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Original Article

Comparison of rectangular array and triangular array arrangements of cylindrical cans in corrugated box

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Abstract

Rectangular and triangular array arrangements of cans in a box were mathematically analyzed. A set of developed equations offers systematic approach of comparing two patterns. In general, a triangular array shows a better economical way for loading cylindrical cans in a box. Sets of best can packing were tabulated which can assist packaging engineers to understand and select a better efficient arrangement of cans in a box. The required smallest volume and least surface area of box obtained from this analysis lead to find the most economical way in arrangement of cylindrical cans in box.

Keywords: rectangular array, triangular array, arrangement, can, corrugated box

1. Introduction

One of primary functions of a package is to offer product protection. The corrugated box is the most common shipping container widely used in food industries for years to contain and protect the canned food throughout the distribution environment. It combines structural and cushioning characteristics required as shipping container at a reasonable price which making it a very desirable shipping container (Fibre Box Association, 1992). The RSC (Regular Slotted Container) is by far the most common accepted style in the industries due to the high efficiency of % box produced per board usage. (Soroka, 1995, Jonson, 1999). One principle of packaging design is to optimize package dimensions in order to minimize material usage requirements. In recent years, researchers have been set to the problem of finding this optimum package dimensions, but most of works are difficult to apply in practice. Maltenfort (1961) and Marcondes (1991) used the same approach in analysis by developing mathematical function of the board area to enclose a given volume and differentiating the function to minimize the corrugated board usage. Since it was based on a given volume not shape of content, this made the analysis impractical to use. An arrangement is a pattern of orienting a number of primary packages in a shipping container (Soroka, 1995). Each arrangement requires different board areas for the same number of primary packages contained. Some patterns may give better efficiency of board usage, while others may provide more stable loads. However, small improvement to can arrangement in the corrugated box can have major impacts on total shipping efficiency and cost through better saving of board usage. This will help food industries to take advantage on producing cost effective product. Surprisingly, no researcher has paid attention to investigate the effect of can arrangement on the size of box. In general, cylindrical can users can arrange cans into rectangular corrugated boxes in either a rectangular array or triangular array. Therefore, consideration of the array that provides the most economical way to save the material or board usage is necessary. Theoretically, to economize cans packing in corrugated box, the corrugated box must be selected with smallest volume and least surface area. The purpose of this paper is to provide an analysis of a rectangular and triangular array arrangement of cylindrical cans in corrugated box and compare the efficiency of board usage between both patterns.

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2. Analysis

Theoretical equations were mathematically developed based on diagrams of cans in a box. They are triangular array and rectangular array arrangement as shown in Figure 1 and 2.

2.1 Box length and width

Rectangular array: In Figure 1, the diagram represents a rectangular array arrangement of 28 cans. If N is the total numbers of cans, L, the corrugated box length and W, the corrugated box width, then:

$$N = mn \quad (1)$$

$$L = nd \quad (2)$$

$$W = md \quad (3)$$

where m is the number of rows, n is the number of columns and d is the can diameter.

Triangular array: Another arrangement involves using a triangular array as shown in Figure 2; however, this is a more complicated case. If a box has the odd numbered m rows of cans (1, 3, 5, 7…), it will contain n columns of cans, whereas the even numbered m rows of cans (2, 4, 6, 8….)

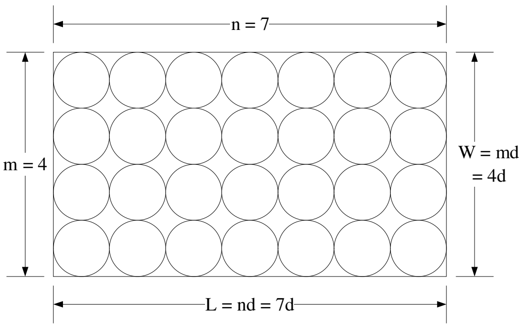

Figure 1. Rectangular array arrangement of cans in corrugated box (top view).

