Songklanakarin J. Sci. Technol.
35 (3), 369-377, May - Jun. 2013

SJST SONGKLANAKARIN JOURNAL OF SCIENCE AND TECHNOLOGY
http://www.sjst.psu.ac.th

Original Article

Time-partitioning heuristic algorithm for optimal production, inventory, and transportation planning with direct shipment

Napaporn Rianthong and Aussadavut Dumrongsiri*

School of Management Technology,
Sirindhorn International Institute of Technology, Thammasat University, Mueang, Pathum Thani, 12000 Thailand.

Received 28 August 2012; Accepted 11 March 2013

Abstract

We developed a mixed integer linear programming model for an integrated decision problem of production, inventory, and transportation planning. Our model combines the direct shipment into the production, inventory, and distribution planning. The objective was to minimize the total operation cost which is comprised of production setup cost, inventory holding cost, transportation cost, and reorder cost. The model is solved to optimality using the leading optimization software, IBM ILOG CPLEX (CPLEX), but the software shows a limited capability to solve large size problems. A time-partitioning heuristic algorithm is proposed to efficiently solve the problem. Numerical experiments are extensively conducted to test the proposed algorithm. In our numerical experiments, the proposed algorithm can solve many large size problems, whereas CPLEX fails to solve them. The numerical experiment shows that a company can gain a significant saving by optimally incorporating the direct shipment. The proposed heuristic algorithm performs well in most cases in terms of a total cost and the computation time.

Keywords: direct shipment, heuristics, mathematical model, production, inventory and transportation planning, time-partitioning

1. Introduction

Due to global economy downturn and high gasoline prices, a distribution strategy called direct shipment is considered a viable method to reduce operation costs. In direct shipment, a manufacturer directly delivers the products to the retailers by bypassing a warehouse, thereby saving the transportation cost from a plant to the warehouse, a material handling cost, and an inventory holding cost at the warehouse. Also, with direct shipment, the operations and coordination are simplified (Chopra and Meindl, 2007).

The applications of direct shipment are mentioned in a number of papers. Burns *et al.* (1985) showed that the optimal shipment size can be calculated using the economic order quantity model for direct shipping. Gallego and Simchi-Levi (1990) showed that direct shipping is at least 94% effective when the minimum economic lot size is at least 71% of the truck capacity. Extensions of this study can be found in Hall (1992) and Jones and Qian (1997). Liu *et al.* (2003) showed from a total number of 11,520 numerical experiments, that using a mixture of direct shipments and traditional shipments (via intermediate stages) can lower the transportation distance in 84.1% of the cases. The average distance saving is 11.5% when it is compared with the distance of the system without the direct shipment. Li *et al*. (2010) pointed out that the effectiveness of direct shipping is at least the square root of the smallest utilization ratio of a vehicle capacity. Barnes-Schuster and Bassok (1997) and Kleywegt *et al*. (2002) analyzed the application of direct shipment in the case of products under stochastic demand. Bertazzi *et al.* (1997) and Bertazzi (2008) studied direct shipment with a discrete shipping time. Jang and Kim (2007) considered direct ship-

* Corresponding author.
Email address: aussadavut@siit.tu.ac.th

ment in a two echelon distribution system under stochastic demand and customer-specific waiting costs. The inventory routing problem for a three-level distribution system is analyzed by Chan and Simchi-Levi (1998) and Li *et al.* (2011). Pishvaee and Rabbani (2011) proposed a mixed integer linear programming model solved by a graph theoretic-based heuristic algorithm for a supply chain network design problem with direct and indirect shipment.

The study of our problem was motivated by a consumer-product manufacturer. The company has an initiative to reduce the quantities of products sent to a warehouse by sending as many products as possible directly to the customers. However, planning the production, inventory and logistics at the same time for large scale problems poses a challenging task to the company. The company seeks a viable solution to help minimize the total cost using the direct shipment. However, leading optimization software such as IBM ILOG CPLEX fails to provide a good result within acceptable computation time.

