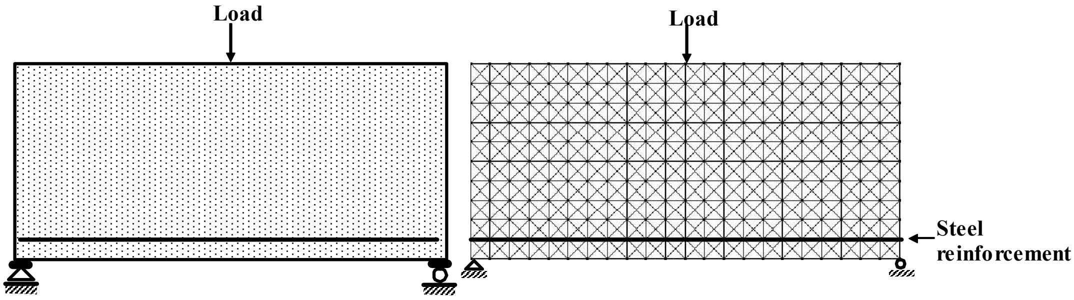


Figure 4. Micro truss model for a reinforced concrete beam

In Figure 4, it can be seen that the horizontal and vertical members resist the normal stress in the respective directions and the diagonal members resist the shear stress. Hence, this model can capture both flexure and shear type of failures in reinforced concrete structures. Further it can be seen that the steel bars can be easily simulated in this model.

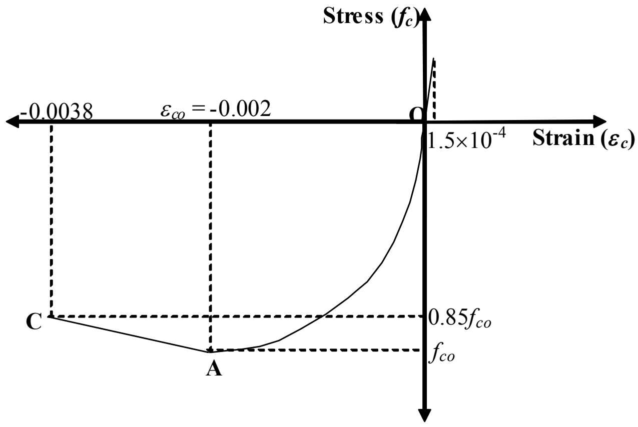
3.2 Development of STM

The STM can be developed by performing a linear analysis on the micro truss model. The compressive and tensile stress paths can be traced by picking up elements having large compressive and tensile forces. The struts can be oriented along the direction of elements having large compressive force and ties along members having large tensile forces.

3.3 Modelling features of reinforced concrete beams for nonlinear analysis

Since the size of truss elements is chosen to be small, the constitutive relations used should represent that of a micro level. That is, the constitutive relations for plain concrete and for bare steel bar should be used. For concrete in compression, the uniaxial stress-strain relation proposed by Hognestad (Park and Paulay, 1975) was used to construct the stress-strain curve. The stress-strain curve for concrete in tension was approximated as a straight line from the origin to the failure point (Figure 5(a)). The failure strain in uniaxial tension was taken as 1.5×10^{-4} and the modulus of elasticity in tension was taken to be the same as that in compression (Pillai and Menon, 1998). The stress-strain relation for steel is described by an elastic-perfectly plastic curve, whose initial slope is the elastic modulus of the material (Figure 5 (b)). Full compatibility between the steel and concrete at their interface is assumed.

The analysis was done using ANSYS 10.0, a finite element analysis software. A macro programme was written for developing the model and for performing the analysis. Geometric nonlinearity will be significant in the micro truss model because the relative effect of geometrical changes will be higher for small size elements. Hence, geometric nonlinearity was taken into consideration. The essential steps for the nonlinear analysis are given below.

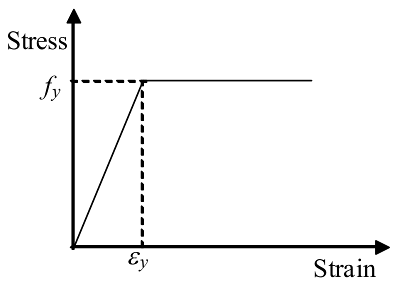


$$\text{OA} = f_c = f_{co}\left[2\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right]$$

