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Original Article

A robust cellular associative memory for pattern recognitions using composite trigonometric chaotic neuron models

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Abstract

This paper presents a robust cellular associative memory for pattern recognitions using composite trigonometric chaotic neuron models. Robust chaotic neurons are designed through a scan of positive Lyapunov Exponent (LE) bifurcation structures, which indicate the quantitative measure of chaoticity for one-dimensional discrete-time dynamical systems. The proposed chaotic neuron model is a composite of sine and cosine chaotic maps, which are independent from the output activation function. Dynamics behaviors are demonstrated through bifurcation diagrams and LE-based bifurcation structures. An application to associative memories of binary patterns in Cellular Neural Networks (CNN) topology is demonstrated using a signum output activation function. Examples of English alphabets are stored using symmetric auto-associative matrix of *n*-binary patterns. Simulation results have demonstrated that the cellular neural network can quickly and effectively restore the distorted pattern to expected information.

Keywords: associative memory, pattern recognition, trigonometric chaotic neuron

1. Introduction

A chaotic neuron model that imitates real biological neuron activities in human nervous systems has served as an elementary constituent inartificial neural networks (ANN), which is a complex and nonlinear information processing system (Guoguang *et al*., 2008). The chaotic neuron typically exhibits rich dynamic behaviors, involving static, periodic, quasi-periodic and chaotic states. Therefore, the characteristic of a specific chaotic neuron type effectively determine the distinctiveness and overall performances of ANNs. Interconnections of chaotic neurons in a cellular topology offer an operative function of associative memories for a variety of applications such as in information recognitions and retrievals (Xia *et al*., 2010). The original and well-studied discrete-time chaotic neuron model proposed by Aihara *et al.* (1990) qualitatively realizes chaotic behaviors of squid giant axon and Hodgkin–Huxley models as follows;

$$x_{n+1} = kx_n - \beta f_1(x_n) + I_0 \quad (1)$$

$$y_{n+1} = f_2(x_{n+1}) \quad (2)$$

where x_{n+1} is an internal state, y_{n+1} is a neuron output, β is a parameter for refractoriness, k is a damping factor of the refractoriness, I_0 is the sum of all input excitations. The functions $f_1(\cdot)$ and $f_2(\cdot)$ are nonlinear feedback and activation functions, respectively. Such a model realizes is a sigmoidal function in both internal feedback state and the output sections and is given by

$$f_1(x) = f_2(x) = \frac{1}{1+e^{-\frac{x}{\varepsilon}}} \quad (3)$$

where ε is a steepness parameter. Figure 1 illustrates the corresponding block diagram of chaotic neuron model in Equation 2. Arrangements of Equation 2 by substituting x_{n+1}

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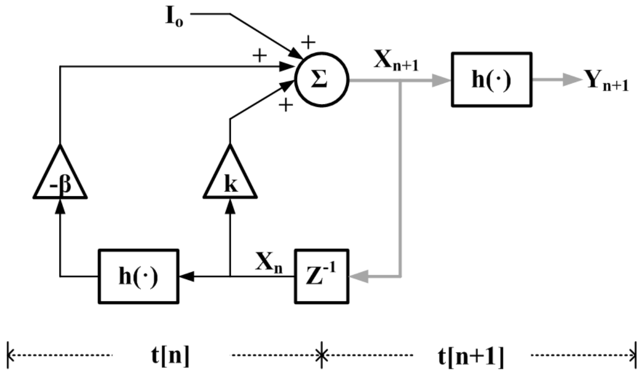

Figure 1. Block diagram of existing chaotic neuron in Equation 2.

into y_{n+1} yield

$$y_{n+1} = f(kx_n - \beta f(x_n) + I_0)$$

$$= f(kx_n + \beta(1 - f(x_n)) + (I_0 - \beta)) \quad (4)$$

It can be considered from Equation 4 that y_{n+1} can alternatively be expressed as $y_{n+1}=f_2(g(x_n)+\theta)$ where $g(x_n)=kx_n+\beta(1-f_1(x_n))$ is an N-shape nonlinear function and $\theta = I_0 - \varepsilon$ is a constant. As a result, the discrete-time chaotic neuron model whose input-output mapping through an *N*-shape sigmoidal function $h(x_n)$ can be described as

$$y_{n+1} = h(x_n) = f_2(k,\beta,I_0,\varepsilon,x_n,f_1(\varepsilon,x_n)) \quad (5)$$

It is apparent in Equation 5 that the overall characteristics of chaotic neurons significantly depend on a type of nonlinear feedback function and system parameters.

