Songklanakarin J. Sci. Technol.
30 (6), 767-774, Nov. - Dec. 2008

Songklanakarin Journal
of Science and Technology
http://rdo.psu.ac.th/sjst

Original Article

Bending of a uniformly loaded square plate resting on unilateral edge supports

Yos Sompornjaroensuk[1]* and Kraiwood Kiattikomol[2]

[1] *Department of Civil Engineering, Faculty of Engineering, Mahanakorn University of Technology, Nong Chok, Bangkok, 10530 Thailand.*

[2] *Department of Civil Engineering, Faculty of Engineering, King Mongkut's University of Technology Thonburi, Bangmod, Thung Khru, Bangkok, 10140 Thailand.*

Received 1 January 2008; Accepted 2 October 2008

Abstract

The objectives of this paper are to analyze the bending behaviors of unilaterally simply supported square plate subjected to the uniformly distributed load, and to examine the extent of receding contacts between the plate and the unilateral supports. In the present problem the mixed boundary conditions exist along the plate edges, which can be written in the form of dual series equations. These equations are further reduced to determine the solution of inhomogeneous Fredholm integral equation of the second kind for an unknown auxiliary function by using the finite Hankel integral transform techniques. Numerical results concerning the extent of receding contact, deflection, bending moment, twisting moment, and support reaction of the plate are given and also compared with the results obtained by other available techniques. From investigations, the conclusions can be stated that (i) the method used is found to be efficient for solving the problem considered, (ii) the extent of contact is independent of the level of loading, but dependent on the values of Poisson's ratio of the plate, and (iii) the support reactions are proportional to the applied load.

Keywords: dual series equations, Fredholm integral equation, Hankel integral transform, mixed boundary conditions, plate bending, receding contact

1. Introduction

Many problems of thin elastic simply supported plates having the right-angle corners anchored by the corner forces are resulting from the twisting moments at the corresponding corners to prevent parts of the plate near and including the corners bent away from the supports upon loading (Timoshenko and Woinowsky-Krieger, 1959). Since no corner forces are provided, the plate corners in general have a tendency to rise up from the supports and the plate is pertained to the natural receding contact problems (Dundurs and Stippes, 1970). This motivates researchers to investigate the actual extent of contact between the plate and the supports. Keer and Mak (1981) first analytically determined the loss of contact at the corner of semi-infinite plate resting on the unilateral supports, while the problem of unilaterally simply supported square plate was treated by Dempsey *et al.* (1984). Both problems were analyzed using the finite Fourier integral transforms in which the dual series equations that obtained from the mixed boundary conditions can be reduced to the Cauchy-type singular integral equation of the first kind. Dempsey and Li (1986) further extended the Fourier integral transform method used in the previous work to formulate the problems of rectangular plates with no sag and two opposite sagged supports. At the same time, Salamon *et al.* (1986) performed the finite element method to model the unilateral supports of square plates by using discrete elastic springs as supports around the plate. Another numerical

*Corresponding author.
Email address: yos@mut.ac.th

method was done by Hu and Hartley (1993) based on the direct boundary element method.

As described above, the corner forces play an important role especially in the case of a simply supported square plate to prevent the separation between the plate corners and the supports upon loading, thus the crucial step is the identification of the correct behaviors of the plate due to an absence of corner forces. For instance, the supports allow the plate to seek its natural contact (Dundurs and Stippes, 1970), and the shear distribution along the supports, which is singular at the tips of the contact intervals is in the order of an inverse square root type, first explained by Keer and Mak (1981). In addition, it is remarkable that for the numerical treatments of square plate cases presented by Salamon *et al.* (1986) and Hu and Hartley (1993), a singular distribution of support reactions does not consider at the transition points of support where the supports change to a free edge. With the best knowledge of the authors, there is only one analytical method (Keer and Mak, 1981; Dempsey *et al.*, 1984; Dempsey and Li, 1986) including singularities at the tips of the contacts to be used in the analysis for this class of problem.

Therefore, an alternative analytical method, the finite Hankel integral transform, is applied in the present work for solving the problem of square plate resting on the unilateral supports under uniformly distributed load. The correct singularity at the points of transition from unilateral support to free edge is taken into an account in the analysis, and the extent of contact is examined. Also the deflections and stress resultants of the plate are provided numerically and graphically in this paper.