However, with direct shipment, the production setup cost can be higher from frequent changeover because the demand is satisfied directly from the production lines. In addition, the inventory holding cost at the retailers can be higher due to the large shipment size of a full truck load. Therefore, research in an integrated decision of production, inventory, and transportation planning will help supply chain managers find the balance of all the related costs and make better decisions. Chandra and Fisher (1944) reported a cost saving of 20% from using an integrated production and distribution approach. Moreover, Martin *et al.* (1993) showed that an annual cost saving of $2,000,000 can be achieved using an optimization model for an integrated decision model in the glass industry. Aliev *et al.* (2007) applied a fuzzy model to an integrated planning problem when the customer demand and the capacities in the production environment are uncertain. Armentano *et al.* (2011) proposed tabu search algorithms for an integrated production and vehicle-routing problem under deterministic demand. The results show that their proposed heuristics are efficient and can provide near optimal solutions within an acceptable time. The existing methodologies proposed for an integrated problem are: Lagrangean relaxation (Fumero and Vercelli, 1999), a two-phase approach (Lei *et al.*, 2006), and the branch and price heuristic (Bard and Nananukul, 2010). An extensive review of mathematical programming model for an integrated planning can be found in Mula *et al.* (2010). A number of studies applied an integrated decision model in industrial cases, such as in the urea fertilizer industry (Haq *et al.*, 1991), the newspaper industry (Song *et al.*, 2002), the ready-mix industry (Garcia and Lozano, 2004), the automotive industry (Jin *et al.*, 2008) and the perishable food industry (Chen *et al.*, 2009).

The remaining part of the paper is organized as follows. In Section 2, the problem motivating our study is represented and the mathematical model is constructed. In Section 3, the time-partitioning heuristic algorithm is proposed. In Section 4, we test the efficiency and the limitation of the proposed method. Section 5 provides concluding remarks with future research directions.

2. Problem Definition and Model

2.1 Problem description

We consider a firm who owns multiple production lines and distributes multiple products to a number of retailers directly from a plant or through a warehouse by a fleet of trucks with limited capacity. The problem is motivated by a consumer-product manufacturer, a Thai subsidiary of a multinational corporation, located near Bangkok. According to Figure 1, the products are scheduled for production at the production lines. At the end of production lines, the finished products (in pallets) are loaded onto the outbound trucks and then shipped to the warehouse. Then, they are transported to the retailers when retailers place orders. With direct shipment, the finished products are sent to the loading area, loaded onto the outbound trucks and then directly shipped to the retailers without going to the warehouse. We assume that the demand at each retailer is deterministic and must be fully satisfied. The manufacturer makes production, inventory and transportation decisions simultaneously in order to minimize the total operation costs. The holding costs are incurred at the loading area, the warehouse, and retailers. The firm incurs a fixed set up cost for a product changeover, a fixed reorder cost for each retailer order, and route-dependent truck costs.

2.2 Mixed integer linear programming model

We formulate a mixed integer linear programming model, and the following are the notations used in this paper. The quantity of product is measured in unit of pallets unless stated otherwise. Decision variables and auxiliary decision variables as shown in Figure 1 are:

$L_{j,t,i}$ = Quantity of product i produced by production line j and sent to the loading area in period t

$Inv_{n,i,t}$ = Quantity of product i inventory at node n at the end of period t

$Tr_{o,d,t}$ = Number of trucks used to transport the product from o to d in period t

$Sh_{i,o,d,t}$ = Quantity of product i being shipped from o to d in period t

$P_{j,t,i}$ = Production quantity of product i from production line j in period t

$f_{j,t,i}$ = 1 if product i is produced from production line j in period t; 0 otherwise

$Or_{o,r,t}$ = 1 if product is ordered from node o by retailer r in period t; 0 otherwise

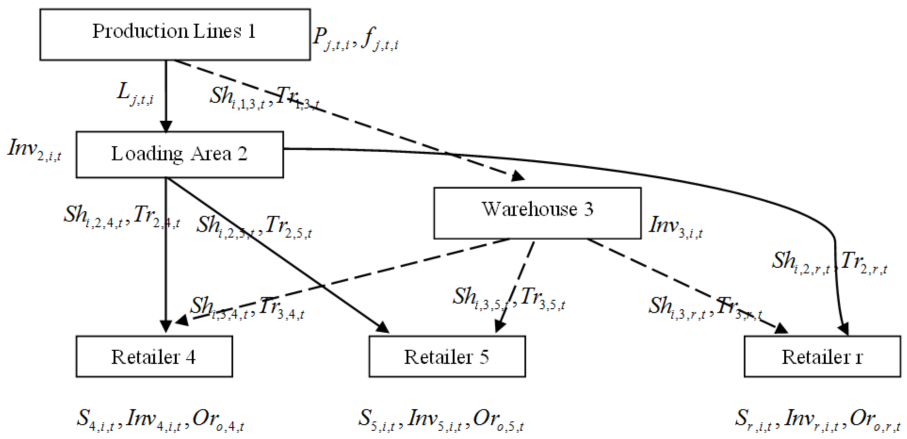

Figure 1. Flow of Products and Decision Variables at each location.

