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*Original Article*

# Preliminary study of the vibration displacement measurement by using strain gauge

Siripong Eamchaimongkol*

*Department of Mechanical Engineering, Faculty of Engineering,
Rajamangala University of Technology Thanyaburi, Pathum Thani, 12110 Thailand.*

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**Abstract**

The purpose of this study is to show the feasibility of the vibration displacement measurement due to harmonic excitation by using strain gauge. The measurement method is based on the principle of vibration due to the harmonic motion of the base. The instrument consists of a seismic mass and a spring. Such spring-mass system is considered a mass ended cantilevered beam. The first mode natural frequency of the spring-mass system is equal to 8.5 Hz. In this paper, the instrument was subjected to the harmonic excitation of the base. The relationships between the displacement and the output voltage were investgated at the frequencies of 12,15 and 19 Hz, respectively. It has been found that the output voltage from the instrument is proportional to the displacement of the base measured by the vibration pickup.

**Keywords:** strain gauge, vibration measurement, vibration displacement

## 1. Introduction

In the operating of machinery vibration always occurs as a result of unbalance of machine components. Vibrations can cause failure a machine. Especially, whenever the natural frequency of a machine coincides with the frequency of the external excitation, there occurs a phenomenon known as resonance, which leads to excessive deflections, and failure. So, the measurement of vibrations is necessary and important. However, the commercial instrument that is available for vibration measurement is expensive. Particularly, the instrument that can be used to analyze the fast Fourier transform (FFT) from the vibration signal. Thus, this paper aims to study the application of strain gauge in which is much cheaper than other sensors in order to measure vibration displacement. The advantage of this method is that it is a simple method and construction. The method of measurement can be described as follow.

* Corresponding author.
Email address: siemkeng@hotmail.com, siripong.e@rmutk.ac.th

## 2. Design of the instrument

The method of measurement is based on the vibration system due to harmonic motion of the base, as shown in Figure 1. The base motion is assumed to have a harmonic motion (Rao, 2005).

Therefore, $$y(t) = Y\sin(\omega t) \tag{1}$$

where $y(t)$ is the displacement of the base, $x(t)$ is the displacement of the seismic mass, $\omega$ is the frequency of the

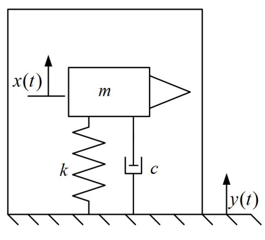


Figure 1. Schematic diagram of a mass, a spring, and a damping system.

harmonic excitation, the relative displacement between the mass and the base $z(t)$ can be shown in Equation (2).

$$z = x - y \tag{2}$$

According to the spring-mass system shown in Figure 1, the equation of motion of mass can be written as

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{3}$$

where $m$ is the seismic mass, $k$ is the stiffness of spring, and $c$ is the damping coefficient.

Hence, the steady-state solution of Equation (3) is given by Equation (4)

$$z(t) = Z\sin(\omega t - \phi) \tag{4}$$

where $Z$ is the amplitude of the relative displacement, $\phi$ is the phase angle between the relative displacement and the base.

The relationship between the amplitude of the relative displacement $(Z)$ and the amplitude of the base displacement $(Y)$ can be expressed as

$$Z = \frac{r^2 Y}{\left[(1-r^2)^2 + (2\varsigma r)^2\right]^{\frac{1}{2}}} \tag{5}$$

where $\varsigma$ is the damping ratio, $r = \left(\frac{\omega}{\omega_n}\right)$ is the frequency ratio of the base frequency to the natural frequency of the system.

The phase angle can be expressed as

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) \tag{6}$$

According to Equation (5), it can be seen that for very large value of $r$, the ratio of $Z$ to $Y$ approaches unity, regardless of the amount of damping. Thus, the relative displacement between the mass and the base is essentially the same as the displacement of the base. Therefore, if the relative displacement can be measured, the base displacement also can be obtained.

As previously mentioned, for very large value of $r$, the amplitude of the relative displacement is the same as those of the base. Hence, we have to design the natural frequency of spring-mass system as low as possible in order that the vibration displacement can be measured at low frequency of excitation.