2. Governing equation and boundary conditions

Based on the Levy-Nadai's approach (Timoshenko and Woinowsky-Krieger, 1959), the deflection of the plate can be taken in the form of single Fourier series. To simplify the analysis, the scaled square plate is then considered as shown in Figure 1, where the coordinates and dimensions are scaled by the factor π/a where a is the actual plate length. Therefore, the equation governing the deflection $w(x,y)$ of the plate under the uniform load q in the scaled coordinates (x,y) can be expressed as

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{qa^4}{\pi^4 D}, \tag{1}$$

where $D=Eh^3/12(1-\nu^2)=$ flexural rigidity, $E=$ Young's modulus, $\nu=$ Poisson's ratio, and $h=$ plate thickness. The symmetry of the geometry and the lateral load at the lines $x=\pi/2$ and $y=\pi/2$ is leading to the symmetry of the deflection function, thus it is necessary to consider only the region bounded by the upper left quadrant of the plate. The boundary conditions are as follows:

$$\frac{\partial w}{\partial y}=0 \quad : 0\le x\le\frac{\pi}{2} \ ; \ y=\frac{\pi}{2}, \tag{2}$$

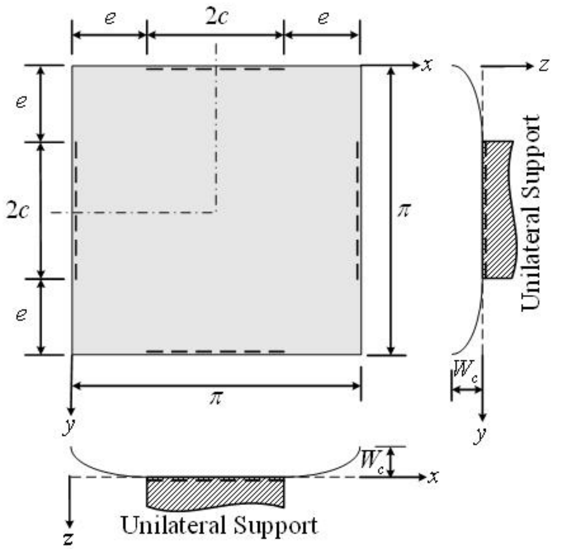

Figure 1. Geometry of square plates on unilateral edge supporters

$$V_y=0 \quad : 0\le x\le\frac{\pi}{2} \ ; \ y=\frac{\pi}{2}, \tag{3}$$

$$M_y=0 \quad : 0\le x\le\frac{\pi}{2} \ ; \ y=0, \tag{4}$$

$$w=\frac{\partial w}{\partial x}=0 \ : e<x\le\frac{\pi}{2} \ ; \ y=0, \tag{5}$$

$$V_y=0 \quad : 0\le x<e \ ; \ y=0, \tag{6}$$

$$w=W_c \quad : x=0 \ ; \ y=0, \tag{7}$$

whereas W_c is the lifted-up deflection at the plate corners, M_y is the bending moment in the direction parallel to the y-axis, and V_y is the supplemented or Kirchhoff shearing force normal to the y-axis. The stress resultants corresponding to the coordinates x,y of the plate can be expressed as (Timoshenko and Woinowsky-Krieger, 1959)

$$M_x=-D\left(\frac{\pi}{a}\right)^2\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right), \tag{8}$$

$$M_y=-D\left(\frac{\pi}{a}\right)^2\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right), \tag{9}$$

$$M_{xy}=D(1-\nu)\left(\frac{\pi}{a}\right)^2\frac{\partial^2 w}{\partial x\partial y}=-M_{yx}, \tag{10}$$

$$V_x=-D\left(\frac{\pi}{a}\right)^3\left[\frac{\partial^3 w}{\partial x^3}+(2-\nu)\frac{\partial^3 w}{\partial x\partial y^2}\right], \tag{11}$$

$$V_y=-D\left(\frac{\pi}{a}\right)^3\left[\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2\partial y}\right], \tag{12}$$

and the corner force R is given by the relation

$$R = 2D(1-\nu)\left(\frac{\pi}{a}\right)^2 \frac{\partial^2 w}{\partial x \partial y} = 2M_{xy}\,, \quad (13)$$

in which M_x, V_x can be explained in the same manner with M_y and V_y, respectively, and M_{xy} represents the twisting moment about the x-axis in the direction of y.

It is remarkable that the corner forces are considered as positive if they act on the plate in the downward direction in order to prevent the plate corners from rising up during bending.

