



Original Article

## On fuzzy $b$ -locally open sets in bitopological spaces

Binod Chandra Tripathy<sup>1\*</sup> and Shyamal Debnath<sup>2</sup>

<sup>1</sup> *Mathematical Sciences Division,  
Institute of Advanced Study in Science and Technology, Guwahati, Assam, 781035 India.*

<sup>2</sup> *Department of Mathematics, Tripura University, Suryamaninagar, Agartala, Tripura, 799022 India.*

Received: 13 August 2014; Accepted: 13 November 2014

### Abstract

In this article we introduce the notion of fuzzy  $b$ -locally open ( $bLO$ ) sets, fuzzy  $bLO^*$  sets, fuzzy  $bLO^{**}$  sets in fuzzy bitopological spaces and obtain several characterizations and some properties of these sets. Also we introduce the notion of fuzzy  $b$ -locally continuous functions on bitopological spaces.

**Keywords:** fuzzy bitopological spaces, fuzzy  $bLO$  sets, fuzzy  $bLO^*$  sets, fuzzy  $bLO^{**}$  sets.

### 1. Introduction and Preliminaries

The notion of fuzzy sets was introduced by L.A. Zadeh in 1965, and thereafter the paper of Chang (1968) paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The notion of fuzzyness has been applied for studying different aspects of mathematics by Tripathy and Baruah (2010); Tripathy and Borgohain (2011); Tripathy *et al.* (2013); Tripathy and Ray (2012); Tripathy and Sarma (2012a) and many workers on sequence spaces in recent years. The notion of bitopological spaces has been investigated from different aspects by Tripathy and Acharjee (2014); Tripathy and Debnath (2013); Tripathy and Sarma (2011; 2012; 2013; 2014) and others. Kandil (1989) introduced the concept of fuzzy bitopological spaces. Later on several authors were attracted by the notion of fuzzy bitopological spaces. The notion of  $b$ -locally open sets in bitopological spaces was introduced by Tripathy and Sarma (2011). In this paper we introduce the concept of  $b$ -locally open sets in fuzzy bitopological spaces.

Let  $(X, \tau)$  be a topological space. Then

#### Definition 1.1 [Andrijevic (1996)].

Let  $A \subset X$ , then  $A$  is said to be  $b$ -open if  $A \subset cl(intA) \cap int(clA)$ , where  $cl(A)$  and  $int(A)$  denote the closure and interior of the set  $A$ .

#### Definition 1.2 [Kuratowski and Sierpinski (1921)].

Let  $A \subset X$ , then  $A$  is said to be locally closed if  $A = G \cap F$ , where  $G$  is an open set in  $X$  and  $F$  is closed in  $X$ .

#### Definition 1.3 [Nasef (2001)].

Let  $A \subset X$ , then  $A$  is said to be  $b$ -locally closed if  $A = G \cap F$ , where  $G$  is  $b$ -open set in  $X$  and  $F$  is  $b$ -closed in  $X$ .

#### Definition 1.4 [Tripathy and Sarma (2011)].

A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $(\tau_1, \tau_2)$ -locally open (in short  $(\tau_1, \tau_2)$ - $LO$ ) if  $A = G \cup F$ , where  $G$  is  $\tau_1$ -closed and  $F$  is  $\tau_2$ -open in  $(X, \tau_1, \tau_2)$ .

#### Definition 1.5 [Tripathy and Sarma (2011)].

A subset  $A$  of a space  $(X, \tau_1, \tau_2)$  is called  $(\tau_1, \tau_2)$ - $bLO$  (in short  $(\tau_1, \tau_2)$ - $bLO$ ) if  $A = G \cup F$ , where  $G$  is  $\tau_1$ - $b$ -closed and  $F$  is  $b$ -open in  $(X, \tau_1, \tau_2)$ .

#### Definition 1.6 [Tripathy and Sarma (2011)].

A subset  $A$  of a space  $(X, \tau_1, \tau_2)$  is called  $(\tau_1, \tau_2)$ - $bLO^*$  if there exists a  $\tau_1$ - $b$ -closed set  $G$  and a  $\tau_2$ -open set  $F$  of

\* Corresponding author.