In this work, the designed instrument consists of a seismic mass and a spring, as shown in Figure 2. The leaf spring, having a length of 70 mm, a width of 10 mm, and a height of 0.2 mm is used as a spring in the instrument. Such spring is fixed at one end and free at the other. The seismic mass having a mass of 4 g is attached to the free end. Therefore, the spring-mass system is considered a cantilevered beam with a lumped mass at the end (Meirovitch, 2001). The first mode natural frequency of the system can be obtained by experimental determination. The strain gauge is used as the displacement sensor for the experiment. It is mounted on the leaf spring, as shown in Figure 2. Thus, when the vibration displacement occurs, the output voltage from the strain gauge varies with the magnitude of the displacement.

## 3. Experimental setup

The experimental setup is shown in Figure 3 a,b. It consists of two main parts. The first is the base excitation setup and the other is the signal conditioning circuit. The detail can be described as follow.

### 3.1 Base excitation setup

The steel beam with a width of 25 mm and a thickness of 2.8 mm is used as the base. The length of the beam can be varied depending on the frequency of the excitation. The rotating unbalance with a mass of 82 g is mounted at the end of the steel beam in order to provide the harmonic excitation, as shown in Figure 3 a,b. The rotating unbalance is driven by a DC motor control circuit. The designed instrument is attached at the middle of the cantilevered beam and the vibration pickup is attached closely to the instrument in order to measure the vibration displacement of the base. According to Figure 3a, the steel beam is fixed at one end and free at the other. Therefore, it can be considered a cantilevered beam with a mass at the end. The natural frequency of a cantilevered beam can be expressed as

$$\omega_n = \sqrt{\frac{k}{m_{eq}}} \tag{7}$$

where $\omega_n$ is the natural frequency of the base system, $k$ is the spring constant of the steel cantilevered beam that can be expressed as

$$k = \frac{3EI}{L^3} \tag{8}$$

where $E$ is the Young's modulus, $I$ is the moments of inertia of areas, $L$ is the length of the cantilevered beam, and $m_{eq}$ is the equivalent mass that can be expressed as

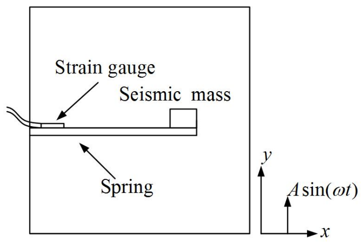


Figure 2. Schematic diagram of designed instrument.

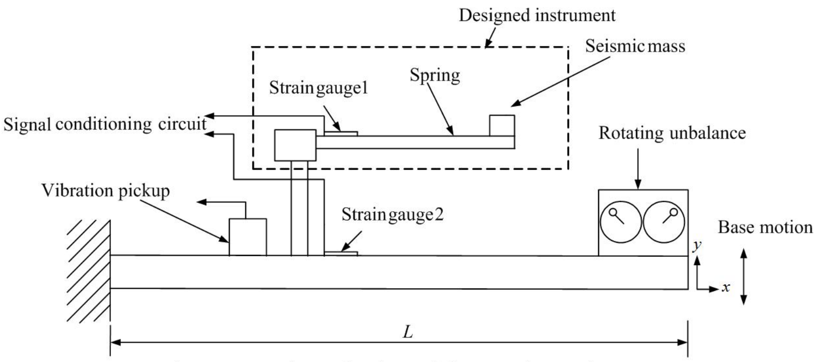


Figure 3 a. Schematic view of the experimental setup.

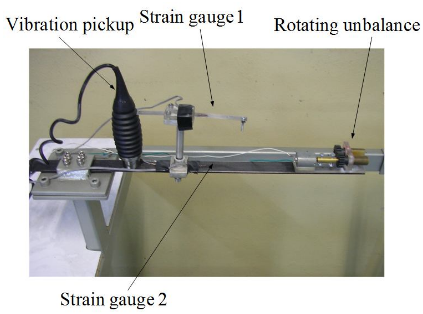


Figure 3 b. Photograph of the experimental setup.

$$m_{eq} = 0.23m_s + M \tag{9}$$

where $m_s$ is the mass of the steel beam, $M$ is the mass of the rotating unbalance (82 g).