The firm requires the following data to make decisions:

$S_{r,i,t}$ = Demand for product i at retailer r in period t

$Pr_{i,j}$ = Production rate of product i at production line j

$Ct_{i,j}$ = Changeover time to product i at production line j

$\overline{P}$ = Upper bound of production quantity for all production lines

$\underline{P}$ = Lower bound of production quantity for all production lines

$\overline{H}$ = Number of available production times (hours) per period per production line

Rc = Fixed reorder cost

$\overline{A}$ = Maximum number of pallets that can be stored in the loading area

$\overline{C}$ = Maximum number of pallets a truck can carry in one container

$h_{n,i}$ = Inventory holding cost of product i per pallet per period at node n

$Vc_{tr,o,d}$ = Variable cost incurred from using truck to ship one container from o to d

$Sc_{i,j}$ = Setup cost incurred from a changeover to product i at production line j

Mathematical Model:

Minimize Total Cost Function z =

$$\sum_{t=1}^{T}\sum_{o\in N_m}\sum_{r\in N_r} Rc\times Or_{o,r,t} + \sum_{t=1}^{T}\sum_{o\in N}\sum_{d\in D_o} Vc_{tr,o,d}\times Tr_{o,d,t} + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{n=2}^{N} h_{n,i}\times Inv_{n,i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{i=1}^{I} f_{j,t,i}\times Sc_{i,j} \quad (1)$$

Subject to:

$$Inv_{2,i,t} = Inv_{2,i,t-1} + \sum_{j=1}^{J} L_{j,t,i} - \sum_{d\in D_2} Sh_{i,2,d,t} \quad \text{for all } i\in Set\ i \text{ and } t\in Set\ t \quad (2)$$

$$Inv_{3,i,t} = Inv_{3,i,t-1} + Sh_{i,1,3,t} - \sum_{d\in D_3} Sh_{i,3,d,t} \quad \text{for all } i\in Set\ i, t\in Set\ t \quad (3)$$

$$Inv_{r,i,t} = Inv_{r,i,t-1} + \sum_{o\in N_m} Sh_{i,o,r,t} - S_{r,i,t} \quad \text{for all } i\in Set\ i, t\in Set\ t, r\in Set\ r \quad (4)$$

$$Inv_{n,i,0} = 0 \quad \text{for all } i\in Set\ i,\ n\in Set\ N \quad (5)$$

$$L_{j,t,i} \le P_{j,t,i} \quad \text{for all } i\in Set\ i, j\in Set\ j, t\in Set\ t \quad (6)$$

$$Sh_{i,1,3,t} = \sum_{j=1}^{J}\left(P_{j,t,i} - L_{j,t,i}\right) \quad \text{for all } i\in Set\ i, t\in Set\ t \quad (7)$$

$$\sum_{i=1}^{I} Inv_{2,i,t} \le \bar{A} \ for \ all \ t \in Set \ t \quad (8)$$

$$\sum_{i=1}^{I} Sh_{i,o,d,t} \le Tr_{o,d,t} \times \bar{C} \quad for \ all \ t \in Set \ t, o \in N, d \in D_o \quad (9)$$

$$P_{j,t,i} \le f_{j,t,i} \times \bar{P} \ for \ all \ i \in Set \ i, j \in Set \ j, t \in Set \ t \quad (10)$$

$$P_{j,t,i} \ge f_{j,t,i} \times \underline{P} \ for \ all \ i \in Set \ i, j \in Set \ j, t \in Set \ t \quad (11)$$

$$\sum_{i=1}^{I} \left(P_{j,t,i} / Pr_{i,j}\right) + \sum_{i=1}^{I} \left(f_{j,t,i} \times Ct_{i,j}\right) \le \bar{H} \ for \ all \ j \in Set \ j, t \in Set \ t \quad (12)$$

$$\sum_{i=1}^{I} Sh_{i,o,r,t} \le \ Or_{o,r,t} \times M \ for \ all \ r \in Set \ N_r, t \in Set \ t, o \in SetN_m \quad (13)$$

The objective function, equation 1, is to minimize total cost, comprised of reorder cost, transportation cost, inventory holding cost, and production setup cost. Equations 2, 3 and 4 calculate the inventories at the loading area, the warehouse, and the retailer. Equation 5 sets the initial condition for inventory. Equation 6 states that the quantity of products sent to the loading area cannot exceed the quantity of products produced from the production lines. Equation 7 calculates the quantity of products to be transported to the warehouse. Equation 8 sets the upper limit on the number of pallets stored at the loading area at the end of a period. Equation 9 calculates the numbers of containers to be transported from the origin node to the destination node. Equation 10 and 11 put constraints on the lower bound and the upper bound of the production batch sizes. Equation 12 stipulates that the sum of total production time must not exceed the total time available in a period. Equation 13 states that if there is a shipment from the warehouse or the loading area to any retailers, the auxiliary binary variable must be one to correctly calculate the reorder cost.