Email address: tripathybc@yahoo.com;  
tripathybc@rediffmail.com

$(X, \tau_p, \tau_2)$  s.t.  $A = G \cup F$ .

**Definition 1.7 [Tripathy and Sarma (2011)].**

A subset  $A$  of a space  $(X, \tau_p, \tau_2)$  is called  $(\tau_p, \tau_2)$ - $bLO^{**}$  if there exists a  $\tau_1$ -closed set  $G$  and a  $\tau_2$ - $b$ -open set  $F$  of  $(X, \tau_p, \tau_2)$  s.t.  $A = G \cup F$ .

Let  $X$  be a non-empty set and  $I$ , the unit interval  $[0, 1]$ . A fuzzy set  $A$  in  $X$  is characterized by a function  $\mu_A : X \rightarrow I$  where  $\mu_A$  is called the membership function of  $A$ , and throughout we denote fuzzy elements by  $x_p, p$  representing the membership grade of  $x$ .

The fuzzy operations  $\wedge$  and  $\vee$  on the fuzzy sets of  $X$  are defined as follows.

- Let  $A = \{x_p : \text{for all } x \in X \text{ and } p \text{ is the membership of } x \in X\}$
- and  $B = \{x_q : \text{for all } x \in X \text{ and } q \text{ is the membership of } x \in X\}$
- $A \wedge B = \{x_r : x \in X \text{ and } r = \min\{p, q\} \text{ for each } x \in X\}$ .
- and  $A \vee B = \{x_r : x \in X \text{ and } r = \max\{p, q\} \text{ for each } x \in X\}$ .

**2. b-locally open sets in fuzzy bitopological spaces**

**Definition 2.1**

A fuzzy set  $a$  of a fuzzy bitopological space  $(X, \tau_p, \tau_2)$  is called  $(\tau_p, \tau_2)$ -fuzzy locally open ( in short  $(\tau_p, \tau_2)$ - $FLO$ ) if  $\alpha = \beta \vee \gamma$ , where  $\beta$  is  $\tau_1$ -fuzzy closed and  $\gamma$  is  $\tau_2$ -fuzzy open in  $(X, \tau_p, \tau_2)$ .

**Definition 2.2**

A fuzzy set  $\alpha$  of a fuzzy bitopological space  $(X, \tau_p, \tau_2)$  is called  $(\tau_p, \tau_2)$ -fuzzy  $b$ -locally open ( in short  $(\tau_p, \tau_2)$ - $FbLO$ ) if  $\alpha = \beta \vee \gamma$ , where  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed and  $\gamma$  is  $\tau_2$ -fuzzy  $b$ -open in  $(X, \tau_p, \tau_2)$ .

**Definition 2.3**

A fuzzy set  $\alpha$  of a fuzzy bitopological space  $(X, \tau_p, \tau_2)$  is called  $(\tau_p, \tau_2)$ -  $FbLO^*$  if there exist a  $\tau_1$ -fuzzy  $b$ -closed set  $\beta$  and a  $\tau_2$ -fuzzy open set  $\gamma$  of  $(X, \tau_p, \tau_2)$  such that  $\alpha = \beta \vee \gamma$ .

**Definition 2.4**

A fuzzy set  $a$  of a fuzzy bitopological space  $(X, \tau_p, \tau_2)$  is called  $(\tau_p, \tau_2)$ -  $FbLO^{**}$  if there exist a  $\tau_1$ -fuzzy closed set  $\beta$  and a  $\tau_2$ -fuzzy  $b$ -open set  $\gamma$  of  $(X, \tau_p, \tau_2)$  such that  $\alpha = \beta \vee \gamma$ .

**Theorem 2.5**

Let  $\alpha$  be a fuzzy set of  $(X, \tau_p, \tau_2)$ . Then if  $\alpha \in (\tau_p, \tau_2)$ - $FLO(X)$ , then

- (a)  $\alpha \in (\tau_p, \tau_2)$ - $FbLO^*(X)$ .
- (b)  $\alpha \in (\tau_p, \tau_2)$ - $FbLO^{**}(X)$ .