AC = linear

f_{co} = Maximum compressive stress

(a) Stress-strain curve for concrete



f_y = Yield stress

ε_y = Yield strain

(b) Stress-strain curve for steel

Figure 5. Constitutive models for the constituent materials

1. The micro truss model for the structure is developed.
2. The expected ultimate load of the structure is applied in a number of load steps.
3. The load corresponding to the load step is applied and the structure is analyzed. Newton-Raphson (N-R) iterations are carried out in each step to arrive at a converged solution. ANSYS software sets the various options related to iterations, like the use of modified N-R iterations and the maximum number of iterations based on the problem parameters.
4. Significant parameters for the given load step, like the load value, displacements, and the axial strains in all the elements, are noted.
5. Those members in which the strain exceeds the failure strain are deactivated (that is, stiffness of the member is made nearly equal to zero, and hence effectively removed - element death options).
6. By removing the deactivated truss members from the original model, the crack pattern can be obtained at any load step.
7. Load corresponding to the next load step is applied and the steps from 3 to 6 are repeated until the analysis shows a divergence. The load corresponding to the last converged solution is taken as the ultimate load.

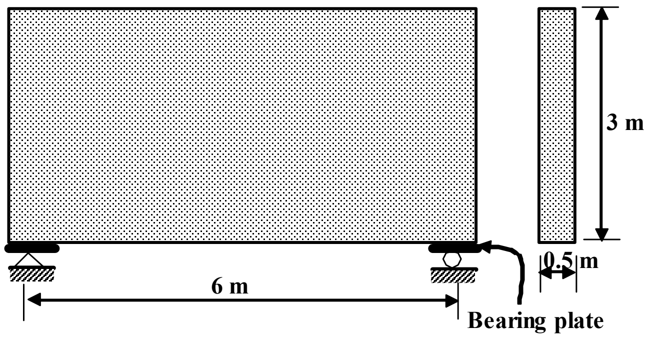


Figure 6. Simply supported deep beam

4. Design example

For illustrating the use of the micro truss model, a simply supported deep beam with a span (L) of 6 m and a depth of 3 m is considered. The details of the beam are shown in Figure 6. The grade of concrete is M30 (compressive strength = 30 MPa) and steel is Fe 415 (yield stress = 415 MPa). The size of the bearing plate is 0.45 m × 0.5 m. The beam is designed to resist a total service load P_S (including dead load) of 1000 kN (load factor as per IS 456:2000 is 1.5). Three different loading conditions (central point load,