3. Formulation of an integral equation governing the plate problem

Utilizing the Levy-Nadai approach, the deflection of the plate automatically satisfies Equation (1), boundary condition of Equation (7), and the conditions of the plate edges ($w = \partial^2 w/\partial x^2 = 0$ at $x = 0,\pi$ and $w = \partial^2 w/\partial y^2 = 0$ at $y = 0,\pi$). Therefore, the total deflection function can be written in the form

$$w = w_p + w_c + W_c\,, \quad (14)$$

where

$$w_p = \frac{2qa^4}{\pi^5 D}\sum_{m=1,3,5,...}^{\infty} m^{-5}[\sin(mx)+\sin(my)]\,, \quad (15)$$

$$w_c = \sum_{m=1,3,5,...}^{\infty}[Y_m \sin(mx) + X_m \sin(my)]\,, \quad (16)$$

and

$$Y_m = \frac{qa^4}{2D}[A_m\cosh(my) + B_m my\sinh(my) + C_m\sinh(my) + D_m my\cosh(my)]\,, \quad (17)$$

$$X_m = \frac{qa^4}{2D}[A_m\cosh(mx) + B_m mx\sinh(mx) + C_m\sinh(mx) + D_m mx\cosh(mx)]\,. \quad (18)$$

Substituting Equation (14) into Equations (2), (3), and (4) leads to the relations of unknown constants A_m, B_m, and C_m in terms of D_m as follows:

$$A_m = \frac{4\nu\eta'}{\pi^5 m^5} + 2D_m\eta'\coth\beta\,, \quad (19)$$

$$B_m = -D_m\coth\beta\,, \quad (20)$$

$$C_m = -\frac{4\nu\eta'}{\pi^5 m^5}\tanh\beta - D_m[2\eta' + \beta(\tanh\beta - \coth\beta)]\,, \quad (21)$$

in which

$$\eta' = \frac{1}{1-\nu}\;;\; \beta = \frac{m\pi}{2}\,. \quad (22a,b)$$

The remaining boundary conditions as given in Equations (5) and (6) are selected to be mixed with respect to the slope and the shear (Dempsey *et al.*, 1984) for the permission of the dual series equations to be reduced easily into the proper form for solution. By substituting Equation (14) into Equations (5b) and (6), the dual series equations can be obtained in the following forms:

$$\sum_{m=1,3,5,...}^{\infty} mP_m\cos(mx) = 0\;;\; e < x \le \frac{\pi}{2}\,, \quad (23)$$

$$\sum_{m=1,3,5,...}^{\infty}\Big\{m^3P_m(1+F_m^{(1)})\sin(mx) + m^3P_m[F_m^{(2)}\sinh(mx) - 2\eta\cosh(mx) + F_m^{(2)}mx\cosh(mx) - \eta\, mx\sinh(mx)]\Big\}$$
$$= \sum_{m=1,3,5,...}^{\infty}[F_m^{(4)}\sin(mx) + F_m^{(5)} + F_m^{(6)}\sinh(mx) - F_m^{(5)}\cosh(mx) + F_m^{(7)}mx\cosh(mx) - F_m^{(8)}mx\sinh(mx)]\;;\; 0 \le x < e\,, \quad (24)$$

where

$$P_m = \frac{2}{\pi^5 m^5} + D_m\coth\beta\,, \quad (25)$$

$$\eta = \frac{1-\nu}{3+\nu}\,, \quad (26)$$

$$1 + F_m^{(1)} = \frac{(3+\nu)\sinh\beta\cosh\beta - (1-\nu)\beta}{(3+\nu)\cosh^2\beta}\,, \quad (27)$$

$$F_m^{(2)} = \eta(2\tanh\beta + \beta\,\mathrm{sech}^2\beta)\,, \quad (28)$$

$$F_m^{(3)} = \eta\tanh\beta\,, \quad (29)$$

$$F_m^{(4)} = \frac{2[(3-\nu)\tanh\beta - (1-\nu)\beta\,\mathrm{sech}^2\beta]}{(3+\nu)\pi^5 m^2}\,, \quad (30)$$

$$F_m^{(5)} = \frac{4}{(3+\nu)\pi^5 m^2}\,, \quad (31)$$

$$F_m^{(6)} = \frac{2[2\tanh\beta + (1-\nu)\beta\,\mathrm{sech}^2\beta]}{(3+\nu)\pi^5 m^2}\,, \quad (32)$$