**Proof:**

(a) Since  $\alpha \in (\tau_p, \tau_2)$ - $FLO(X)$ , so there exists a  $\tau_1$ -fuzzy closed set  $\beta$  and a  $\tau_2$ -fuzzy open set  $\gamma$  such that  $\alpha = \beta \vee \gamma$ .

Since  $\beta$  is  $\tau_1$ -fuzzy closed, we have  $\text{int}(\text{cl}(\beta)) \leq \beta$  and  $\text{cl}(\text{int}(\beta)) \leq \beta$

Therefore,  $\text{int}(\text{cl}(\beta)) \wedge \text{cl}(\text{int}(\beta)) \leq \beta$ .

Hence  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed.

Thus we have,  $\alpha = \beta \vee \gamma$ , where  $\beta$  is  $\tau_1$  fuzzy  $b$ -closed and  $\gamma$  is  $\tau_2$ -fuzzy open.

Hence  $\alpha \in (\tau_p, \tau_2)$ - $FbLO^*(X)$ .

(b) Since  $\alpha \in (\tau_p, \tau_2)$ - $FLO(X)$ , so there exists a  $\tau_1$ -fuzzy closed set  $\beta$  and a  $\tau_2$ -fuzzy open set  $\gamma$  such that  $\alpha = \beta \vee \gamma$ .

Since  $\gamma$  is  $\tau_2$ -fuzzy open, we have  $\gamma \leq \text{int}(\text{cl}(\gamma))$  and  $\gamma \leq \text{cl}(\text{int}(\gamma))$

Therefore,  $\gamma \leq \text{cl}(\text{int}(\gamma)) \vee \text{int}(\text{cl}(\gamma))$ .

Hence  $\gamma$  is  $\tau_2$ -fuzzy  $b$ -open.

Now, we have,  $\alpha = \beta \vee \gamma$ , where  $\beta$  is  $\tau_1$ -fuzzy closed and  $\gamma$  is  $\tau_2$ -fuzzy  $b$ -open.

Hence  $\alpha \in (\tau_p, \tau_2)$ - $FbLO^{**}(X)$ .

**Remark 2.6**

The converse is not necessarily true. It is clear from the following example:

**Example 2.7**

Let  $X = \{a, b, c\}$  and consider the fuzzy sets on  $X$  are  $\alpha_1 = \{a_{0.3}, b_{0.8}, c_0\}$ ,  $\alpha_2 = \{a_{0.4}, b_{0.9}, c_0\}$  and  $\alpha = \{a_{0.4}, b_{0.5}, c_0\}$ . Let  $\tau_1 = \{\underline{0}, \underline{1}, \alpha_1, \alpha_2, \alpha_1 \vee \alpha_2, \alpha_1 \wedge \alpha_2\}$  and  $\tau_2 = \{\underline{0}, \underline{1}, \alpha_1\}$  be two fuzzy topologies on  $X$ .

Then for  $\tau_1$ ,  $\text{cl}(\alpha) = \alpha_2'$ ,  $\text{int}(\text{cl}(\alpha)) = \alpha_2$ ,  $\text{int}(\alpha) = \alpha_2$ ,  $\text{cl}(\text{int}(\alpha)) = \text{cl}(\alpha_2) = \alpha_2'$ .

Therefore,  $\text{int}(\text{cl}(\alpha)) \wedge \text{cl}(\text{int}(\alpha)) = \alpha_2 \wedge \alpha_2' = \alpha_2 \leq \alpha$ .

Thus  $\alpha$  is  $\tau_1$ -fuzzy  $b$ -closed set in  $(X, \tau_p, \tau_2)$ .

Then  $\lambda = \alpha \vee \alpha_1 \in (\tau_p, \tau_2)$ - $FbLO^*(X)$  but  $\lambda = \alpha \vee \alpha_1 \notin (\tau_p, \tau_2)$ - $FLO(X)$ , because  $\alpha$  is not  $\tau_1$ -fuzzy closed set in  $(X, \tau_p, \tau_2)$ .

