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Original Article

Three cumulative link models for ordinal responses: A review of methods and power comparisons

Veeranun Pongsapukdee* and Sujin Sukgumphaphan

Department of Statistics, Faculty of Science,
Silpakorn University, Snam Chandra Palace Campus, Nakhon Pathom, 73000 Thailand.

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Abstract

Power assessing the goodness of fits of three cumulative link models for ordinal response data with interaction term of explanatory variables is investigated. The three link models are the cumulative logit link models, the cumulative probit link models, and the cumulative complementary log-log link models. The simulations have been conducted for the models with three response categories, K=3, and two explanatory variables, namely X_1~Ber(0.5) and X_2~N(0,1). Data were simulated under the sample sizes of 600, 800, 1000, and 1200. Given the set of parameters, of $\alpha_1 = \log\ (\frac{p_1}{p_2 + p_3})$, $\alpha_2 = \log\ (\frac{p_1 + p_2}{p_3})$, $\beta_1 = \log 2$, $\beta_2 = \log 3$, and $\beta_{12} = 0.0 - 4.5$ (increment 0.3) from which it follows that the true model parameters are corresponding to the proportions, p_1, p_2, and p_3 of the response categories, Y = 1, 2, 3 in the sample data, respectively. The response's outcomes y_j, j = 1,..., n, are then computed through the correctly specified models under each sample size. Each condition was carried out for 1,000 repeated simulations using the developed macro program.

The results show that the cumulative logit link models generally improve the model fits when the sample sizes and β_{12} are increased; however, results from the power using the score statistic, under the alternative model with indicators, are probably stable and the power plots remain around the diagonal line. The cumulative probit link models, show that all the power plots improve the model fits as the power approach 1 quite rapidly for every statistic as the β_{12} and sample sizes are large. The last results, from the cumulative complementary log-log link models, show that every power plot is approaching 1 slowly. Therefore, overall the results reveal in general that the cumulative probit link models give the best power of the tests for every test statistic. The likelihood ratio statistic and the Wald statistic perform better than the score statistic does. Although, the score statistic gives the minimum power among the three test statistics, it still performs quite stably for every link function and sample size. Thus, this statistic may not have high power, but it still has significantly high power against the minimal null hypothesis.

Keywords: cumulative logit models, interaction terms, goodness of fits, link functions

1. Introduction

For statistical analyses, generally data are from one of the three sampling frameworks: historical data, experimental data, and sample survey data. Historical data are observation data which involve no randomization and so it is often difficulty to assume that they are representative of a convenient population. Experimental data are drawn from studies that involve randomization, which have good coverage of the possibilities of treatments for the restricted protocol population. Examples include studies where subjects are administered different dosages of drug therapies. In sample survey studies, subjects are randomly chosen, probably with equal or unequal probabilities, from a larger study population and have very good coverage of the larger population. Moreover, some sampling designs may be a combination of sample

*Corresponding author.
Email address: veeranun@su.ac.th

survey and experimental data processes, which we randomly select a study population and then randomly assign treatments to the resulting study subjects (Stokes *et al.*, 2000). Frequently, categorical data are presented in the form of contingency tables. Data sets with categorical categories, particularly those in the behavioral and biomedical sciences as well as in the social sciences, industrial quality control, human genetics, ecology, marketing, and econometric are mostly concerned with the analysis of categorical response measures, regardless of whether any accompanying explanatory variables, X's are also categorical or are continuous. Categorical response, Y, can be dichotomous, polychotomous, ordinal, nominal, discrete counts, or grouped survival times. The statistical analyses, which has the task of trying to understand the data involves looking for patterns in the data. The data are examined and combined with the models of interest under a process called fitting a statistical model. Thus, for attention to the statistical modeling, data concerned should come from the representative of the larger population, either from sample survey or experiments or including a combination of these two types and randomly generated works.