$$F_m^{(7)} = \frac{2\eta\tanh\beta}{\pi^5 m^2}\,, \quad (33)$$

$$F_m^{(8)} = \frac{2\eta}{\pi^5 m^2}\,. \quad (34)$$

The solution techniques similar to those used by Stahl and Keer (1972); Kiattikomol *et al.* (1974); Kiattikomol *et al.* (1985) and Sompornjaroensuk and Kiattikomol (2008) for the problems of plate having anchors at the corners are applied to the present work, except that, the order of singularity is assumed to be an inverse square root in the shear instead of the moment (Keer and Mak, 1981; Dempsey *et al.*, 1984; Dempsey and Li, 1986; Sompornjaroensuk and Kiattikomol, 2006). This is due to the nature of contact problems in which the singularity in the moments cannot be allowed at the transition points from support to no contact (Dundurs and Stippes, 1970). By introducing the function P_m in the form of a finite Hankel integral transform

$$m^2 P_m = \int_0^e t\varphi(t) J_1(mt)dt\,;\; m = 1,3,5,...\,, \quad (35)$$

which automatically satisfies the first dual series equations in Equation (23), and $\varphi(\;)$, $J_1(\;)$ are the unknown auxiliary function and Bessel function of the first kind and order 1, respectively.

Utilizing the identity that presented by Stahl and Keer (1972) as

$$\sum_{m=1,3,5,...}^{\infty} J_1(mt)\cos(mx) = \frac{1}{2t} - \frac{xH(x-t)}{2t(x^2-t^2)^{1/2}} + \int_0^{\infty} \frac{I_1(ts)\cosh(xs)}{\exp(\pi s)+1} ds\,;$$
$$x + t < \pi\,, \quad (36)$$

where $H(\;)$ is the Heaviside unit step function and $I_n(\;)$ is the modified Bessel function of the first kind and order n. Also using the identities given in Gradshteyn and Ryzhic (1980) and Abramowitz and Stegun (1964), hence, the second dual series equations given in Equation (24) are reduced to the following inhomogeneous Fredholm integral equation of the second kind,

$$\Psi(\rho) + \int_0^1 K(\rho,r)\Psi(r)dr = f(\rho)\,;\; 0 \le \rho, r \le 1\,, \quad (37)$$

where

$$\Psi(\rho) = \varphi(e\rho)\,;\; \Psi(r) = \varphi(er)\,, \quad (38)$$

$$K(\rho,r) = 2e^2 r \Bigg\{ \sum_{m=1,3,5,...}^{\infty} [-\frac{4}{\pi}\eta m - \eta m L_1(me\rho) - \eta m^2 e\rho L_0(me\rho)$$
$$+ mF_m^{(1)} J_1(me\rho) + m(F_m^{(2)} - F_m^{(3)}) I_1(me\rho)$$
$$+ m^2 F_m^{(3)} e\rho I_0(me\rho)] J_1(mer)$$
$$- \int_0^{\infty} s[\exp(\pi s)+1]^{-1} I_1(se\rho) I_1(ser) ds \Bigg\}\,, \quad (39)$$

$$f(\rho) = 2\sum_{m=1,3,5,...}^{\infty} [F_m^{(4)} J_1(me\rho) + (F_m^{(6)} - F_m^{(7)}) I_1(me\rho) + mF_m^{(7)} e\rho I_0(me\rho)$$
$$+ (F_m^{(8)} - F_m^{(5)}) L_1(me\rho) - mF_m^{(8)} e\rho L_0(me\rho)]\,, \quad (40)$$

in which $L_n(\;)$ is the modified Struve function of order n, and ρ, r are the dummy variables.

Since the deflection at the plate corners W_c has remained to be determined, it can be obtained by substituting Equations (19) to (21), together with Equation (25) and imposing $y = 0$ in Equation (14) leading to

$$w(x,0) = \frac{qa^4}{(1-\nu)D} \sum_{m=1,3,5,...}^{\infty} P_m \sin(mx) + W_c\,;\; 0 \le x \le \frac{\pi}{2}\,. \quad (41)$$

Using the function P_m presented by Equation (35) and the identity that shown below (Stahl and Keer, 1972),

$$\sum_{m=1,3,5,...}^{\infty} m^{-2} J_1(mt)\sin(mx) = \begin{Bmatrix} \frac{1}{4}\left[\frac{x}{t}(t^2-x^2)^{\frac{1}{2}} + t\sin^{-1}\left(\frac{x}{t}\right)\right]; & x < t \\ \frac{\pi}{8}t & ; \; x \ge t \end{Bmatrix}\,,$$
$$x + t < \pi\,, \quad (42)$$

it is found that the quantity of W_c can immediately be determined by applying the boundary condition given in Equation (5a). Therefore,

$$W_c = -\frac{qa^4\pi e^3}{8(1-\nu)D} \int_0^1 \rho^2 \Psi(\rho) d\rho\,. \quad (43)$$