Next let  $\tau_1 = \{\underline{0}, \underline{1}, \alpha_1\}$  and  $\tau_2 = \{\underline{0}, \underline{1}, \alpha_1, \alpha_2, \alpha_1 \vee \alpha_2, \alpha_1 \wedge \alpha_2\}$  be two fuzzy topologies on  $X$  and  $\beta = \{a_{0.4}, b_{0.9}, c_0\}$  be a fuzzy set on  $X$ .

Then for  $\tau_2$ ,  $\text{int}(\text{cl}(\beta)) = \underline{1}$ ,  $\text{cl}(\text{int}(\beta)) = \underline{1}$ .

Therefore,  $\beta \leq \text{int}(\text{cl}(\beta)) \vee \text{cl}(\text{int}(\beta)) = \underline{1}$ .

Thus  $\beta$  is  $\tau_2$ -fuzzy  $b$ -open set in  $(X, \tau_p, \tau_2)$ .

Then  $\lambda_1 = \alpha_1' \vee \beta \in (\tau_p, \tau_2)$ - $FbLO^{**}(X)$  but  $\lambda_1 = \alpha_1' \vee \beta \notin (\tau_p, \tau_2)$ - $FLO(X)$ , because  $\beta$  is not  $\tau_2$ -fuzzy open set in  $(X, \tau_p, \tau_2)$ .

**Theorem 2.8**

Let  $\alpha$  be a fuzzy subset of a fuzzy bitopological space  $(X, \tau_p, \tau_2)$ . If  $\alpha \in (\tau_p, \tau_2)$ - $FbLO^*(X)$  then  $\alpha \in (\tau_p, \tau_2)$ - $FbLO(X)$ .

**Proof.**

Let  $\alpha \in (\tau_p, \tau_2)$ - $FbLO^*(X)$ , then there exists a  $\tau_1$ -fuzzy  $b$ -closed set  $\beta$  and a  $\tau_2$ -fuzzy open set  $\gamma$  such that  $\alpha = \beta \vee \gamma$ .

Since  $\gamma$  is  $\tau_2$ -fuzzy open, we have  $\gamma \leq \text{int}(\text{cl}(\gamma))$  and  $\gamma \leq \text{cl}(\text{int}(\gamma))$ .

Hence  $\gamma \leq \text{int}(\text{cl}(\gamma)) \vee \text{cl}(\text{int}(\gamma))$ . Thus  $\gamma$  is  $\tau_2$ -fuzzy  $b$ -open set in  $(X, \tau_p, \tau_2)$ .

Then there exists a  $\tau_1$ -fuzzy  $b$ -closed set  $\beta$  and a  $\tau_2$ -fuzzy  $b$ -open set  $\gamma$  such that  $\alpha = \beta \vee \gamma$ .

Therefore,  $\alpha \in (\tau_p, \tau_2)$ - $FbLO(X)$ .

**Remark 2.9**

The converse of the above theorem is not always true. It follows from the following example:

**Example 2.10**

Let  $X = \{a, b, c\}$  and consider the fuzzy sets on  $X$ ,  $\alpha_1 = \{a_{0.3}, b_0, c_{0.8}\}$ ,  $\alpha_2 = \{a_{0.4}, b_0, c_{0.4}\}$ ,  $\alpha = \{a_{0.4}, b_0, c_{0.5}\}$  and  $\beta = \{a_{0.4}, b_0, c_{0.9}\}$ . Let  $\tau_1 = \tau_2 = \{\underline{0}, \underline{1}, \alpha_1, \alpha_2, \alpha_1 \vee \alpha_2, \alpha_1 \wedge \alpha_2\}$  be fuzzy topologies on  $X$ .

Then  $\alpha$  is a  $\tau_1$ -fuzzy  $b$ -closed set and  $\beta$  is a  $\tau_2$ -fuzzy  $b$ -open set. Thus  $\lambda = \alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$  but  $\lambda \notin (\tau_1, \tau_2)\text{-FbLO}^*(X)$  because  $\beta$  is not  $\tau_2$ -fuzzy open in  $X$ .