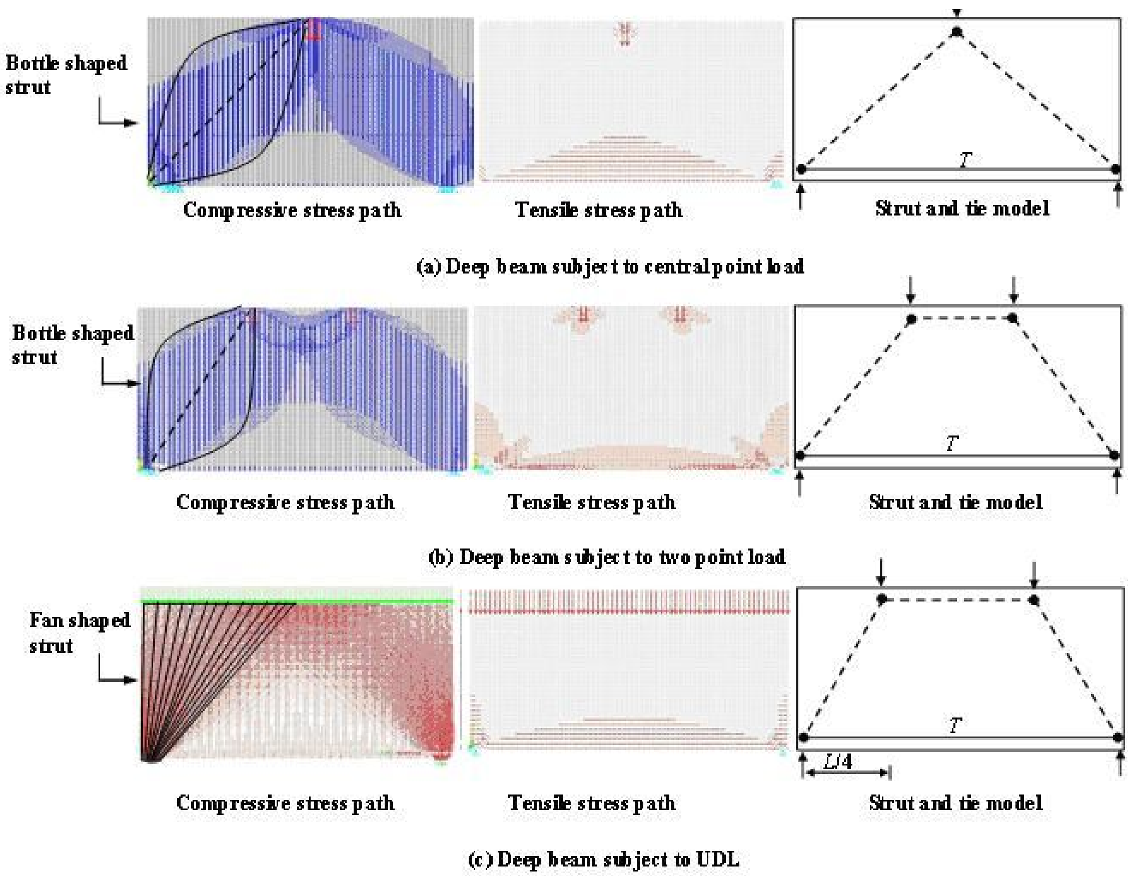


Figure 7. STM for deep beam

two point load and UDL) are considered for the design.

4.1 Development of STM

The STM are developed using the results obtained from the linear analysis performed on the micro truss model and are shown in Figure 7. It can be seen that, the type of strut that should be used to develop the STM can be obtained from the micro truss model. The STM for the deep beam subject to UDL was developed by replacing the load by equivalent concentrated loads (Nagarajan et al 2007). In Figure 7, struts are shown as dotted lines and ties are shown as solid lines. The beams are designed for an ultimate load P_U of 1500 kN (1.5 × service load) using the procedure given in SP-208 (2003) and the results of the design are summarized in Table 1. Once the force in the tie T is known, the area of tensile reinforcement A_{st} can be determined as:

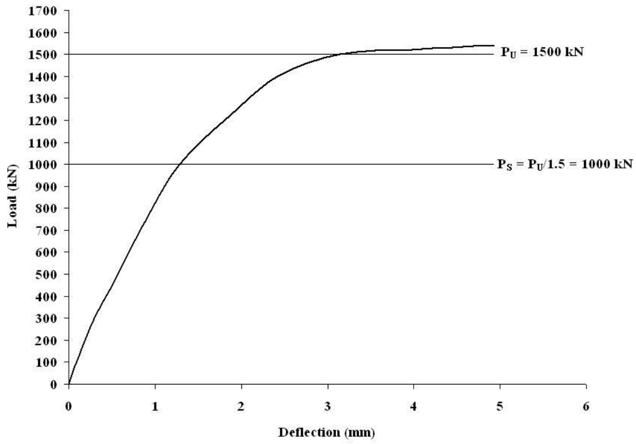
$$A_{st} = \frac{T}{\gamma_s f_y} \tag{4}$$

where γ_s is the partial (material) safety factor for steel, which is equal to 1.15 as per IS 456 code recommendations and f_y is the yield stress of the steel used.

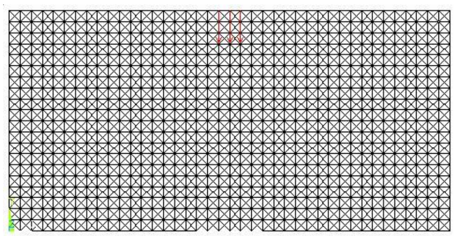
4.2 Nonlinear analysis of deep beams

For the analysis, the length of both horizontal and vertical elements in the micro truss was taken as 10 mm (a = 10 mm, k = 1, $\theta = 45^o$). This length was found to be small enough for simulating the crack propagation properly. The load deflection curves and the crack pattern at 1000 kN (service stage limit) and at ultimate load for the deep beams are shown in Figures 8 to 10. The ultimate loads of the beams obtained from the analysis are given in Table 2.

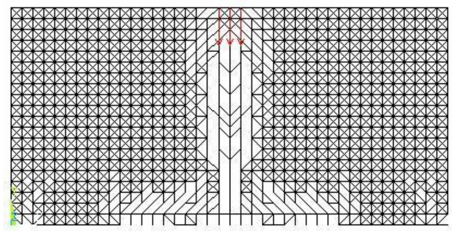
From Table 2, it can be seen that deep beams designed using STM have a capacity larger than the design load. This shows that STM gives a reasonable amount of safety from the design point of view. It can be seen that up to the service stage, the load deflection relation is almost linear and from the crack pattern obtained at 1000 kN it can be seen that the major portion of the beam remains un-cracked. Hence it can be concluded that, beams designed using STM behaves satisfactorily under service loads. The crack pattern at ultimate load for the beams shows a typical flexure failure that is ties fail before strut fails and this is one of the inherent assumptions of STM.