It is interesting to note that the unknown constants presented by Equations (19) to (21) are related to the function $\Psi(\rho)$ by using Equations (25), (35) and (38), thus the deflection function as given in Equation (14) can be expressed in terms of function $\Psi(\rho)$ after performing Equation (37). However, the correct value of $\Psi(\rho)$ is still constrained with the condition of zero corner force due to the unilateral supports capable of exerting forces in one direction only, which differ from the problems of simply supported square plate. Therefore, the zero corner force condition can be determined by setting Equation (13) to be zero and after changing the variable $t = er$, $0 \le r \le 1$ with using Equation (35), that results as

$$e^2 \int_0^1 T(er) r \Psi(r) dr = B\,, \quad (44)$$

where

$$T(er) = \sum_{m=1,3,5,...}^{\infty} (\eta'' \tanh\beta - \beta\,\mathrm{sech}^2\beta) J_1(mer)\,, \quad (45)$$

$$B = 2\sum_{m=1,3,5,...}^{\infty} \left[\frac{1}{\pi^5 m^3}(\tanh\beta - \beta\,\mathrm{sech}^2\beta)\right]\,, \quad (46)$$

and

$$\eta'' = \frac{1+\nu}{1-\nu}\,. \quad (47)$$

4. Numerical results and Discussions

All physical quantities of the plate can be evaluated when the function $\Psi(\rho)$ is known by transforming Equation (37) to a system of linear algebraic equations with using the Simpson's rule as explained in Sompornjaroensuk and Kiattikomol (2006, 2007) and then, the function $\Psi(\rho)$ is solved numerically. However, the correct value of $\Psi(\rho)$ as well as the noncontact length e can be found by iteration until Equation (44) is satisfied. After that the deflection and stress resultants of the plate are, respectively, computed from Equation (14) and Equations (8) to (12). The results are compared with the other analytical (Dempsey *et al.*, 1984; Dempsey and Li, 1986) and numerical (Salamon *et al.*, 1986; Hu and Hartley, 1993) solutions as shown in Tables 1 to 3 and presented graphically in Figures 2 to 4.

It revealed that the extent of contact e/π is only depended on the Poisson's ratio ν of the plate as seen in Table 1. The free edge deflections near the corner and the deflec-

Table 1. The noncontact lengths for different values of the Poisson's ratio.

ν	e/π				
	Dempsey *et al.* (1984)	Dempsey and Li (1986)	Salamon *et al.* (1986)	Hu and Hartley (1993)	Present
0.10	0.307	0.300	0.3078	-	0.3015
0.15	-	-	0.2974	0.300	0.2922
0.30	0.268	0.262	0.2663	0.250	0.2626
0.50	0.224	0.218	0.2248	-	0.2188

Table 2. Deflections of square plates resting on unilateral edge supports.

ν	$\frac{x}{n}$	$w(x,0)/(qa^4/10^3D)$			$w(x,\pi/2)/(qa^4/10^3D)$			$w(x,x)/(qa^4/10^3D)$	
		Dempsey and Li (1986)	Hu and Hartley (1993)	Present	Dempsey and Li (1986)	Hu and Hartley (1993)	Present	Dempsey and Li (1986)	Present
0.10	0.0	-1.12	-	-1.1346	0.00	-	0.0000	-1.12	-1.1346
	0.1	-0.488	-	-0.4815	1.52	-	1.5307	0.278	0.2726
	0.2	-0.100	-	-0.0969	2.85	-	2.8549	1.74	1.7469
	0.3	0.000	-	0.0000	3.84	-	3.8517	3.18	3.1878
	0.4	0.000	-	0.0000	4.46	-	4.4641	4.26	4.2681
	0.5	0.000	-	0.0000	4.66	-	4.6699	4.66	4.6699
0.15	0.0	-	-	-1.0329	-	-	0.0000	-	-1.0329
	0.1	-	-	-0.4225	-	-	1.5011	-	0.2930
	0.2	-	-	-0.0754	-	-	2.8012	-	1.7158
	0.3	-	-	0.0000	-	-	3.7812	-	3.1277
	0.4	-	-	0.0000	-	-	4.3839	-	4.1908
	0.5	-	-	0.0000	-	-	4.5867	-	4.5867
0.30	0.0	-0.756	-0.7702	-0.7647	0.00	0.0000	0.0000	-0.756	-0.7647
	0.1	-0.283	-0.2923	-0.2721	1.43	1.4296	1.4299	0.347	0.3447
	0.2	-0.030	-0.0347	-0.0282	2.67	2.6709	2.6715	1.64	1.6393
	0.3	0.000	0.0000	0.0000	3.61	3.6101	3.6108	2.98	2.9820
	0.4	0.000	0.0000	0.0000	4.19	4.1896	4.1904	4.00	4.0043
	0.5	0.000	0.0000	0.0000	4.38	4.385	4.3857	4.38	4.3857
0.50	0.0	-0.479	-	-0.4817	0.00	-	0.0000	-0.479	-0.4817
	0.1	-0.135	-	-0.1262	1.37	-	1.3678	0.394	0.3930
	0.2	-0.001	-	-0.0014	2.56	-	2.5582	1.57	1.5691
	0.3	0.000	-	0.0000	3.46	-	3.4618	2.85	2.8535
	0.4	0.000	-	0.0000	4.02	-	4.0211	3.84	3.8410
	0.5	0.000	-	0.0000	4.21	-	4.2100	4.21	4.2100