In this article, based on simulated data we empirically study and discuss modeling the ordinal response variable, primarily through the use of multinomial generalized linear models, or shortly multinomial GLMs (McFadden, 1974), under three different link functions and also review some relevant basic methods. GLMs appeared on the statistical scene in the path breaking article of Nelder and Wedderburn, 1972; McCullagh and Nelder, 1983; 1989. They generalize the classical linear models based on the normal distribution to involve two aspects: a variety of distributions from continuous to discrete or categorical, exponential family distribution models, and they also involve transformations of mean, through the link functions, linking the systematic part of models to the mean of one of the distributions. GLMs are now a mature and well known data-analytic methodology. We give particular interest in using the multinomial GLMs and applying for ordinal responses, which will be also called cumulative models. Our aims include investigating the performance of the cumulative models under different link functions applied to the most popular cumulative logit models, known as proportional odds models (Walker and Duncan, 1967; McCullagh, 1980). Statistical analyses for assessing the model fits are using goodness-of-fit-statistics: the likelihood ratio statistics, Wald statistic, and the score statistic under the alternative model with indicators (Lipsitz, *et al.*, 1996) and evaluating the power of tests among the three link functions: logit, probit, and complementary log-log links when the response categories are ordinal count data. All models contain the main effects and those contain the interaction effects. Meanwhile, we review some of the most important basic curves with the shapes for the ordinal response categories having cumulative link models (Section 2) and that for the simplicity in binary responses with a single explanatory variable having the following model formulas:

The logit model for the binary response:

$$\text{logit}\,[P(Y = 1 \mid x)] = \alpha + \beta x. \quad (1)$$

The appropriate link function is the log odds transformation, called the logit link,

$$\log \frac{P(Y=1\mid x)}{1-P(Y=1\mid x)} = \alpha + \beta x.$$

The probit model for the binary response:

$$\Phi^{-1}[P(y = 1\mid x)] = \alpha + \beta x. \quad (2)$$

Where, Φ is the standard normal cdf and the link function is called the probit link function, $\Phi^{-1}(.)$, α and β are model parameters.

The complementary log-log model for the binary response:

$$\log[-\log\{1- P(y=1 \mid x)\}] = \alpha + \beta x. \quad (3)$$

Then, $P(y = 1 \mid x) = 1 - \exp[-\exp(\alpha + \beta x)$.

This link function is called the complementary log-log link, $\log[-\log\{1-P(y=1 \mid x)\}]$, since the log-log link applies to the complement of $P(y = 1 \mid x)$. It is asymmetric, and $P(y = 1 \mid x)$ approaching 0 fairly slowly but approaching 1 quite sharply. On the other hand, the logit and probit links are symmetric about 0.5.

To summarize the above GLMs basic ideas, the GLMs differ from the conventional general linear model (of which, for example, regression model is a special case) in two major respects. First, the distribution of the response variable can be explicitly non-normal, i.e. it can be binomial, Poisson, nominal or ordered multinomial or even product multinomial etc. Second, the response values are predicted from a linear combination of explanatory variables, which are also generalized to mixed categorical and continuous or either of them, and connected to the response variable via a link function. In the general linear model the response variable values are expected to follow the normal distribution, and the link function is a simple identity function. For GLMs the response variable follows the exponential family distribution models, and the most often used link functions are logit, probit, complementary-log-log, and log links.

2. The three cumulative link models for ordinal responses

Consider a multinomial response variable, Y with ordered categorical outcomes, denoted by 1, 2, …, K, and let x_i denotes a p-dimensional vector of explanatory variables.

Three models that simultaneously use all cumulative functions for the cumulative logit link model, the cumulative probit link model, and the cumulative complementary-log-log link model are shown in the following Sections, 2.1-2.3, respectively.

2.1 The cumulative logit link model:

$$\text{logit}\ [P(Y \le j \mid \mathbf{x}_i)]$$

$$= \log \frac{P(Y \le j \mid \mathbf{x}_i)}{1 - P(Y \le j \mid \mathbf{x}_i)}$$

$$= \log [\frac{P(Y \le j \mid \mathbf{x})}{P(Y > j \mid \mathbf{x})}]$$

$$= \log \frac{\pi_1 + \ldots + \pi_j}{\pi_{j+1} + \ldots + \pi_K}$$

$$= \alpha_j + \beta' \mathbf{x}_i, \ j = 1, \ldots, K\text{-}1, \ i = 1, \ldots, p. \quad (4)$$

Where, $P(Y \le j \mid x) = \pi_1 + \pi_2 + \ldots\ldots + \pi_j$, $\pi_1 + \pi_2 + \ldots\ldots + \pi_K = 1$, $j = 1, \ldots, K$. K denotes the number of response categories; p denotes the number of explanatory variables.