**Theorem 2.11**

Let  $\alpha$  be a fuzzy subset of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . If  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$  then  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Proof:**

Can be established following standard techniques.

**Remark 2.12**

The converse of the above theorem is not always true. It follows from the following example:

**Example 2.13**

Let  $X = \{a, b, c\}$  and consider the fuzzy sets on  $X$  are  $\alpha_1 = \{a_0, b_{0.3}, c_{0.8}\}$ ,  $\alpha_2 = \{a_0, b_{0.4}, c_{0.4}\}$ ,  $\alpha = \{a_0, b_{0.4}, c_{0.5}\}$  and  $\beta = \{a_0, b_{0.4}, c_{0.9}\}$ . Let  $\tau_1 = \tau_2 = \{\underline{0}, \underline{1}, \alpha_1, \alpha_2, \alpha_1 \vee \alpha_2, \alpha_1 \wedge \alpha_2\}$  be a fuzzy topologies on  $X$ .

Then  $\alpha$  is a  $\tau_1$ -fuzzy  $b$ -closed set and  $\beta$  is a  $\tau_1$ -fuzzy  $b$ -open set. Thus  $\lambda = \alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$  but  $\lambda \notin (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$  because  $\alpha$  is not  $\tau_1$ -fuzzy closed in  $X$ .

**Theorem 2.14**

Let  $\alpha$  and  $b$  be any two fuzzy subsets of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . If  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}(X)$  and  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed and  $\tau_2$ -fuzzy  $b$ -open, then  $\alpha \wedge \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Proof:**

Since  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}(X)$ , then there exists a  $\tau_1$ -fuzzy  $b$ -closed set  $\alpha_1$  and a  $\tau_2$ -fuzzy  $b$ -open set  $\alpha_2$  such that  $\alpha = \alpha_1 \vee \alpha_2$ .

We have,  $\alpha \wedge \beta = (\alpha_1 \vee \alpha_2) \wedge \beta = (\alpha_1 \wedge \beta) \vee (\alpha_2 \wedge \beta)$ . Since  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed and  $\tau_2$ -fuzzy  $b$ -open, then  $(\alpha_1 \wedge \beta)$  is  $\tau_1$ -fuzzy  $b$ -closed and  $(\alpha_2 \wedge \beta)$  is  $\tau_2$ -fuzzy  $b$ -open.

Thus there exist a  $\tau_1$ -fuzzy  $b$ -closed set  $(\alpha_1 \wedge \beta)$  and a  $\tau_2$ -fuzzy  $b$ -open set  $(\alpha_2 \wedge \beta)$  such that  $\alpha \wedge \beta = (\alpha_1 \wedge \beta) \vee (\alpha_2 \wedge \beta)$ . Hence  $\alpha \wedge \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Theorem 2.15**

Let  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$  and  $\beta$  be a  $\tau_1$ -fuzzy closed and  $\tau_2$ -fuzzy open subsets of  $(X, \tau_1, \tau_2)$ , then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$ .

**Proof:**

Can be established following standard techniques.

**Theorem 2.16**

Let  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$  and  $\beta$  be a  $\tau_1$ -fuzzy closed

and  $\tau_2$ -fuzzy open subsets of  $(X, \tau_1, \tau_2)$ , then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$ .

**Proof:**

Can be established following standard techniques.

**Theorem 2.17**

Let  $\alpha$  be a fuzzy subset of a fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . Then  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$  if and only if  $\alpha = \beta \vee \tau_2\text{-int}(\alpha)$ , for some  $\tau_1$ -fuzzy  $b$ -closed set  $\beta$ .

**Proof:**

Let  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$ . Then  $\alpha = \beta \vee \gamma$ , where  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed and  $\gamma$  is  $\tau_2$ -fuzzy open set in  $(X, \tau_1, \tau_2)$ . Since  $\beta \leq \alpha$  and  $\tau_2\text{-int}(\alpha) \leq \alpha$ , we have  $\beta \vee \tau_2\text{-int}(\alpha) \leq \alpha$  (1)

Further,  $\tau_2\text{-int}(\alpha) \leq \gamma$ , therefore,  $\beta \vee \tau_2\text{-int}(\alpha) \vee \gamma = \alpha$  (2)

From (1) and (2), we have,  $\alpha = \beta \vee \tau_2\text{-int}(\alpha)$ .