3. Time-partitioning Heuristic

The proposed heuristic for this problem is based on the time-partition approach. The planning horizon is always partitioned into three periods: the initial period, called period 0, the current period, called T_1, and the future period, called T_2. The period 0 is the period just before the current period. The decisions in period 0 will affect the current period as it is the initial state of the current period. The current period is the immediate period that a factory has to produce enough products. In the current period, the firm tries to meet all the current demands using the inventory at the warehouse and at retailers given by decision variables in period 0 and the finished goods from production lines produced within this current period. The demand for the current period is the actual demand to be satisfied from production and inventory this period. The future period represents all future periods after the current period combined. The demand for future period is the sum of demands of all periods after the current period. The production plan for the future period is temporary and not fixed. The plan for the future period is devised to make sure that it is possible to satisfy the future demand at low costs, given the past and current decisions. The future plan indicates which products are to be produced later, not at the current periods. This future plan will facilitate smooth production operations in the future. As the current period moves from period 1 to the end, the production plan of each period is determined based on the plan of previous period indexed by period 0 as the initial status. After the algorithm determines the plan for all periods, all plans are combined as one production plan for all periods. The main idea of procedure of algorithm is stated as follows:

3.1 Steps showing the main idea of proposed heuristic

Step 1: Initial the variables and parameters

$$Let \ t = 1, \ T_1 = 1, T_2 = 2, Inv_{2,i,0} = 0, Inv_{3,i,0} = 0, Inv_{r,i,0} = 0$$
$$for \ all \ i \in I, r \in N_r$$

Step 2: Combine the demand and production time into two periods

$$Let \ Set \ T_1 = \{t\}, Set \ T_2 = \{t+1, ..., T\},$$
$$S_{r,i,T_1} = S_{r,i,t}, \ S_{r,i,T_2} = \sum_{k \in SetT_2} S_{r,i,k}$$
$$\bar{H}_{T_1} = total \ production \ time \ available \ in \ period \ t = \bar{H}_t,$$
$$\bar{H}_{T_2} = total \ production \ time \ available \ in \ period \ T_2 = \sum_{k \in Set \ T_2} \bar{H}_k$$

Step 3: Solve the disaggregated model for two periods.

Solve the model with two periods (i.e. T_1 and T_2) to optimality using an optimization software tool such as IBM ILOG CPLEX. Record value of decision variables in period but discard those in period .

Step 4: Use the current decision variables as initial conditions for next current period.

Set $Inv_{2,i,0} = Inv_{2,i,T_1}, Inv_{3,i,0} = Inv_{3,i,T_1}, Inv_{r,i,0} = Inv_{r,i,T_1}$

for all $i \in I, r \in N_r$,

Set $t = t + 1$

Step 5: Check a terminal condition.

If $t = T$, *then go to Step* 6; *otherwise go to Step* 2.

Step 6: Consolidate the production plan found in Step 3 from each of current period into one production plan for all periods.

4. Numerical Experiment

We conducted several numerical experiments to show the benefits from incorporating the direct shipment within the supply chain and also tested the performance of the proposed heuristic. Unless stated otherwise, input data used in the experiment is provided in Table 1. The data in Table 1 is hypothetical but approximated from the company motivated the problem.

In the first experiment, the total cost of production, inventory and distribution using both direct shipment and warehouse options is compared with the total cost using only the direct shipment option. This will show the cost saving from optimally incorporating the direct shipment in the distribution strategy. Using four planning periods, we assume that a manufacturer produces five products on two production lines and then distributes the products to a number of retailers by a fleet of trucks with limited capacity. To represent a wide range of situations, we vary the number of retailers, R∈ {5, 10, 15, 20, 30, 50, 100} each of which is tested for five replications. The demands of the five replications are randomized using the different means of {40, 60, 80, 100, 120} pallets per period. Therefore, a total of 35 problem instances are generated. The optimization software tool called IBM ILOG CPLEX version 12.4 is used to solve the problems to optimality and then the results are shown in Table 2. In this numerical experiment, we observe that the firm could generate an average cost saving of 18.64% by optimally incorporating direct shipment within the supply chain.