This model has the same effect of vector β for each logit. Each cumulative logit has its own intercept and the $\{\alpha_j\}$ are increasing in j, since P(Y < j | x) increases in j for fixed **x** (Agresti, 2002). Thus, the logit is an increasing function of this probability and the effect is independent of the cut point. The Model (4) also satisfies

$$\text{logit}\ [P(Y < j \mid x_1)] - \text{logit}\ [P(Y < j \mid x_2)] = \beta'(x_1 - x_2).$$

The odds of making response ≤ j at $\mathbf{X} = \mathbf{x}_1$ are exp $[\beta'(\mathbf{x}_1 - \mathbf{x}_2)]$ times the odds at $\mathbf{X} = \mathbf{x}_2$. The log cumulative odds ratio is proportional to the distance between $\mathbf{X}_1$ and $\mathbf{X}_2$. Similarly to the simple case in Model (1), the same proportionality constant applies to each logit and thus, McCullagh (1980) called a proportional odds Model, (4). In many applications of the probit and logit models, in particular those involving decision making, the latent dependent or response variable may represent the probability that an event occurs or the preference level a decision maker has for several alternative outcomes. In probit, the distribution of the underlying variable is assumed to be normal, while in logit, it is assumed to be based on the logistic curve (Aldrich and Nelson, 1984). Complementary log-log models represent a third alternative to logit models and probit analyses. The complementary log-log models are frequently used when the probability of an event is very small or very large. Unlike logit and probit the complementary log-log function is asymmetric and is often applied to some studies in biological science and survival analysis.

2.2 The cumulative probit link model:

$$\Phi^{-1}[P(Y < j \mid \mathbf{x})] = \alpha_j + \beta' \mathbf{x}. \quad (5)$$

This cumulative link model links the cumulative probabilities to the linear predictors. These cumulative probit models provide fits similar to those for cumulative logit models, and their parameter interpretation is simpler.

Moreover, an underlying extreme value distribution for Y implies an alternative model of the cumulative complementary log-log model in Section 2.3.

2.3 The cumulative complementary log-log link model:

$$\log[-\log\{1 - P(Y < j \mid \mathbf{x})\}] = \alpha_j + \beta' \mathbf{x}. \quad (6)$$

The ordinal model using this cumulative complementary log-log link is sometimes called a proportional hazards model (Agresti, 2002), since it results from a generalization of the proportional hazards model survival data to handle grouped survival times (Prentice and Gloeckler, 1987).

The cumulative logit models; particularly, those in the form of the proportional odds models (Walker and Duncan, 1967; McCullagh, 1980) and the continuation-ratio models for ordinal response (Fienberg, 1980) have been the primary focus in epidemiological and biomedical applications (Amstrong and Sloan, 1989; Bercedis and Harrell, 1990; Lipsitz *et al.*, 1996; Cole *et al.*, 2003) while other forms of models for the analysis of ordinal outcomes have received less attention.

Hence, for example, when K=3, and j = 1, 2 (for K-1 = 2), the Model (4) consists of two simultaneously equations, under the cumulative link-functions for solving the model parameters, in the followings:

$$\log [\frac{P(Y \le j \mid \mathbf{x})}{P(Y > j \mid \mathbf{x})}] = \log [\frac{\pi_{i1}}{\pi_{i2} + \pi_{i3}}] = \alpha_1 + \beta' \mathbf{x}, \ \text{for } j = 1,$$

$$\log [\frac{P(Y \le j \mid \mathbf{x})}{P(Y > j \mid \mathbf{x})}] = \log [\frac{\pi_{i1} + \pi_{i2}}{\pi_{i3}}] = \alpha_2 + \beta' \mathbf{x}, \ \text{for } j = 2.$$

Where, α_j are the intercept parameters,

$\beta = (\beta_1, \beta_2, \ldots, \beta_p)'$ is a vector of coefficients corresponding to **x**'s, and $P(Y \le j \mid x) = \pi_1 + \pi_2 + \ldots + \pi_j$, and $P(Y > j \mid x) = \pi_{j+1} + \pi_{j+2} + \ldots + \pi_K$, $j = 1, \ldots, K\text{-}1$.