Conversely, given that  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed, we have,  $\tau_2\text{-int}(\alpha)$  is  $\tau_2$ -open. Thus there exist a  $\tau_1$ -fuzzy  $b$ -closed set  $\beta$  and a  $\tau_2$ -open set  $\tau_2\text{-int}(\alpha)$  in  $(X, \tau_1, \tau_2)$  such that  $\alpha = \beta \vee \tau_2\text{-int}(\alpha)$ . Hence  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$ .

**Theorem 2.18**

Let  $\alpha$  and  $\beta$  be any two fuzzy sets of the fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . If  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}(X)$  and  $\beta$  is either  $\tau_1$ -fuzzy  $b$ -closed or  $\tau_2$ -fuzzy  $b$ -open, then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Proof:**

Since  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}(X)$ , then there exists a  $\tau_1$ -fuzzy  $b$ -closed set  $\alpha_1$  and a  $\tau_2$ -fuzzy  $b$ -open set  $\alpha_2$  such that  $\alpha = \alpha_1 \vee \alpha_2$ . We have,  $\alpha \vee \beta = (\alpha_1 \vee \alpha_2) \vee \beta = (\alpha_1 \vee \beta) \vee \alpha_2$ .

If  $\beta$  is  $\tau_1$ -fuzzy  $b$ -closed, then  $(\alpha_1 \vee \beta)$  is also  $\tau_1$ -fuzzy  $b$ -closed.

Hence  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

Let  $\beta$  be  $\tau_2$ -fuzzy  $b$ -open, then  $\alpha \vee \beta = \alpha_1 \vee (\alpha_2 \vee \beta)$ , where  $\alpha_2 \vee \beta$  is  $\tau_2$ -fuzzy  $b$ -open.

Thus  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Theorem 2.19**

Let  $\alpha$  and  $\beta$  be any two fuzzy sets of the fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . If  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$  and  $\beta$  is either  $\tau_1$ -fuzzy closed or  $\tau_2$ -fuzzy open, then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$ .

**Proof:**

Can be established following standard techniques.

**Theorem 2.20**

Let  $\alpha$  and  $b$  be any two fuzzy sets of the fuzzy bitopological space  $(X, \tau_1, \tau_2)$ . If  $\alpha \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$  and  $\beta$  is either  $\tau_1$ -fuzzy closed or  $\tau_2$ -fuzzy open, then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$ .

**Proof:**

Can be established following standard techniques.

**Theorem 2.21**

If  $\alpha, \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$  then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Proof:**

Let  $\alpha, \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ . Then there exist  $\tau_1$ -fuzzy  $b$ -closed sets  $\alpha_1, \beta_1$  and  $\tau_2$ -fuzzy  $b$ -open sets  $\alpha_2, \beta_2$  such that  $\alpha = \alpha_1 \vee \alpha_2$  and  $\beta = \beta_1 \vee \beta_2$ .

We have  $\alpha \vee \beta = (\alpha_1 \vee \alpha_2) \vee (\beta_1 \vee \beta_2) = (\alpha_1 \vee \beta_1) \vee (\alpha_2 \vee \beta_2)$ , where  $(\alpha_1 \vee \beta_1)$  is  $\tau_1$ -fuzzy  $b$ -closed set and  $(\alpha_2 \vee \beta_2)$  is  $\tau_2$ -fuzzy  $b$ -open set.

Hence  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}(X)$ .

**Theorem 2.22**

If  $\alpha, \beta \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$  then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}^*(X)$ .

**Proof:**

Can be established following standard techniques.

**Theorem 2.23**

If  $\alpha, \beta \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$  then  $\alpha \vee \beta \in (\tau_1, \tau_2)\text{-FbLO}^{**}(X)$ .

**Proof:**

Can be established following standard techniques.