Table 1. Input data.

Input data		
Production setup cost	$ 570	
Reorder cost	$ 31.67	
Unit holding cost at each location	Loading area = $ 0.19	
	Warehouse = $ 0.06	
	Retailer = $ 0.47	
Transportation cost	Plant to warehouse	= $ 31.67
	Warehouse to retailer	= $ 95
	Loading area to retailer	= $ 95
Production setup time	2 hrs	
Total working hours per week	168 hrs	
Pallet space	10 sq.ft (1 pallets)	
Loading area space	600 sq.ft (60 pallets)	
Container space	400 sq.ft (40 pallets)	

Table 2. Impact of the number of retailers on the average cost saving from optimally incorporating the direct shipment with 5 products, 4 periods and $R \in \{5,10,15,20,30,50,100\}$.

No. of Retailers	Avg. total cost ($) from CPLEX		Avg. of % Saving	SD of % saving	Min. of % saving	Max. of % saving
	Direct	No direct				
5	26,383	29,631	10.53	1.60	8.42	12.48
10	46,708	56,121	15.74	3.57	11.17	19.79
15	66,455	82,620	18.87	2.39	15.75	21.52
20	86,651	108,909	19.76	2.35	16.25	22.04
30	127,403	161,172	20.33	2.10	17.65	22.54
50	205,940	265,110	21.87	1.55	19.43	23.32
100	410,335	537,443	23.40	0.92	21.87	24.18
Average	138,554	177,287	18.64	2.07	15.79	20.84
SD	133,816	176,921	4.31	0.84	4.65	3.94

In the next experiment, the heuristic solutions are compared to the optimal solutions obtained by CPLEX to show the effectiveness of the proposed heuristic. Due to complexity of the problem, we test the problems with five retailers, five products, four periods and two production lines as the starting point so that CPLEX can find the optimal solution in most of the experiments. To show the impact of the number of retailers on the performance of the proposed heuristic, we vary the number of retailers, R∈ {5, 10, 15, 20, 30, 50, 100}. Similarly, each of them is tested five times using randomly generated demand with the mean of {40, 60, 80, 100, 120}. The result is shown in Table 3. We define the optimality gap as the difference, expressed as percentage, between the optimal total cost given by CPLEX and the total cost obtained from the proposed heuristic algorithm. In this numerical experiment, the results show that the proposed heuristic can solve the problem on average 327 times faster than that of CPLEX and the average optimality gap is 0.44%.

Table 4 shows the performance of the proposed heuristic by varying the number of products, I ∈ {5, 10, 15, 20, 30, 50, 90}. Similarly, each of them is tested five times using randomly generated demand with the mean of {40, 60, 80,100, 120}. The number of retailers is five, and the planning horizon is four periods. The results show that, within 10,000 seconds, CPLEX can optimally solve only small size problems, I ∈ {5, 10, 15, 20}. However, in this numerical experiment, the proposed heuristic can solve problems on average 727 times faster than that of CPLEX, and the average optimality gap is 0.43%. We also observe that CPLEX fails to give optimal decisions within 10,000 seconds fifteen times out of thirty-five instances (43%), but the heuristic algorithm can solve all of them within 118.62 seconds in the worst cases. In Table 4, the cases when CPLEX fails to solve problems are also shown.

Table 5 reports on the impact of the number of production lines on the performance of both methods. We vary the number of production lines, J∈ {2, 3, 4 5}. Similarly, each of them is tested five times using randomly generated demand with the mean of {40, 60, 80, 100, 120}. In this numerical experiment, the results show that the proposed heuristic algorithm can solve the problems on average 994 times faster than that of CPLEX, and the average optimality gap is 1.01%.

To test the effect of demand pattern to performance of the heuristic algorithm, we construct four patterns of demand: a constant demand, a zigzag demand, a seasonal demand with an upward trend and a seasonal demand with a downward trend. The value of constant demand is determined by the midpoint of zigzag demand. The zigzag demand is shifted upward and downward constantly each period to construct the seasonal demand with an upward trend and the seasonal demand with a downward trend respectively. To allow CPLEX to solve the problem to optimality with a larger number of periods (T=8), the problem size is reduced to four retailers, and three products. The experiments are repeated four times with the different means of demand. Figure 2 plots an example of four demand patterns of product i=1 faced by the retailer r=4.