The Model (4) for any K ≥ 3 is often called the proportional odds model (McCullagh, 1980). It is based on the assumption that the effects of the explanatory variables X_1, ..., X_p are the same for all categories, on the logarithmic scale. It probably also represents the most widely used ordinal categorical model at the present time (Pongsapukdee and Sukgumphaphan, 2007).

Similarly to the above Model (4), we apply the use of logit, probit and complementary log-log links for the case of two explanatory variables, we then have Model (7) to (12).

For evaluating the likelihood ratio statistic or model chi-square or change of deviances statistic, the Model (7) to (12) are fitted. Each cumulative link model has its own intercept. The $\{\alpha_j\}$ are increasing in j, since $P(Y \leq j \mid \mathbf{x})$ increases in j for fixed **x**, and the link function is an increasing function of this probability. Each cumulative link function uses all K response categories (Lawal, 2003).

The three cumulative link models (main effects or minimal models):

$$\mathbf{log}\left(\frac{\pi_{i1}+...+\pi_{ik}}{\pi_{i,k+1}+...+\pi_{iK}}\right) = \alpha_j + \beta_1 x_{1i} + \beta_2 x_{2i},\ j=1,2,\ K=3,\quad i=1,2,...n. \qquad ..... (7)$$

$$\Phi^{-1}[(\pi_{i,1}+...+\pi_{i,(K-1)})] = \alpha_j + \beta_1 x_{1i} + \beta_2 x_{2i},\ j=1,2,\ K=3,\quad i=1,2,...n. \qquad ..... (8)$$

$$\mathbf{log}\ [-\mathbf{log}\ \{1-(\pi_{i,1}+...+\pi_{i,(K-1)})\}] = \alpha_j + \beta_1 x_{1i} + \beta_2 x_{2i},\ j=1,2,\ K=3,\quad i=1,2,...n. \qquad ..... (9)$$

The three cumulative links with two-factor- interaction models (Interaction effects or alternative models):

$$\mathbf{log}\left(\frac{\pi_{i1}+...+\pi_{ik}}{\pi_{i,k+1}+...+\pi_{iK}}\right) = \alpha_j + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_{12} x_{1i} x_{2i},\ j=1,2,\ K=3,\quad i=1,2,...n. \qquad .... (10)$$

$$\Phi^{-1}[(\pi_{i,1}+...+\pi_{i,(K-1)})] = \alpha_j + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_{12} x_{1i} x_{2i},\ j=1,2,\ K=3,\quad i=1,2,...n. \qquad .... (11)$$

$$\mathbf{og}\ [-\mathbf{log}\ \{1-(\pi_{i,1}+...+\pi_{i,(K-1)})\}] = \alpha_j + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_{12} x_{1i} x_{2i},\ j=1,2,\ K=3,\ i=1,2,..,n. \qquad ..... (12)$$

For evaluating the score tests, the following alternative models, which are similar to Model (10) to (12) except that for adding the indicator term, are fitted and compared with Model (7) to (9), respectively.

$$L_{ik} = \alpha_k + \beta_1 x_{2i} + \beta_2 x_{2i} + \beta_{12} x_{1i} x_{2i} + \sum_{r=1}^{D-1} I_{ir}\gamma\ ,$$

k=1, …, K-1. i =1, …, n, and r = 1, …, D-1.

Where, L_{ik} stands for each of the three choices of link functions.