(a) Load-deflection at mid span



(b) Crack pattern at service stage



(c) Crack pattern at ultimate load

Figure 8. Results of deep beam subject to central point load

Table 1. Summary of design for deep beams

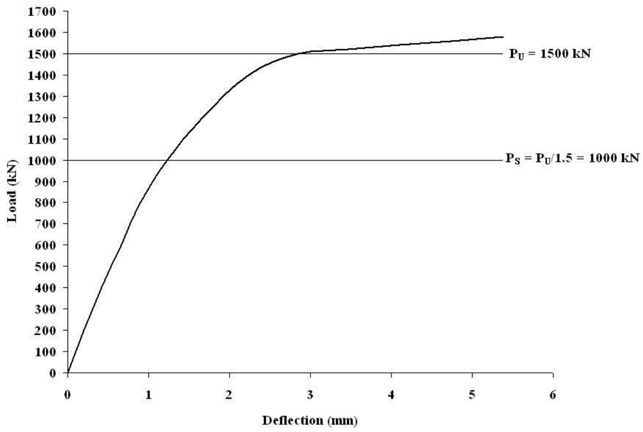
Load type	Height of truss (m)	A_{st} (mm^2)
Central point load	2.87	2093.28
Two point load	2.91	1427.67
UDL	2.93	1062.54

Table 2. Ultimate load of the beams

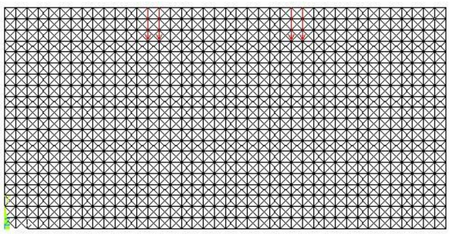
Load type	Ultimate load (kN)
	Analysis
Central point load	1540
Two point load	1580
UDL	1560

5. Conclusions

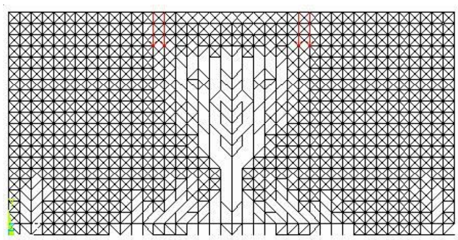
The micro truss model is a simplified method to simulate the behaviour of reinforced concrete structures. It is based on the framework method and uses lattice mesh to simulate reinforced concrete. The model can be considered as a generalization of STM. STM can be easily developed



(a) Load-deflection at mid span

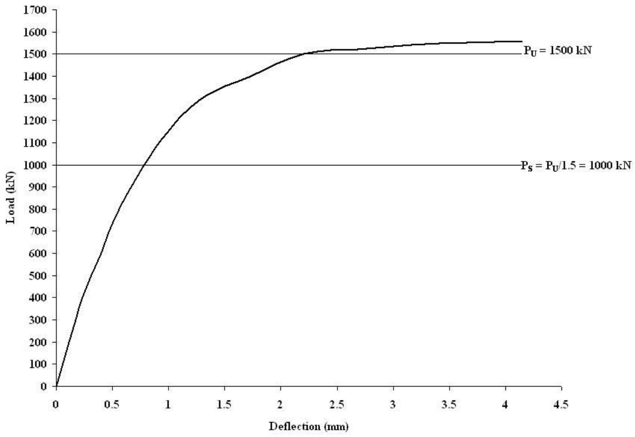


(b) Crack pattern at service stage

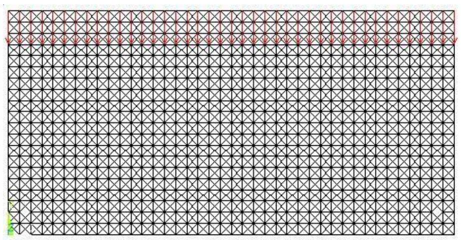


(c) Crack pattern at ultimate load

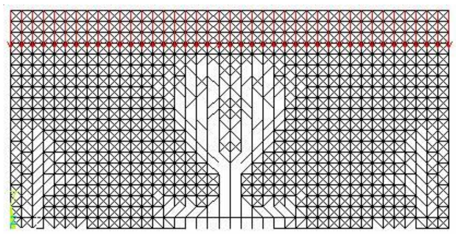
Figure 9. Results of deep beam subject to two point load



(a) Load-deflection at mid span



(b) Crack pattern at service stage



(c) Crack pattern at ultimate load

Figure 10. Results of deep beam subject to UDL

using the micro truss model. Further, micro truss can be used to predict the nonlinear response of concrete structures and it can be used to trace the crack pattern. The method is simple and faster than the two dimensional/three dimensional finite element method used for nonlinear analysis of reinforced concrete structures, since it uses only truss elements and simple constitutive laws for materials. It is shown that structures designed using STM behaves satisfactorily under service loads and will have a capacity greater than the design load.

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