In this paper, two strain gauges (strain gauge 1 and strain gauge 2) are used as the sensor to measure the vibration displacement. One (strain gauge 1) is mounted on the leaf spring in order to detect the waveform signal from the instrument and the other (strain gauge 2) is mounted on the base in order to detect the waveform signal from the base. The assumptions of the experiment are as follow.

- The harmonic vibration only occurs in the y-axis.
- All materials behave both elastically and linearly.
- The vibration system is considered an undamped single degree of freedom system.

### 3.2 Signal conditioning circuit

The signal conditioning circuit is shown in Figure 4. Such circuit can be divided into two important parts. One is the Wheatstone bridge circuit that is used to detect the resistant change of the strain gauge and the other is the differential amplifier circuit that is used to amplify the signal from the Wheatstone bridge circuit. This circuit is used as the signal conditioning circuit to process the signal in both strain gauge 1 and strain gauge 2. Both strain gauges have a gauge length of 5 mm, a gauge factor of 2.1, and a resistance of 120 $\Omega$. Each of them is mounted at the leg of the Wheatstone bridge circuit. Hence, when the vibration occurs, the signal voltage between $A$ and $B$ (as shown in Figure 4) is amplified by the differential amplifier resulting in the output voltage at $V_o$ (Floyd, 1996). In this work, the designed instrument was subjected to the harmonic excitation of the base at the frequencies of 12, 15 and 19 Hz, respectively. The root mean square voltage (Vrms) is measured from the circuit and is compared to the vibration displacement measured by the vibration pickup.

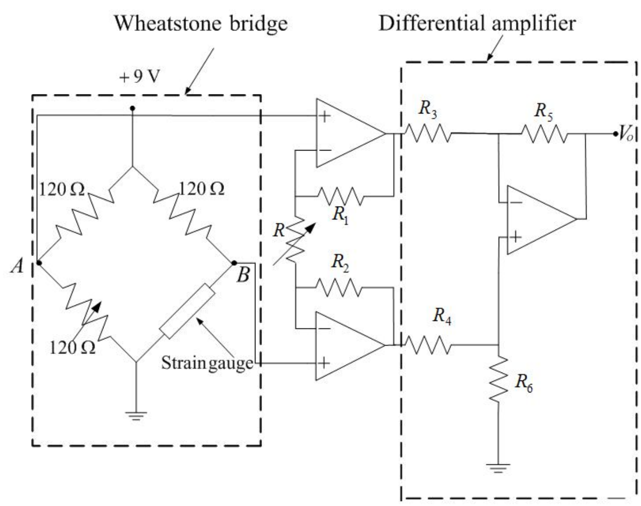


Figure 4. Signal conditioning circuit.

Table 1. Length of the steel cantilevered beam at various frequencies of the base excitations.

| Excitation frequency (Hz) | Length of the steel beam, $L$ (m) | Spring constant, $k$ $\left(\frac{N}{m}\right)$ | Mass of the steel beam, $m_s$ (kg) | Equivalent mass, $m_{eq}$ (kg) |
|---|---|---|---|---|
| 12 | 0.34 | $7.33\times10^{2}$ | $1.86\times10^{-1}$ | $1.25\times10^{-1}$ |
| 15 | 0.30 | $1.07\times10^{3}$ | $1.64\times10^{-1}$ | $1.20\times10^{-1}$ |
| 19 | 0.26 | $1.64\times10^{3}$ | $1.42\times10^{-1}$ | $1.15\times10^{-1}$ |

## 4. Experimental procedure

In practice, it is difficult to control the base excitation frequency with different amplitudes. It has to use the mechanical shaker that can vary both the frequency and the displacement simultaneously. Unfortunately, such device is expensive. Therefore, in this work, the rotating unbalance is used as the vibration excitation source. In order to provide the sufficiently large amplitude of the excitation for the experimentation, the natural frequency of the steel beam system was adjusted to coincide with the excitation frequency. This can be accomplished with varying the length of the steel cantilevered beam. For example, in order to investigate the relationship between the displacement and the output voltage at the excitation frequency of 12 Hz, the natural frequency of the steel beam system is also approximately 12 Hz. The length of the steel cantilevered beam calculated by Equation (7), (8) is approximately 34 cm; therefore, the length of the cantilevered beam is varied in order to have a length of 34 cm. In a similar way, the relationships between the displacement and the output voltage at the excitation frequency of 15 Hz and 19 Hz were investigated by the same procedure. The lengths of the steel cantilevered beam at the frequency of 12, 15 and 19 Hz are summarized in Table 1.