A particular consideration on Equation 2 points that the nonlinear feedback function $f_1(\cdot)$ significantly sets dynamical behaviors of both a single neuron and an overall ANN structure. Consequently, several alternative functions have been suggested over the past decade. Zhou and Chen (2000) proposed the use of a hyperbolic tangent function in coupled neurons for a search in global minima of the energy with transient chaos. Xiu *et al*. (2004) proposed a chaotic neuron with a combination of Gauss and sigmoidal functions for multi-valued associative memory. Tanaka and Hiura (2005) introduced a piecewise sine map for well-defined optimization problems. Zhou *et al*. (2010) suggested a non-monotonic Gaussian function with strong approximation ability due to compact support and symmetry properties. Jung *et al*. (2011) realized an approximated three-piecewise empirical equation for VLSI implementations of ANNs. Xiu and Liu (2010) proposed symmetric polynomial and smooth hysteretic functions for associative memories, respectively.

Although most existing chaotic neurons have successfully been realized in particular applications, nonlinear functions employed are relatively complicated as a number of system parameters are involved. The setting of appropriate parameter values or additional parameter controller is consequently required in order to sustain the desirable performances of ANNs. This rises to the question of whether there are other chaotic neurons composed by simple nonlinear feedback function and small number of parameters, but potentially offers rich dynamic behaviors, of the simple form

$$x_{n+1} = f_1(\alpha_1 x_n) + f_2(\alpha_2 x_n) + ... + f_m(\alpha_m x_n) \quad (6)$$

where *m* is a positive integer and the final input-output mapping described as

$$y_{n+1} = h(x_n) = f_n(\sum_{\alpha=1}^{m} f_m(\alpha_1,\alpha_2,...,\alpha_m,x_n)) \quad (7)$$

where *n* is a positive integer, α_1 and α_2 are parameters necessitated for setting the bifurcation regions in mathematical functions. Note that the functions $f_1(\cdot)$ to $f_m(\cdot)$ may not be identical as long as desirable dynamic behaviors are achieved.

2. Proposed Robust Composite Trigonometric Chaotic Neuron Models

On the contrary to single modal chaotic maps, i.e. logistics map, and other two or multi modal chaotic maps based on polynomial functions, which comprises several mathematical terms, trigonometric chaotic neuron models, i.e. sine and cosine maps, are found to be potential results in terms of chaotic dynamics and a simple neuron structure in which a single input excitation and two arbitrary parameters are required. In fact, Maclaurin series reveal that both sine and cosine offer relative complex functions in the form

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... \quad (8)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... \quad (9)$$

Equations 8 and 9 indicate that complex chaotic behaviors can be achieved if these functions are combined in the form described in Equation 6.

In order to investigate the complex behaviors of chaotic maps, the bifurcation diagram is realized as a tool for qualitative measure while the Lyapunov Exponent (*LE*) is realized as another tool for quantitative measure. On the one hand, the bifurcation diagram shows a period doubling that accompanies the onset of chaos, and also represents the sudden appearance of a qualitatively different solution for a nonlinear system as some parameter is varied. On the other hand, the LE characterizes the rate of separation of infinitesimally close trajectories, and can be described as

$$LE = \lim_{t\to\infty} \lim_{\Delta x_o \to 0} \frac{1}{t} \ln \frac{|\Delta x(t)|}{|\Delta x_o|} \quad (10)$$

where Δx_o is an initial separation of the two trajectories in phase space. Typically, the cases where *LE*<0 and *LE* = 0 indicate that the orbit attracts to a stable fixed point or stable periodic orbit, a neutral fixed point, respectively. In the particular case where *LE*>0, the orbit is unstable and chaotic.

Figure 2 shows the bifurcation diagram of the existing chaotic neuron in Equation 2. It can be seen that the small region of chaos is apparent; indicating that the chaotic behaviors cannot be sustained over parameter spaces and are also vulnerable to parameter changes. Figure 3 shows the Bifurcation diagrams of sine and cosine maps with single parameter α_1; (a) $x_{n+1} = \sin(\alpha_1 x_n)$, (b) $x_{n+1} = \cos(\alpha_1 x_n)$, (c) $x_{n+1} = \sin(\alpha_1 x_n) + \cos(\alpha_1 x_n)$, and (d) $x_{n+1} = \sin(\alpha_1 x_n) - \cos(\alpha_1 x_n)$. It can be seen from Figure 3 that the chaotic behaviors cover a wider range than that of Figure 2. However, a single parameter $á_1$ may not be sufficient for sustaining chaos over the region. This paper therefore proposes the use of composite trigonometric chaotic neuron as;