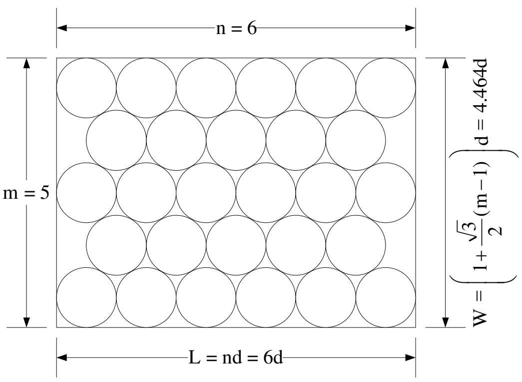

Figure 2. Triangular array arrangement of cans in corrugated box (top view).

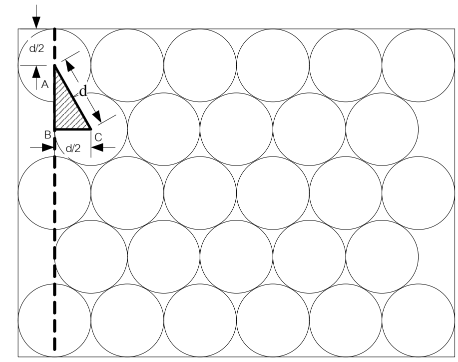

Figure 3. Diagram for deriving width of triangular array arrangement (top view).

will make up (n-1) columns of cans. Thus,

if m is odd number:

$$N = mn - \frac{(m-1)}{2} \quad (4)$$

and if m is even number:

$$N = mn - \frac{m}{2} \quad (5)$$

Therefore, in Figure 2, m = 5 and n = 6 so that N = 28.

In order to derive the equation of width of triangular array, can no. 1 is the first can in the first row and can no. 2 is the first can in the second row (Figure 3). Let the center points of the top surface of can no.1 and 2 be designated as point A and point C respectively. Points A and C are then joined with a straight line and the radius of can no.2 is constructed parallel to the horizontal. The point where the radius intersects the circumference of can no. 2 is called point B. Points A and B are then joined with a straight vertical line, then a right triangle ABC is constructed. AC represents the hypotenuse. AB and BC represent the two legs of triangle ABC. AC is equal to the diameter of can (d) and BC is equal to the radius of the can (d/2). By the Pythagorean Theorem (Benice, 1976), the length of AB can be calculated in term of can diameter as follows:

$$AB^2 = AC^2 - BC^2$$

and $$AB^2 = d^2 - (d/2)^2 = \frac{3d^2}{4}$$

Then $$AB = \frac{\sqrt{3}}{2}d \quad (6)$$

If the right triangles are constructed by the previous method as many as possible in a triangular array of any length, the maximum number of triangle will be (m-1) triangles. In addition, there will be two radii necessary to make up the complete box width in every case.

Therefore, the equation for the width (W) of any triangular array is:

$$W = (m-1)\left(\frac{\sqrt{3}}{2}d\right) + d \qquad (7)$$

and the corrugated box length (L) is :

$$L = nd \qquad (8)$$

2.2 Box surface area

The general equation for surface area of a box containing either array is S = 2 (hL + hW + WL) where h is the can height

For a rectangular array:

$$W = md$$

$$L = nd$$

Therefore,

$$S = 2d(hn+hm+Nd) \qquad (9)$$

For a triangular array:

$$S = 2d[(0.13397 + 0.86603m)(h + nd) + hn] \qquad (10)$$

2.3 Box volume

Again, the general equation for either rectangular or triangular array is V = hLW

For a rectangular array:

$$V = hmnd^2 = Nhd^2 \qquad (11)$$

For a triangular array:

$$V = hnd^2 (0.13397 + 0.86603m) \qquad (12)$$

3. Results and Discussion

The preceding equations derived can be used to compare between a rectangular array and triangular array. The equations for volume (V), surface area (S), and number of cans (N) are then applied to examples in Figure 1 and 2.

If N = 28.