The displacement of the base excitation can be controlled by a slight adjustment of the excitation frequency. This is based on the fact that whenever the vibration system vibrates around the resonance frequency, slight variation of the excitation frequency results in the dramatic change of the displacement. Hence, the displacement of the excitation at any excitation frequency can be controlled by a slight adjustment of the DC motor speed around the excitation frequency. For example, in order to control the displacement at the excitation frequency of 12 Hz, the DC motor speed is adjusted to around 12 Hz; it may be 11.7 Hz, 11.8 Hz, or 11.9 Hz. This means that the relationship between the displacement and the output voltage at the frequency of 12 Hz is only approximately 12 Hz, but not exactly 12.00 Hz. Similarly, the relationships between the displacement and the output voltage at the frequency of 15 Hz and 19 Hz were investigated with the same procedure.

## 5. Experimental results and discussions

The first mode natural frequency of the spring-mass system can be experimentally determined by initiating the initial displacement to the spring-mass system. The waveform signal from the signal conditioning circuit that is measured by an oscilloscope (time division is 50 ms/div) can be shown in Figure 5. It is obvious that the waveform signal from the circuit is the harmonic function with the natural frequency $(\omega_n)$ of 8.5 Hz. As the frequency of the excitation of the base is approximately equal to 8.0 Hz $\left(r = \dfrac{\omega}{\omega_n} < 1\right)$, the comparison of the waveform signal from the spring-mass system (strain gauge 1) with those from the base motion (strain gauge 2) measured by an oscilloscope (time division is 50 ms/div) can be shown in Figure 6. It can be seen that the phase angle of the waveform from the spring-mass system (strain gauge 1) is the same as those from the base (strain gauge 2). This implies that both signals are in phase. When the frequency of the excitation is slightly increase to 10 Hz, the comparison of both signals can be shown in Figure 7 (time division is 200 ms/div). It can be seen from the figure that the phenomenon of beats occurs in the spring-mass system whereas the waveform signal from the base is still a harmonic function. The phenomenon of beats in the spring-mass system caused

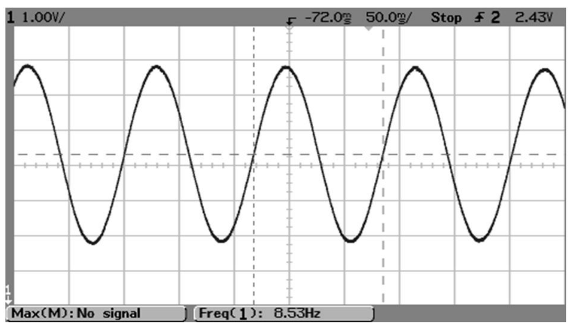


Figure 5. Waveform signal of the first mode natural frequency of the spring-mass system.

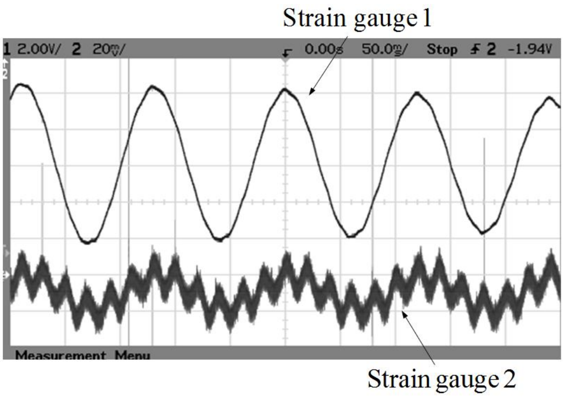


Figure 6. Waveform signal taken from the spring-mass system (strain gauge 1) and from the base (strain gauge 2) at the frequency of 8.0 Hz.