$$x_{n+1} = \cos(\alpha_1 x_n) + \sin(\alpha_2 x_n) \quad (11)$$

where α_1 and α_1 are parameters associated with the frequency of sine and cosine functions. Figure 4 shows the corresponding block diagram of the proposed chaotic neuron in Equation 11. Figure 5 show the plots of 2-D bifurcation structures between α_1 and α_2 of the proposed chaotic neuron described in Equation 11. This bifurcation structure is obtained from LE values, i.e. the white region represents non-chaotic behaviors while the blue region represents the chaotic behaviors. It is apparent in Figure 5 that the proposed chaotic neuron offers attractive features on dynamic behaviors for applications where highly chaotic behaviors are required.

3. Cellular Chaotic Neuron Networks Using Sinusoidal Chaotic Neuron Models

The associative memory dynamics has demonstrated that a CNN is a promising approach in information processing such as memory recalling or pattern recognition (Chua and Yang, 1998; Osana, 2012). Figure 6 shows a finite-size 2-D CNN structure in which neurons are arranged in M×N matrix size. It is seen in Figure 6(a) that each neuron is identified by the position in the grid and communicates directly to the neighborhoods through the *r*-neighborhood. In such a topology, the dynamics of the i^{th} neurons in the position range [1, M×N] can be described as (Sudo *et al.*, 2009)

$$x_{i,n+1} = f(n_{i,n+1} + y_{i,n+1}) \quad (12)$$

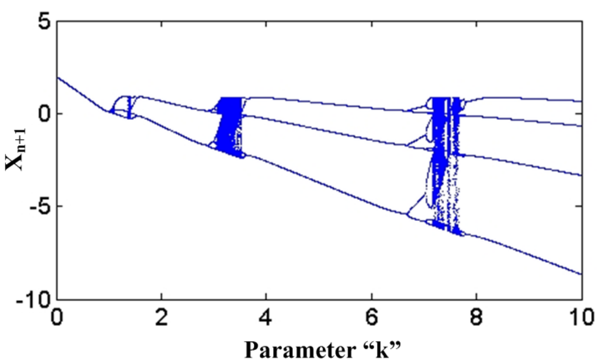

Figure 2. Bifurcation diagram of the existing chaotic neuron in Equation 2.

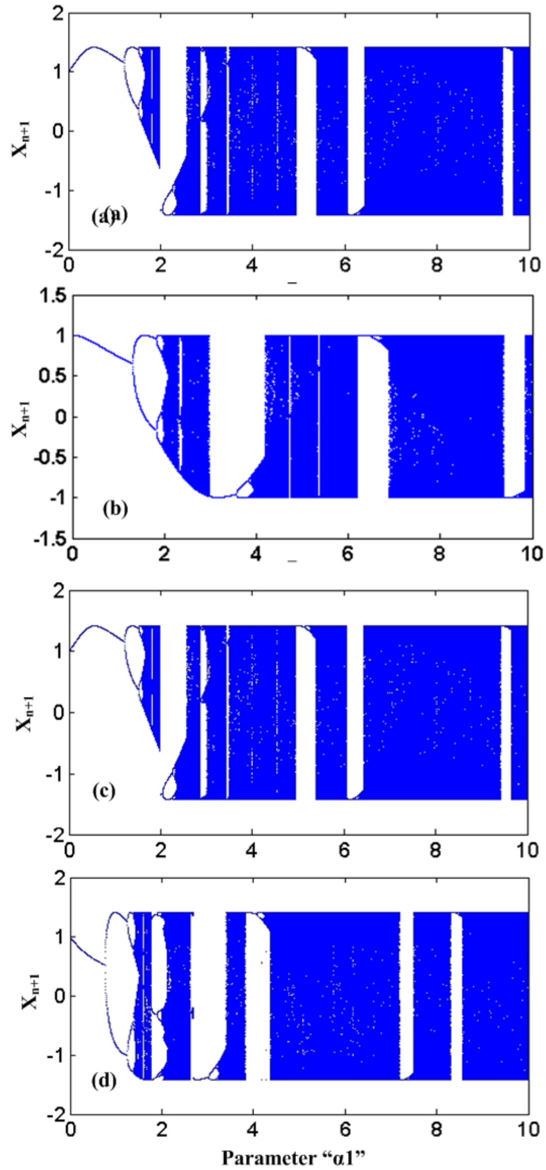

Figure 3. Bifurcation diagrams of sine and cosine maps with single parameter α_1; (a) $x_{n+1} = \sin(\alpha_1 x_n)$, (b) $x_{n+1} = \cos(\alpha_1 x_n)$, (c) $x_{n+1} = \sin(\alpha_1 x_n) + \cos(\alpha_1 x_n)$, and (d) $x_{n+1} = \sin(\alpha_1 x_n) - \cos(\alpha_1 x_n)$.