3. Simulation and Statistical Analyses

From the models in Section 2, the simulations have been conducted for the three response categories or K=3, and for two explanatory variables. Data were simulated under the sample sizes of 600, 800, 1000, and 1200 units. Due to the samples needed to achieve the power 0.90-0.95, when using the Bernoulli (0.5) explanatory variable, the units would be around 1000-2000 (Shieh, 2001). When either the association parameter (β_{12}) increases or the sample size is small, the sparseness in the contingency tables may occur (Sukgumphaphan and Pongsapukdee, 2008). Thus, we choose the moderate to large sample sizes. The explanatory variables are X_1~Bernoulli(0.5) and X_2~Normal(0,1). Given the set of parameters $\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$ of $\alpha_1 = \log\ (\frac{p_1}{p_2+p_3})$, $\alpha_2 = \log\ (\frac{p_1+p_2}{p_3})$, which are the different intercepts when X's values are zeroes, $\beta_1 = \log 2$, $\beta_2 = \log 3$, and β_{12} = 0.00-4.5(increment 0.3) are fixed, from which it follows that the true model parameters are corresponding to the proportions, p_1, p_2, and p_3 of the response categories, Y = 1, 2, 3 in the sample data, respectively. The response's outcomes y_j, j = 1,..., n, are then computed through the correctly specified models under each sample size. Each condition is carried out for 1,000 repeated simulations using the developed macro program run with the Minitab Release 11 (command syntax) and 15 (power plots) on Pentiums IV.

Statistical analyses for assessing goodness-of-fits of the models using several statistics are performed for each combination of the model conditions. The likelihood ratio statistics, Wald statistic, and the score statistic from the partition of the deciles (D=10) data are processed and evaluated through the models in Section 2. Under the deciles data we have compared the null main effect models and the alternative interaction effect models for every condition.

All the statistics are computed using the following formulae:

G_M = -2 [ln(LO-ln(LM)] (The likelihood ratio or model chi-square statistic: Change of Deviances).

$W = (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta_0})' [cov(\hat{\boldsymbol{\beta}})]^{-1} (\hat{\boldsymbol{\beta}} - \boldsymbol{\beta_0})$ (The Wald statistic).

$\hat{\boldsymbol{\beta}}$ denotes the estimated vector of $\boldsymbol{\beta}$, and $\boldsymbol{\beta}_0$ is the null hypothesized parameters.

The indicator for the (D-1) grouped data follows Lipsitz *et al.* (1996). That is

$$I_{ir} = \begin{cases} 1 & \textit{if } \hat{\mu}_i \textit{ is in region } r, \\ 0 & \textit{if otherwise.} \end{cases}$$

i =1, …, n, and r = 1, …, D-1.

Where, $\hat{\mu}_i = \sum_k^K s_k p_{ik}$, s_k = k, p_{ik} = the probability of response k, $\sum_k^K p_{ik}$ =1, i =1, …, n.

The alternative model:

$$L_{ik} = \alpha_k + \beta_1 x_{2i} + \beta_2 x_{2i} + \beta_{12} x_{1i} x_{2i} + \sum_{r=1}^{D-1} I_{ir}\gamma \text{ , k=1, …, K-1}$$

The null model:

$$L_{ik} = \alpha_k + \beta_1 x_{2i} + \beta_2 x_{2i} + \beta_{12} x_{1i} x_{2i} \text{ ,}$$

which is corresponding to the above alternative model parameters,

$$\gamma_{1,1} = \ldots = \gamma_{D-1,1} = \gamma_{1,2} = \ldots = \gamma_{D-1,2} = \gamma_{1,K-1} = \ldots = \gamma_{D-1,K-1}$$
= 0, D=10, K=3.

The power of the test is the percentage corresponding to the rejection of H_0 when H_0 is false in 1,000 simulations. Whereas, n is the sample size, LO is the likelihood function for the model containing only the main effects. LM is the likelihood function for the model containing the interaction effects. L_{ik} is for each of the three choices of link functions relating the elements of the probabilities of Y to the explanatory variables.