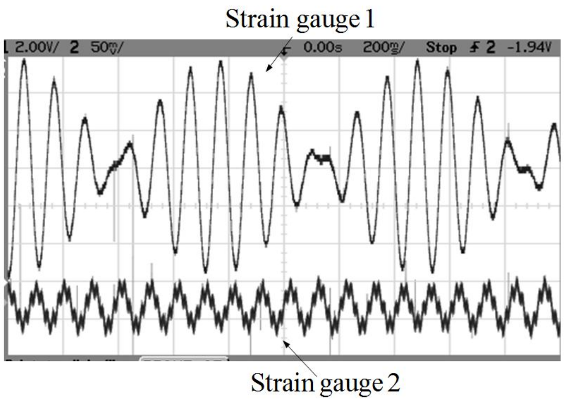


Figure 7. Waveform signal taken from the spring-mass system (strain gauge 1) and from the base (strain gauge 2) at the frequency of 10 Hz.

by that the frequency of excitation is closed to, but not exactly equal to the natural frequency of the system (Kelly, 2000). When the frequency of the excitation is increased to 12 Hz $\left( r = \dfrac{\omega}{\omega_n} > 1 \right)$, both waveform signals can be depicted in Figure 8. It can be seen from the figure that the phase difference between both signals is almost equal to 180°. This implies that both signals are out of phase. It is caused by the fact that when the frequency of vibration is larger than the natural frequency $(r > 1)$, the phase difference between the displacement of spring-mass system and the displacement of the base is equal to 180°.

In addition, the above results can be verified by the theoretical calculation expressed in Equation (6). The calculation from Equation (6) is shown plotted in Figure 9 for several values of damping ratio $(\zeta)$. It can be seen in Figure 9 that for an undamped system $(\zeta = 0)$, the phase angle is 0 for $0 < r < 1$ and 180° for $r > 1$. This implies that the experimental results are in agreement with the theoretical calculation.

As discussed above, the experiment shows the waveform signal from the spring-mass system and the waveform signal from the base at various frequencies. Next, the experiment concern with the relation between the output voltage and the displacement that is obtained by the vibration pickup. In this part, the instrument is subjected to the harmonic excitation of the base at the frequencies of 12, 15 and 19 Hz, respectively.

Obviously, Figure 10, 11 and 12 show that the output voltage of the circuit increase in proportion to the vibration displacement at each frequency of excitations. In other words, output voltage is proportional to the vibration displacement, which is an advantage of the proposed instrument. However, it can be seen from these figures that the slope of each curve are not equal at all. This implies that the output voltage does not only depend on the displacement of vibration but also the frequency of vibration. Therefore, in order to apply this method to measure the vibration displacement, the frequency of excitation must be concerned.

## 6. Conclusions

The measurement of vibration displacement by using strain gauges is able to measure the displacement of vibration at various frequencies. Also, the output voltages from the instrument is proportional to the vibration displacement. Besides, the other advantage of the proposed method is that it is not expensive with simple construction. Nevertheless, the proposed method is frequency dependent. In order to apply this device in practical measurements, the frequency of vibration must be concerned. It may be applied as a look up table technique in practical measurements. However, this paper is a preliminary study. The excitation of the base in this experiment is assumed to be harmonic excitation whereas the practical operating of machinery is not harmonic excitation. In this case, the measurement can be obtained by using the principle of superposition.

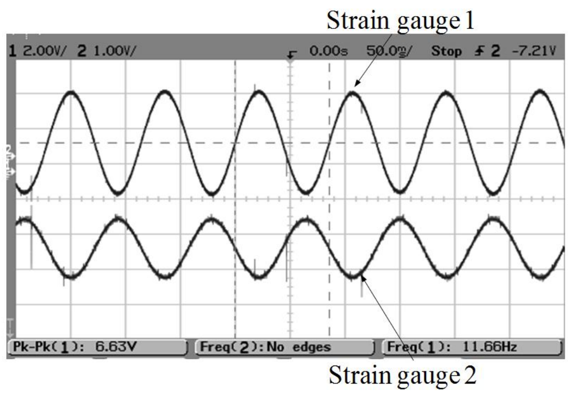


Figure 8. Waveform signal taken from the spring-mass system (strain gauge 1) and from the base (strain gauge 2) at the frequency of 12 Hz.