tions along the center line and diagonal line of the plate are shown in Figure 2 corresponding with the numerical results given in Table 2. It can be seen that their magnitudes are increasing with decreasing Poisson's ratio.

The bending and twisting moments are presented in Figure 3. From the obtained results, with increasing the Poisson's ratio ν , both of the bending moments $M_x(x,\pi/2)$ and $M_y(x,\pi/2)$ are decreasing, but the twisting moments $M_{xy}(x,x)$ are increasing. In addition, the distribution of $M_{xy}(x,x)$ along the diagonal line of the plate increases in magnitude for all values of ν up to some certain maximum value and then decreases to zero at the center of the plate. It is important to note that the twisting moments are vanished at the plate corner for all values of the Poisson's ratio due to the condition of zero corner force. This satisfies the behaviors of plates with no anchoring at the plate corners.

Figure 4 illustrates the distributions of support reaction along the unilateral support. It can be observed that they are singular at the end point $x = e$ of the support when changed to a free edge (Dempsey *et al.*, 1984; Dempsey and

Table 3. Bending moments, twisting moments, and support reactions of square plates on unilateral edge supports.

v	$\frac{x}{n}$	$M_x(x,\pi/2)/(qa^2/100)$			$M_y(x,\pi/2)/(qa^2/100)$			$\frac{M_{xy}(x,x)}{(qa^2/100)}$	$V_y(x,0)/(qa/10)$		
		Dempsey and Li (1986)	Hu and Hartley (1993)	Present	Dempsey and Li (1986)	Hu and Hartley (1993)	Present		Dempsey and Li (1986)	Hu and Hartley (1993)	Present
0.10	0.0	0.00	-	0.0000	0.00	-	0.0000	0.0000	0.00	-	0.0000
	0.1	2.26	-	2.2699	1.44	-	1.4442	1.8633	0.00	-	0.0000
	0.2	3.56	-	3.5661	2.72	-	2.7255	2.1525	0.00	-	0.0000
	0.3	4.20	-	4.2040	3.70	-	3.7111	1.2763	-	-	0.0000
	0.4	4.46	-	4.4696	4.32	-	4.3280	0.3601	5.16	-	5.1095
	0.5	4.53	-	4.5383	4.53	-	4.5383	0.0000	4.97	-	4.9107
0.15	0.0	-	-	0.0000	-	-	0.0000	0.0000	-	-	0.0000
	0.1	-	-	2.2715	-	-	1.5312	1.8305	-	-	0.0000
	0.2	-	-	3.6060	-	-	2.8487	2.0541	-	-	0.0000
	0.3	-	-	4.2909	-	-	3.8460	1.1942	-	-	10.7430
	0.4	-	-	4.5920	-	-	4.4643	0.3346	-	-	4.9146
	0.5	-	-	4.6738	-	-	4.6738	0.0000	-	-	4.7784
0.30	0.0	0.00	0.0000	0.0000	0.00	0.0000	0.0000	0.0000	0.00	0.000	0.0000
	0.1	2.31	2.3087	2.3098	1.78	1.7786	1.7775	1.6937	0.00	0.000	0.0000
	0.2	3.75	3.7558	3.7569	3.21	3.2113	3.2100	1.7404	0.00	0.000	0.0000
	0.3	4.57	4.5728	4.5732	4.16	4.2521	4.2513	0.9586	5.28	6.780	5.2843
	0.4	4.97	4.9731	4.9731	4.88	4.8811	4.8808	0.2634	4.48	4.590	4.4507
	0.5	5.09	5.0914	5.0913	5.09	5.0914	5.0913	0.0000	4.47	4.488	4.4339
0.50	0.0	0.00	-	0.0000	0.00	-	0.0000	0.0000	0.00	-	0.0000
	0.1	2.42	-	2.4235	2.09	-	2.0912	1.4002	0.00	-	0.0000
	0.2	4.03	-	4.0270	3.69	-	3.6886	1.2766	0.00	-	0.0000
	0.3	5.00	-	5.0047	4.81	-	4.8064	0.6652	3.99	-	3.9739
	0.4	5.52	-	5.5221	5.47	-	5.4655	0.1794	4.24	-	4.0104
	0.5	5.68	-	5.6830	5.68	-	5.6830	0.0000	4.08	-	4.0644