Table 6 shows the result of computation by CPLEX and the proposed heuristic algorithm. In our numerical result, it is found that the demand patterns affect the computation time and the average optimality gap. The computation time of the heuristic algorithm is relatively unchanged. However, the seasonal demand with an upward trend causes a larger effect to the average optimality gap (2.72%).

From all of the numerical experiments conducted, it can be concluded that the proposed heuristic algorithm performs well in most cases. However, the proposed heuristic algorithm also shows some limitation. The worst case performance of the proposed heuristic algorithm in terms of the average computation time and the average total cost is found when the proposed heuristic algorithm is applied to the seasonal demand with an upward trend. However, for this case, the average optimality gap is 2.72% which is still considered acceptable. Also, as the number of retailers or the number of products decreases, the average optimality gap

Table 3. Impact of the number of retailers on the performance of the two approaches with 5 products, 4 periods.

No. Retailer	Avg. of the CPU time (sec)		Time Required by CPLEX/Heuristic	Avg. gap (%)	SD. gap (%)	Mingap (%)	Maxgap (%)
	CPLEX	Heuristic					
5	79.25	0.53	149.53	0.92	0.56	0.00	1.50
10	105.27	0.50	211.38	0.88	0.44	0.40	1.36
15	326.53	0.94	346.64	0.46	0.48	0.02	1.16
20	521.70	0.75	697.45	0.33	0.31	0.01	0.71
30	1227.37	1.66	739.38	0.31	0.20	0.08	0.60
50	217.94	2.43	89.69	0.16	0.16	0.02	0.30
100	240.61	4.16	57.84	0.00	0.00	0.00	0.01
Average	388.38	1.57	327.41	0.44	0.31	0.08	0.81
SD	398.35	1.34	283.23	0.35	0.20	0.15	0.56

Table 4. Impact of the number of products on the performance of the two approaches with 5 retailers and 4 periods.

No. Products	Avg. of the CPU time (sec)		Time Required by CPLEX/Heuristic	Avg. gap (%)	SD. gap (%)	Mingap (%)	Maxgap (%)
	CPLEX	Heuristic					
5	79.25	0.53	149.53	0.92	0.56	0	1.5
10	594.91	0.77	776.65	0.31	0.35	0	0.69
15	2091.04	1.61	1295.56	0.16	0.18	0	0.45
20	1228.45	1.79	684.76	0.31	0.51	0.00	1.21
30	>10000	1.92	-	-	-	-	-
30	>10000	0.85	-	-	-	-	-
30	>10000	7.63	-	-	-	-	-
30	out-of-memory	66.22	-	-	-	-	-
30	out-of-memory	30.42	-	-	-	-	-
50	>10000	1.78	-	-	-	-	-
50	>10000	118.62	-	-	-	-	-
50	>10000	3.73	-	-	-	-	-
50	>10000	8.01	-	-	-	-	-
50	>10000	1.25	-	-	-	-	-
90	>10000	2.08	-	-	-	-	-
90	>10000	27.96	-	-	-	-	-
90	>10000	12.63	-	-	-	-	-
90	>10000	2.12	-	-	-	-	-
90	>10000	9.09	-	-	-	-	-
Average	998.41	1.18	726.62	0.43	0.40	0.00	0.96
SD	866.88	0.62	469.39	0.34	0.17	0.00	0.48

Table 5. Impact of the number of production lines on the performance of the two approaches with 5 products, 5 retailers and 4 periods.

Production line	Avg. of the CPU time (sec)		Time Required by CPLEX/Heuristic	Avg. gap (%)	SD. gap (%)	Mingap (%)	Maxgap (%)
	CPLEX	Heuristic					
2	79.25	0.53	149.53	0.92	0.56	0	1.50
3	227.90	0.69	330.29	0.92	0.56	0	1.50
4	615.85	0.50	1231.70	0.69	0.64	0	1.32
5	1561.82	0.69	2263.51	1.49	1.15	0	2.40
Average	621.21	0.60	993.76	1.01	0.73	0	1.68
SD	666.63	0.10	969.84	0.34	0.28	0	0.49

tends to be larger. However, this also means for the larger size problems where the heuristic algorithm is most needed, the proposed heuristic algorithm performs better in terms of the total cost.