For a Rectangular array:

$$V = 28\ hd^2$$

$$S = (22h + 56d)d$$

For a Triangular array:

$$V = 26.785hd^2$$

$$S = (20.928h + 53.569d)d$$

In this particular example, a rectangular and triangular array both containing the same number of cans was analyzed using equations in Table 1. It was found that the triangular array is more efficient. By comparing the volumes and surface areas of both arrays, the triangular array indicates smaller dimensions of box for any chosen values of d and h. In order to compare the efficiency between the two arrays, the efficiency index, e is proposed. e is defined as the ratio of the areas of the N circumscribing squares (Nd^2) to the top surface area of the container (WL). For all rectangular arrays, e is equal to 1, and in triangular arrays e is no more than 1.155. Therefore, the better can arrangement

Table 1. Summary of equations for rectangular and triangular arrays.

Type of Arrangement		
Rectangular Array	Triangular Array	
	m = odd number	m = even number
N = mn	N = mn - (m - 1)/2	N = mn - m/2
L = nd	L = nd	
W = md	$W = (m-1)\left(\frac{\sqrt{3}}{2}d\right) + d$	
$V = Nhd^2$	$V = hnd^2(0.13397 + 0.86603m)$	
S = 2d(hn + hm + Nd)	S = 2d((0.13397 + 0.86603m)(h + nd) + hn)	
$\varepsilon = \frac{Nd^2}{mnd^2} = 1$	$\varepsilon = \frac{Nd^2}{WL} = \frac{N}{n(L/d)} \le 1.155$	

Table 2. Best packing arrangements for any number of cans from 20 to 40.

N	Type of Arrangement	m	n	ε	Recommended For	N	Type of Arrangement	m	n	ε	Recommended For
10	R	5	2	1.000	V, (S, 0d”h/dd”1.976)	17	T	11	2	0.880	
10	T	4	3	0.926	(S, 1.976d”h/d)	17	T	3	6	1.037	V, S
11	T	2	6	0.982		17	T	2	9	1.012	
11	T	3	4	1.007	V, S	18	R	2	9	1.000	
11	T	7	2	0.888		18	R	3	6	1.000	
12	R	6	2	1.000		18	T	12	2	0.855	
12	R	3	4	1.000	V, S	18	T	7	3	0.968	
12	T	8	2	0.850		18	T	5	4	1.008	V, S
13	T	5	3	0.971	(S, 0.236d”h/d)	18	T	4	5	1.001	
13	T	2	7	0.995	V, (S, 0d”h/dd”0.236)	19	T	2	10	1.018	V, S
14	R	2	7	1.000		20	R	2	10	1.000	
14	T	9	2	0.883		20	R	4	5	1.000	(S, 1.196d”h/d)
14	T	4	4	0.973	(S, 5.464d”h/d)	20	T	13	2	0.878	
14	T	3	5	1.025	V, (S, 0d”h/dd”5.464)	20	T	8	3	0.944	
15	R	3	5	1.000	(S, 0.038d”h/d)	20	T	3	7	1.046	V, (S, 0d”h/dd”1.196)
15	T	10	2	0.853		21	R	3	7	1.000	(S, 0.165d”h/dd”0.479)
15	T	6	3	0.938		21	T	14	2	0.857	
15	T	2	8	1.005	V, (S, 0d”h/dd”0.038)	21	T	6	4	0.985	(S, 0.479d”h/d)
16	R	4	4	1.000	V, S	21	T	2	11	1.023	V, (S, 0d”h/dd”0.165)
22	R	2	11	1.000		27	R	3	9	1.000	
22	T	4	6	1.019	V, S	27	T	18	2	0.859	
23	T	15	2	0.876		27	T	6	5	1.013	(S, 0.095d”h/d)
23	T	9	3	0.967		27	T	2	14	1.034	V, (S, 0d”h/dd”0.095)
23	T	5	5	1.030	(S, 0.366d”h/d)	28	R	2	14	1.000	
23	T	3	8	1.052	V, (S, 0d”h/dd”0.366)	28	R	4	7	1.000	
23	T	2	12	1.027		28	T	11	3	0.966	
24	R	2	12	1.000		28	T	8	4	0.991	
24	R	3	8	1.000		28	T	5	6	1.045	V, S
24	R	4	6	1.000	V, S	29	T	19	2	0.874	
24	T	16	2	0.858		29	T	3	10	1.061	V, S
25	R	5	5	1.000	(S, 1.097d”h/d)	29	T	2	15	1.036	
25	T	10	3	0.948		30	R	2	15	1.000	
25	T	7	4	1.009	(S, 0.113d”h/dd”1.097)	30	R	3	10	1.000	
25	T	2	13	1.031	(S, 0d”h/dd”0.113)	30	R	5	6	1.000	(S, 2.032d”h/d)
26	R	2	13	1.000		30	T	20	2	0.859	
26	T	17	2	0.875		30	T	12	3	0.950	
26	T	4	7	1.032	(S, 0.527d”h/d)	30	T	4	8	1.042	V, (S, 0d”h/dd”2.032)
26	T	3	9	1.057	V, (S, 0d”h/dd”0.527)	31	T	2	16	1.038	V, S
32	R	2	16	1.000		35	T	2	18	1.042	
32	R	4	8	1.000		36	R	2	18	1.000	
32	T	21	2	0.873		36	R	3	12	1.000	
32	T	9	4	1.009		36	R	4	9	1.000	
32	T	7	5	1.033	(S, 0.366d”h/d)	36	R	6	6	1.000	(S, 11.075d”h/d)
32	T	3	11	1.065	V, (S, 0d”h/dd”0.366)	36	T	24	2	0.860	
33	R	3	11	1.000		36	T	8	5	1.020	V, (S, 0d”h/dd”11.075)
33	T	22	2	0.860		37	T	2	19	1.044	V, S
33	T	13	3	0.966		38	R	2	19	1.000	
33	T	6	6	1.032	(S, 5.464d”h/d)	38	T	25	2	0.872	
33	T	5	7	1.056	V, (S, 0d”h/dd”5.464)	38	T	15	3	0.965	
33	T	2	17	1.040		38	T	5	8	1.064	(S, 0.060d”h/d)
34	R	2	17	1.000		38	T	4	10	1.056	
34	T	4	9	1.050	V, S	38	T	3	13	1.070	V, (S, 0d”h/dd”0.060)
35	R	5	7	1.000	(S, 0.811d”h/d)	39	R	3	13	1.000	
35	T	23	2	0.873		39	T	26	2	0.861	
35	T	14	3	0.952		39	T	11	4	1.009	
35	T	10	4	0.995		39	T	7	6	1.049	V, S
35	T	3	12	1.068	V, (S, 0d”h/dd”0.811)	39	T	6	7	1.045	