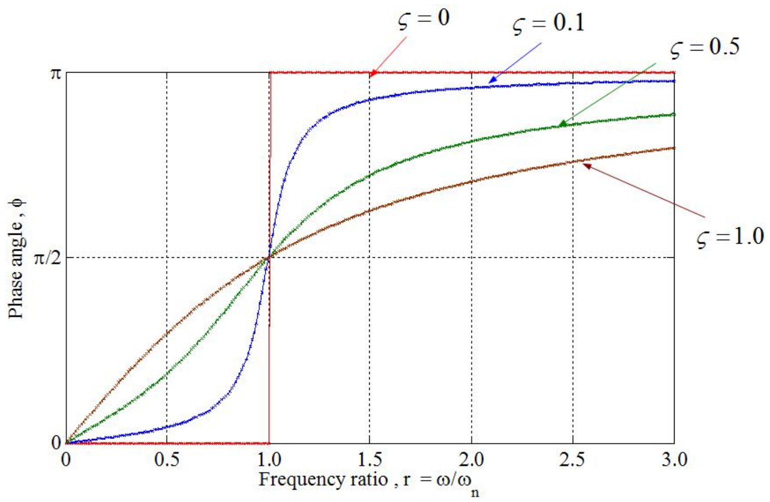


Figure 9. Phase angle for several values of *r*.

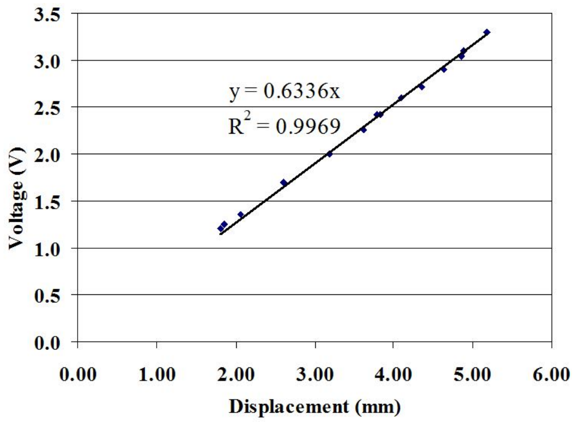


Figure 10. Displacement versus output voltage (Vrms) at the frequency of 12 Hz.

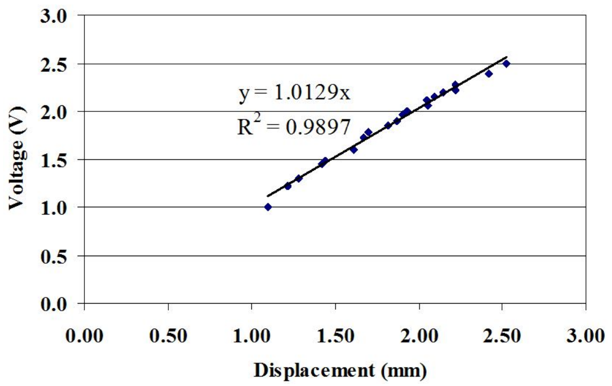


Figure 11. Displacement versus output voltage (Vrms) at the frequency of 15 Hz.

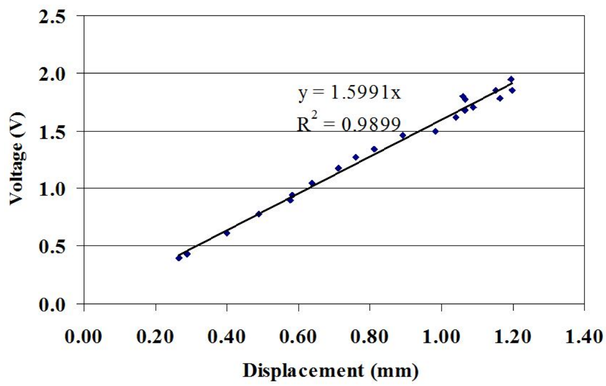


Figure 12. Displacement versus output voltage (Vrms) at the frequency of 19 Hz.

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