5. Conclusion

In this research, we developed a mixed integer linear programming model for an integrated planning of production, inventory and transportation. We consider a manufacturer who produces multiple products on multiple production lines and distributes the products to a number of retailers directly from a plant or through a warehouse by a fleet of vehicles. The objective is to minimize the total operation cost, comprised of production setup cost, inventory holding cost, transportation cost and reorder cost. The model is solved to optimality using IBM ILOG CPLEX version 12. The numerical experiments show that an average cost saving of 18.64% could be achieved by optimally incorporating direct shipment within the supply chain. In addition, a time-partitioning

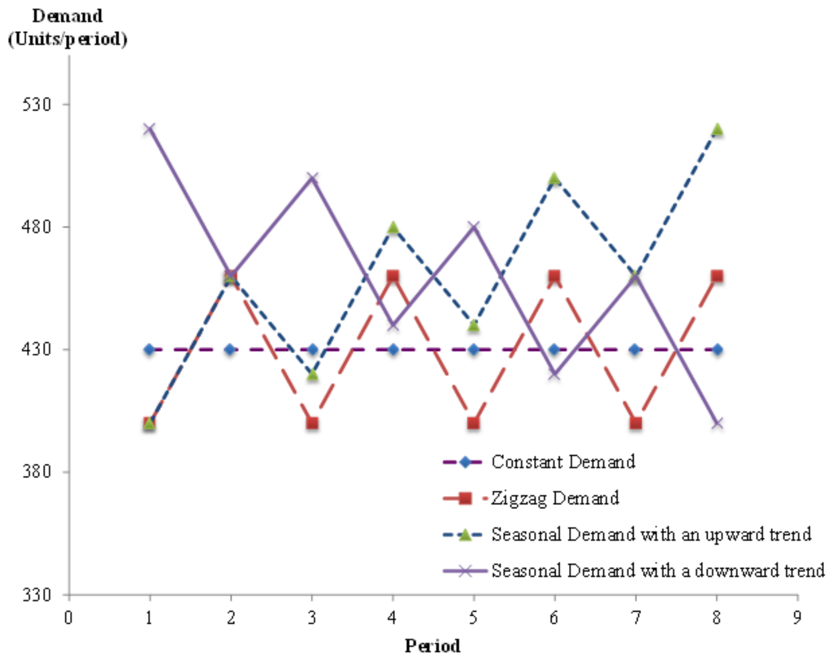

Figure 2. Example of four demand patterns of the product i=1 faced by the retailer r=4.

Table 6. Impact of the demand pattern to the computation time and total cost performed by CPLEX and the proposed heuristic algorithm.

Demand Pattern	Average CPU time (sec)		Average ratio of time Required by CPLEX/Heuristic	Average total cost gap (%) from the optimal cost
	CPLEX	Heuristic		
Constant demand	21.81	0.48	47.00	0.00
Zigzag demand	134.82	0.43	295.02	0.01
Seasonal demand with an upward trend	12.92	0.54	25.28	2.72
Seasonal demand with an downward trend	14.99	0.48	31.77	0.57

heuristic algorithm is proposed to efficiently solve large-scale problems. The solutions from the heuristic algorithm are compared to optimal solutions obtained by CPLEX to show the effectiveness of the proposed heuristic method. The numerical results show that the proposed heuristic performs well in most cases. From the numerical experiment, by varying the number of retailers, the average computation time is 994 times faster than that of CPLEX and the average optimality gap is 1.10%. Similarly, by varying the number of products in numerical experiments, the average computation time is 727 times faster than that of CPLEX and the average optimality gap is 0.43%. The proposed heuristic algorithm also shows some limitations. The worst case performance in terms of computation time (25.29 times faster than CPLEX's) and total cost (the average optimality gap of 2.72%) is found when the heuristic algorithm is applied to a seasonal demand with an upward trend. Also, as the number of retailer is smaller or the number of product decreases, the average optimality gap seems to increase. In the future study, these limitations will be studied and improved.

Acknowledgments

This work was supported by the Higher Education Research Promotion and National Research University Project of Thailand, Office of the Higher Education Commission, Thammasat University Research Fund (2011) and the Low Carbon Scholarship program at Sirindhorn International Institute of Technology supported by the Bangchak Petroleum PCL.

References

Aliev, R.A., Fazlollahi, B., Guirimov, B.G. and Aliev, R.R. 2007. Fuzzy-genetic approach to aggregate production-distribution planning in supply chain management. Information Sciences. 177, 4241-4255.

Armentano, V.A., Shiguemoto, A.L. and Lokketangen, A. 2011. Tabu search with path relinking for an integrated production-distribution problem. Computers & Operations Research. 38, 1199-1209.