Table 2. (Continued)

N	Type of Arrangement	m	n	ε	Recommended For
39	T	2	20	1.045	
40	R	2	20	1.000	
40	R	4	10	1.000	
40	R	5	8	1.000	
40	T	16	3	0.953	V, S

in a box will indicate the higher efficiency index.

In general, the triangular array will give a more efficient volume compared to a rectangular array having the same number of cans under the following conditions:

1) For triangular arrays in which m is odd and $n \geq 4$

2) For triangular arrays in which $m = 2$, $n \geq 8$; $4 \leq m \leq 14$, $n \geq 5$; $m \geq 16$, $n \geq 4$

To illustrate the benefit of this analysis, Table 2 is constructed to determine the best packing arrangements for any number of cans from 20 to 40. In Table 2 R and T denote a rectangular and triangular array respectively. e is the efficiency index. In the last column of Table 2 labeled "Recommended for" V represents the arrangement for the least volume and S without parentheses denotes the arrangement for the least surface area of the six sides of the corrugated box. S within parentheses is accompanied by range and can height (h) for the arrangement produces the least surface area.

4. Conclusions

Determining the best packing arrangement of cans in the box is of importance to the food industries because it is essentially related to the costs of product and packaging. From analysis, it is possible to find the most economical way in packing corrugated boxes with cylindrical cans by selecting the box with smallest volume and least surface area. Moreover, using the analysis outlined in this paper with other approaches to determine the optimum dimensions of packages (Maltenfort (1961) and Marcondes (1991)) will strengthen the success of board or cost saving. There are some considerations of this analysis to be noted. The most efficient can arrangement selected by this method may not be the best fit on the pallet or warehouse space. It may not be effective in protection and transportation or compatible with packaging regulations and marketing needs. Thus, in packaging design, the packaging engineers must also integrate cost saving through a better design, optimum production and material handling as well as performance, packaging regulations and marketing demands made upon the package.

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