4. Results

In this section the results of the comparisons in terms of power plots between the power of the tests and the parameter β_{12} are evaluated for each test statistic, likelihood ratio statistic, Wald statistic, and the score statistic from the alternative model using indicators and the partition of the deciles (D=10) data. Four sample sizes that are simulated and compared for each of the three cumulative link models for ordered response categories have been performed. Firstly, the results from the first link function, the cumulative logit link models show that the power 1, using the likelihood ratio statistic, the power 2, using the Wald statistic, both of them perform well and improve the models fitted when the sample sizes and β_{12} are increased; however, results from the power 3, using the score statistic under the alternative model with indicators, are probably more stable and the power plots remain around in the diagonal line (Figure 1). Secondly, the results from the cumulative probit link models, show that all the power plots are probably giving very much improved the model fits as the power plots approach 1 quite rapidly for the power 1, power 2, and the power 3, the last one, respectively, as the β_{12} and sample sizes are large. In these plots it is found that the power 3 in Figure 2 can give more power than that of power 3 in Figure 1. The final results, from the cumulative complementary log-log link models show that all power plots are approaching 1 slowly (Figure 3). Therefore, overall, the power 1 and 2 perform better than the power 3 does. In addition, the power 3 does vary less dependently upon the three link functions and all sample sizes (Figure 1-3).

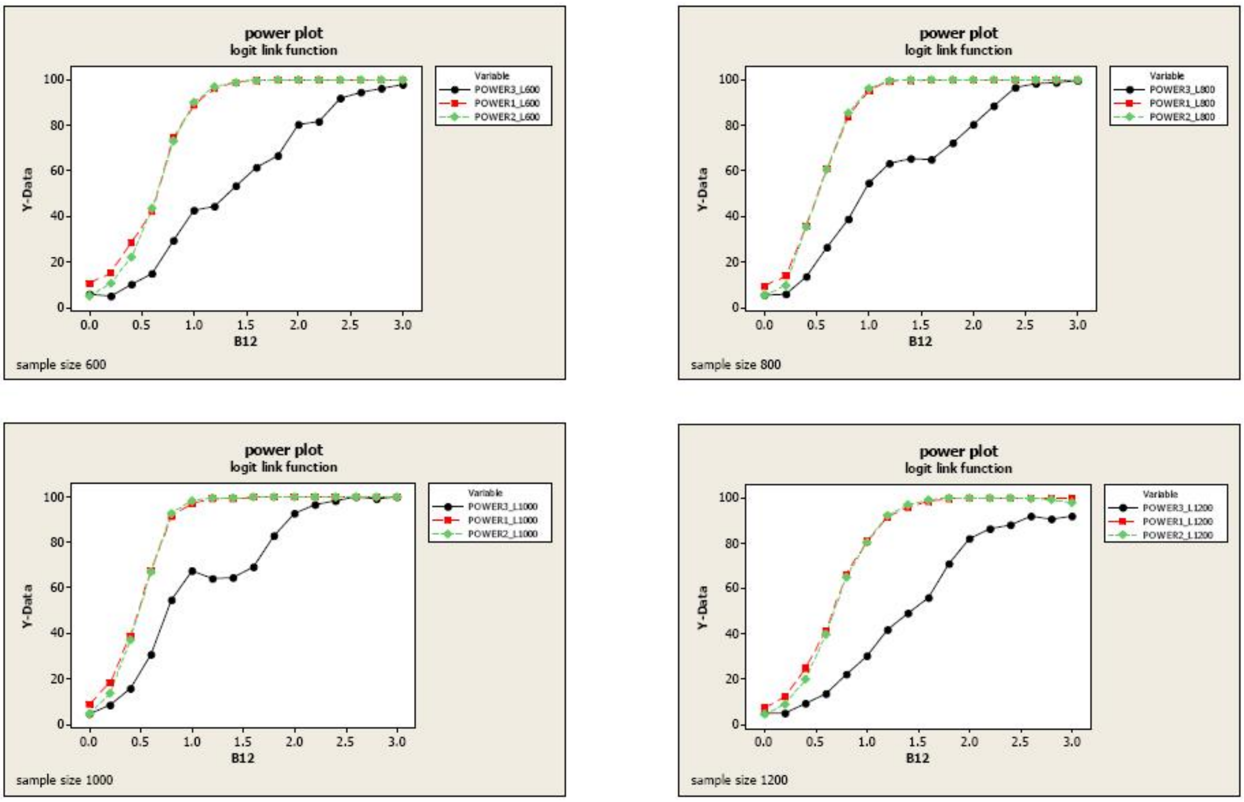

Figure 1. Power Plots of Cumulative Logit Models Classified by Statistics in each Sample size of 600, 800, 1000, and 1200