**Definition 2.24**

Let  $(X, \tau_1, \tau_2)$  and  $(Y, \rho_1, \rho_2)$  be two bitopological spaces and  $f: X \rightarrow Y$  be a mapping. Then  $f$  is said to be fuzzy locally continuous if the inverse image of each FLO-set of  $Y$  is FLO in  $X$ .

**Definition 2.25**

Let  $(X, \tau_1, \tau_2)$  and  $(Y, \rho_1, \rho_2)$  be two bitopological spaces and  $f: X \rightarrow Y$  be a mapping. Then  $f$  is said to be fuzzy  $b$ -locally continuous if the inverse image of each FbLO-set of  $Y$  is FLO in  $X$ .

From the definition it is obvious that every fuzzy locally continuous function is fuzzy  $b$ -locally continuous but the converse may not be true (by Theorem 2.5 and Theorem 2.7).

**References**

- Andrijevic, D. 1996. On  $b$ -open sets. *Matematički Vesnik*. 48, 59-64.
- Azad, K.K. 1981. On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity. *Journal of Mathematical Analysis Applications*. 82, 14-32.
- Chang, C.L. 1968. Fuzzy topological spaces. *Journal of Mathematical Analysis Applications*. 24, 182-90.
- Kandil, A. 1989. Biproximities and fuzzy bitopological spaces. *Simon Stevin*. 63, 45-66.
- Kelly, J.C. 1963. Bitopological Spaces. *Proceedings of the London Mathematical Society*. 13, 71-89.
- Kuratowski, C. and Sierpinski, W. 1921. Sur les difference de deux ensembles fermes. *Tohoku Mathematical Journal*. 20, 22-25.
- Nasef, A.A. 2001.  $b$ -locally closed sets and related topics. *Chaos Solitons & Fractals*. 12, 1909-1915.
- Tripathy, B.C. and Acharjee, S. 2014. On  $(\gamma, \delta)$ -Bitopological semi-closed set via topological ideal. *Proyecciones Journal of Mathematics*. 33(3), 345-357.
- Tripathy, B.C. and Baruah, A. 2010. Nörlund and Riesz mean of sequences of fuzzy real numbers, *Applied Mathemaatics Letters*. 23, 651-655.
- Tripathy, B.C. and Borgogain, S. 2011. Some classes of difference sequence spaces of fuzzy real numbers defined by Orlicz function. *Advances in Fuzzy Systems*. Article ID216414, 6 pages.
- Tripathy, B.C. and Debnath, S. 2013.  $\gamma$ -open sets and  $\gamma$ -continuous mappings in fuzzy bitopological spaces. *Journal of Intelligent and Fuzzy Systems*. 24(3), 631-635.
- Tripathy, B.C., Paul, S. and Das, N.R. 2013. Banach's and Kannan's fixed point results in fuzzy 2-metric spaces. *Proyecciones Journal of Mathematics*. 32(4), 363-379.
- Tripathy, B.C. and Ray, G.C. 2012. On Mixed fuzzy topological spaces and countability. *Soft Computing*. 16(10), 1691-1695.
- Tripathy, B.C. and Sarma, B. 2012a. On  $I$ -convergent double sequences of fuzzy real numbers. *Kyungpook Mathematical Journal*. 52(2), 189-200.
- Tripathy, B.C. and Sarma D.J. 2011. On  $b$ -locally open sets in bitopological spaces. *Kyungpook Mathematical Journal*. 51(4), 429-433.
- Tripathy, B.C. and Sarma D.J. 2012. On pairwise  $b$ -locally open and pairwise  $b$ -locally closed functions in bitopological spaces. *Tamkang Journal of Mathematics*. 43(4), 533-539.
- Tripathy, B.C. and Sarma D.J. 2013. On weakly  $b$ -continuous functions in Bitopological spaces, *Acta Scientiarum Technology*. 35(3), 521-525.
- Tripathy, B.C. and Sarma D.J. 2014. Generalized  $b$ -closed sets in Ideal bitopological spaces. *Proyecciones Journal of Mathmathematics*. 33(3), (2014), 315-324.