Li, 1986), according to each case of the plate with various Poisson's ratio values as listed in Table 1. At this point, it can be concluded from the results presented in Figure 4 that the support reactions are proportional to the level of the applied loads (Dundurs and Stippes, 1970). This feature is opposed to the advancing contact problems (Dundurs *et al.*, 1974; Sompornjaroensuk and Kiattikomol, 2006) because the extent of receding contact is only depended on the values of the Poisson's ratio of the plate. The variation for the distribution of stress resultants with Poisson's ratio is tabulated and also compared with the results obtained by other methods as shown in Table 3.

5. Conclusions

This paper presents the natural receding contact between the square plate and the unilateral supports under the uniformly distributed load using the method of finite Hankel integral transform incorporating the square root shear singularity at the points of discontinuous support. The solution is first set up by utilizing the Levy-Nadai approach. The mixed boundary conditions resulting from the discontinuity of the boundary conditions are written in the form of dual series equations, and can further be reduced to the inhomogeneous Fredholm integral equation of the second kind in terms of an unknown auxiliary function, in which this function can conveniently be solved numerically using the Simpson's rule. The extent of contact between the plate and the unilateral supports and the physical quantities of deflections, bending and twisting moments, and support reactions are calculated and compared with results obtained by other investigators. It is seen that the present results are in close agreement with the results obtained by analytical methods (Dempsey *et al.*, 1984; Dempsey and Li, 1986), in which the inverse square root shear singularities have been included in their analysis, and in good agreement when compared with the numerical method (Hu and Hartley, 1993). However, the singularity is excluded in the latter results. This is the highlight of the present problem. From the analysis, the following conclusions can be drawn: the extent of contact is independent of the level of loading but dependent on the values of Poisson's ratio, the support reactions are proportional to the applied loads, and the proposed method is found to be efficient for solving the problem of plates with mixed boundary conditions.

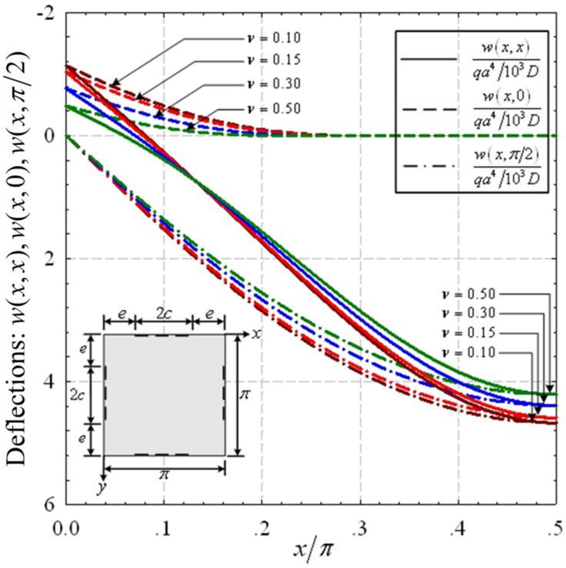

Figure 2. Deflections of square plates on unilateral edge supports.

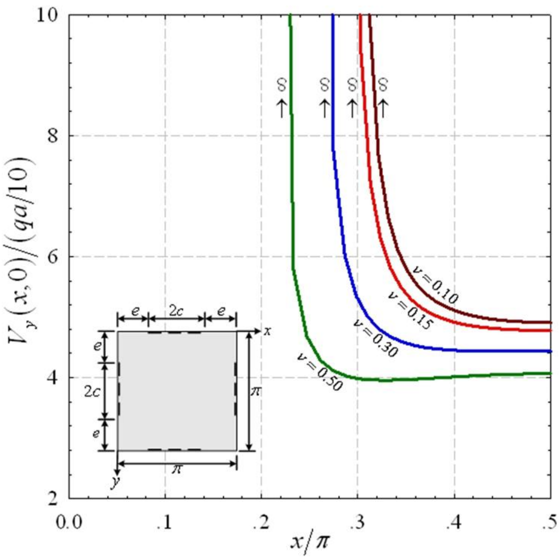

Figure 4. Support reactions of square plates on unilateral edge supports.