Bard, J.F. and Nananukul, N. 2010. A branch and price algorithm for an integrated production and inventory routing problem. Computers & Operations Research. 37, 2202-2217.

Barnes-Schuster, D. and Bassok, Y. 1997. Direct shipping and the dynamic single-depot/multi-retailer inventory system. European Journal of Operational Research. 101, 509-518.

Bertazzi, L. 2008. Analysis of Direct Shipping Policies in an Inventory routing Problem with Discrete Shipping Times. Management Science. 54, 784-762.

Bertazzi, L. and Speranza, M.G. 1997. Minimization of logistic costs with given frequencies. Transportation Research B. 31(4), 327-340.

Burns, L.D., Hall, R.W., Blumenfeld, D.E. and Daganzo, C.F. 1985. Distribution strategies that minimize transportation and inventory costs. Operational Research. 33, 469-490.

Chan, L.M.A. and Simchi-Levi,D. 1998. Probabilistic analyses and algorithms for three-level distribution systems. Management Science. 44, 1562-1576.

Chandra, P. and Fisher, M.L. 1994. Coordination of production and distribution planning. European Journal of Operational Research. 72, 503-517.

Chen, H.K., Hsueh, C.F. and Chang, M.S. 2009. Production scheduling and vehicle routing with time window for perishable food products. Computers & Operations Research. 36, 2311-2319.

Chopra, S., Meindl, P.J., 2007. Supply chain management: strategy, planning, and operation. Pearson/Prentice Hall , Upper Saddle River, N.J., U.S.A, pp.389.

Fumero, F. and Vercellis, C. 1999. Synchronized development of production, inventory, and distribution schedules. Transportation Science. 33, 330-340.

Gallego, G. and Simchi-Levi, D. 1990. On the Effectiveness of Direct Shipping Strategy for the One Warehouse Multi-Retailer R-Systems. Management Science. 36, 240-243.

Garcia, J.M. and Lozano, S. 2004. Production and vehicle scheduling for ready-mix operations. Computer and Industrial Engineering. 46, 803-816.

Hall, R. 1992. A note on bounds for direct shipping cost. Management Science. 38, 1212-1214.

Haq, A.N., Vrat, P. and Kanda, A. 1991. An integrated production-inventory-distribution model for manufacturer of urea: a case. International Journal of Production Economics. 25, 39-49.

Jang, W. and Kim, D. 2007. Integrated production and allocation policies with one direct shipping option. European Journal of Operational Research. 181, 716-732.

Jin, M., Luo, Y. and Eksioglu, S.D. 2008. Integration of production sequencing and outbound logistics in the automotive industry. International Journal of Production Economics. 113,776-774.

Jones, P. and Qian, T. 1997. Fully loaded direct shipping strategy in one warehouse/N retailer systems without central inventories. Transportation Science. 31, 193-195.

Kleywegt, A., Nori, V. and Savelsbergh, M. 2002. The stochastic inventory routing problem with direct deliveries. Transportation Science. 36, 94-118.

Lei, L., Liu, S., Ruszczynski, A. and Park, S. 2006. On the integrated production, inventory, and distribution routing problem. Institute of Industrial Engineers (IIE) Transactions. 38, 955-970.

Li, J., Chen, H. and Chu, F. 2010. Performance evaluation of distribution strategies for the inventory routing problem. European Journal of Operational Research. 202, 412-419.

Li, J., Chu, F. and Chen, H. 2011. A solution approach to the inventory routing problem in a three level distribution system. European Journal of Operational Research. 210, 736-744.

Liu, J., Li, C. and Chan, C. 2003. Mixed truck delivery systems with both hub-and –spoke and direct shipment. Transportation Research Part E. 39, 325-339.

Martin, C.H., Dent, D.C. and Eckhart, J.C. 1993. Integrated production, distribution, and inventory planning at Libber-Owens-Ford. Interfaces. 23, 68-78.

Mula, J., Peidro, D., Diaz-Madronero, M. and Vicens, E. 2010. Mathematical programming models for supply chain production and transport planning. European Journal of Operational Research. 204, 377-390.

Pishvaee, M. and Rabbani, M. 2011. A graph theoretic-based heuristic algorithm for responsive supply chain network design with direct shipment and indirect shipment. Advances in Engineering Software. 42, 57-63.

Song, S.H., Lee, K.S. and Kim, G.S. 2002. A practical approach to solving a newspaper logistics problem using digital map. Computers & Industrial Engineering. 43, 315-330.