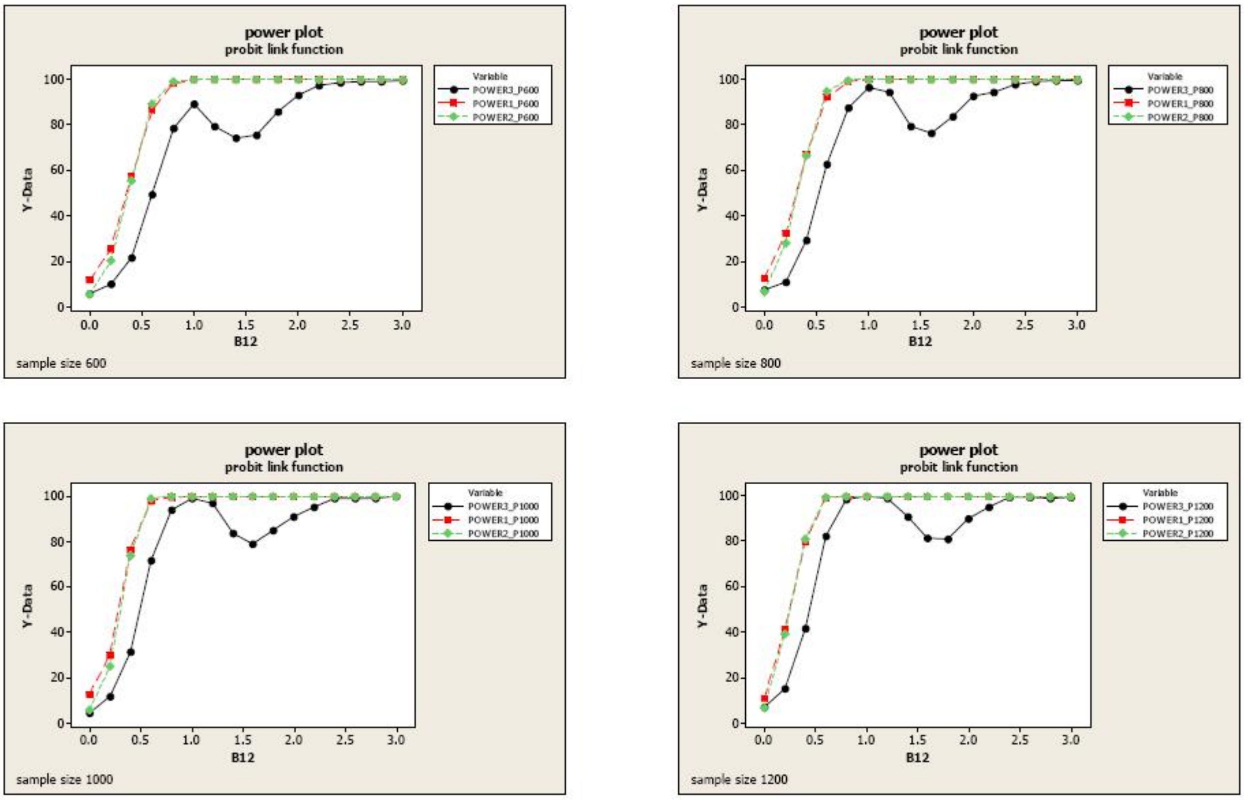

Figure 2. Power Plots of Cumulative Probit Models Classified by Statistics in each Sample size of 600, 800, 1000, and 1200