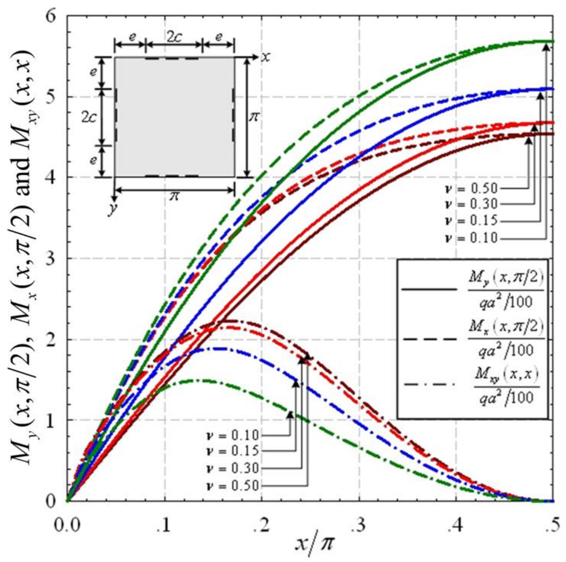

Figure 3. Bending and twisting moments of square plates on unilateral edge supports.

References

Abramowitz, M. and Stegun, I.A. 1964. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, Dover Publications, New York.

Dempsey, J.P. and Li, H. 1986. Rectangular plates on unilateral edge supports: Part 2-implementation; concentrated and uniform loading. Journal of Applied Mechanics, 53, 151-156.

Dempsey, J.P., Keer, L.M., Patel, N.B. and Glasser, M.L. 1984. Contact between plates and unilateral supports. Journal of Applied Mechanics, 51, 324-328.

Dundurs, J. and Stippes, M. 1970. Role of elastic constants in certain contact problems. Journal of Applied Mechanics, 37, 965-970.

Dundurs, J., Kiattikomol, K. and Keer, L.M. 1974. Contact between plates and sagged supports. Journal of the Engineering Mechanics Division, 100, 445-456.

Gradshteyn, I.N. and Ryzhik, I.M. 1980. Table of Integrals, Series, and Products, Academic Press, New York.

Hu, C. and Hartley, G.A. 1993. Boundary element analysis of thin plates unilaterally edge supported. Engineering Analysis with Boundary Element, 12, 47-55.

Keer, L.M. and Mak, A.F. 1981. Loss of contact in the vicinity of a right-angle corner for a simply supported, laterally loaded plate. Journal of Applied Mechanics, 48, 597-600.

Kiattikomol, K., Keer, L.M. and Dundurs, J. 1974. Application of dual series to rectangular plates. Journal of the Engineering Mechanics Division, 100, 433-443.

Kiattikomol, K., Porn-anupapkul, S. and Thonchangya, N. 1985. Application of dual series equations to a square plate with varying corner supported lengths. Proceedings of The Recent Advances in Structural Engineering, Bangkok, Thailand, Mar. 14-15, 115-130.

Salamon, N.J., Pawlak, T.P. and Mahmoud, F.F. 1986. Plates in unilateral contact with simple supports: pressure loading. Journal of Applied Mechanics, 53, 141-145.

Sompornjaroensuk, Y. and Kiattikomol, K. 2006. Singularity of advancing contact problems of plate bending. The Tenth East Asia-Pacific Conference on Structural Engineering and Construction, Bangkok, Thailand, Aug. 3-5, 193-198.

Sompornjaroensuk, Y. and Kiattikomol, K. 2007. Dual-series equations formulation for static deformation of plates with a partial internal line support. Theoretical and Applied Mechanics, 34, 221-248.

Sompornjaroensuk, Y. and Kiattikomol, K. 2008. Exact analytical solutions for bending of rectangular plates with a partial internal line support. Journal of Engineering Mathematics, 62, 261-276

Stahl, B. and Keer, L.M. 1972. Vibration and buckling of a rectangular plate with an internal support. Quarterly Journal of Mechanics and Applied Mathematics, 25, 467-478.

Timoshenko, S.P. and Woinowsky-Krieger, S. 1959. Theory of Plates and Shells, 2nd ed., McGraw-Hill, Singapore.