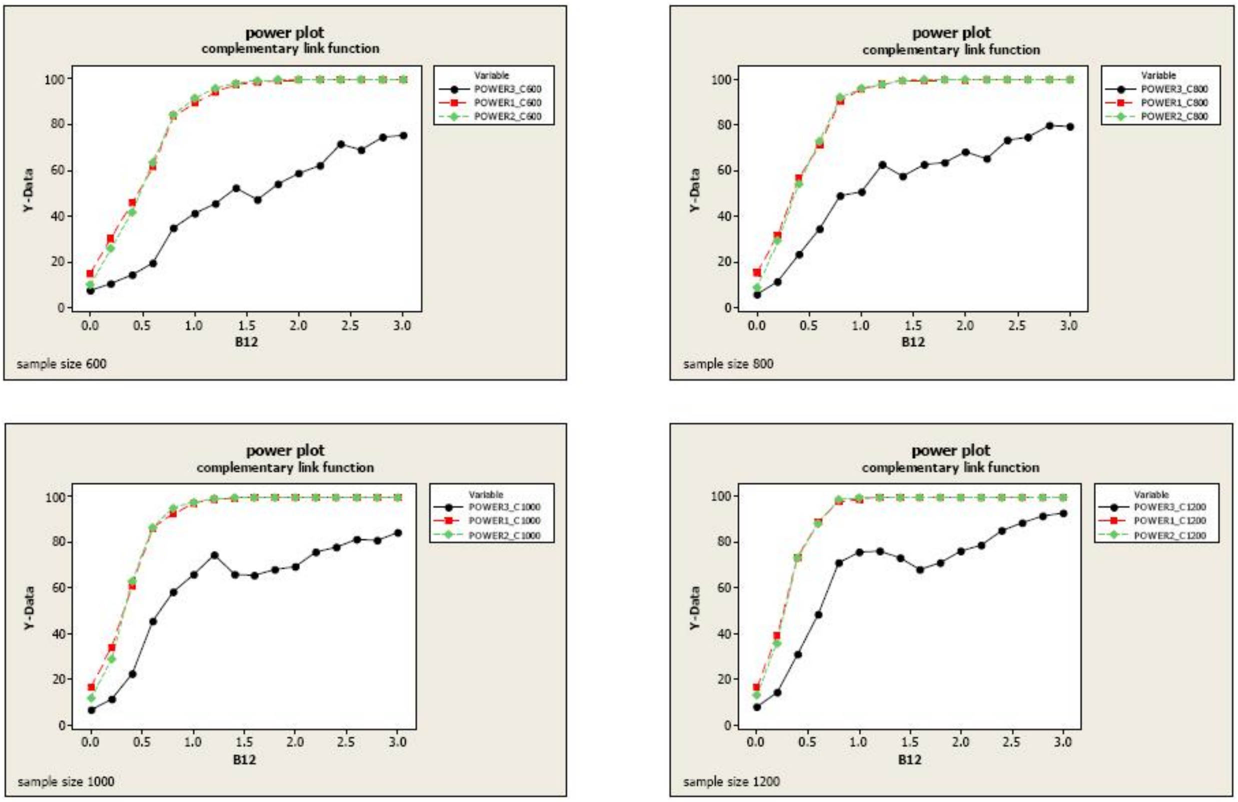

Figure 3. Power Plots of Cumulative Complementary-log-log Models Classified by Statistics in each Sample size of 600, 800, 1000, and 1200

5. Conclusion and Discussion

The results are concluded for the use of the three link functions applied to the cumulative GLMs for the ordinal response variable. All the power plots increase as the β_{12} and sample sizes are increased. The results also reveal in general, that the cumulative probit link models give the best power of the tests for every test statistic. The likelihood ratio statistic and the Wald statistic perform better than the score statistic does. For results when the samples are large, the score test of the indicator variable do still confirm those results in Lipsitz *et al.* (1996), where samples are much smaller. This probably dues to the same strategy grouping effects. However, in stead of partitioning the subjects in to 10 groups, as we and the Lipsitz *et al.* (1996) did, we find in Xie *et al.* (2008) that they proposed new method of partitioning the subjects using differing number of groups by clustering in the continuous covariates' space and the power of the tests were improved. Although, all power plots show that the power 3, the score statistic, gives the minimum power among the three test statistics. It still performs quite stably for every link function and sample size. Thus, this statistic may not have high power, but it is probably able to be used safely and has statistically significant results against the minimal null hypothesis.

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