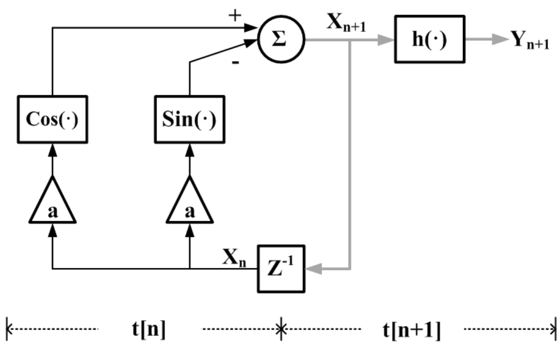

Figure 4. Proposed composite trigonometric chaotic neuron model.

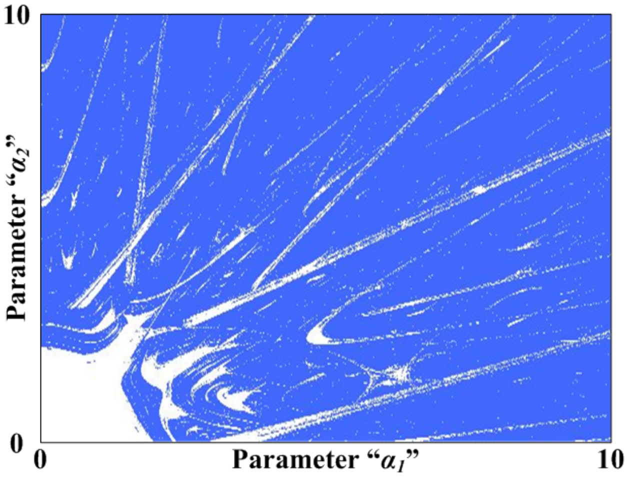

Figure 5. Bifurcation structure obtained from LE of the composite $x_{n+1}=\sin(\alpha_1 x_n)+\cos(\alpha_2 x_n)$.

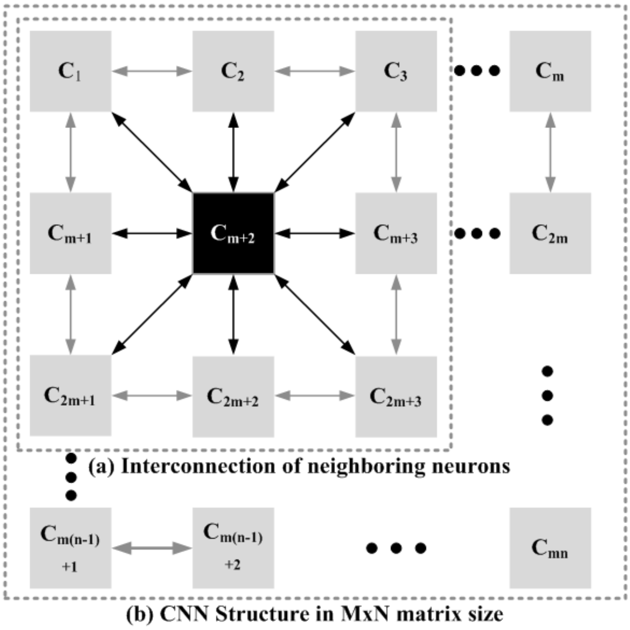

Figure 6. Finite-size 2-D CNN in which neurons are arranged in M rows and N columns.

$$n_{i,n+1} = kn_{i,n} + \sum_{j}^{N} w_{ij} x_{j,n} \tag{13}$$

$$y_{i,n+1} = \cos(\alpha_1 x_n) + \sin(\alpha_2 x_n) + I_i \tag{14}$$

where $x_{i,n}$ is the output of the i^{th} chaotic neuron, the function $f(\cdot)$ realized in this paper has been designated as a hyperbolic tangent function. The functions $n_{i,n}$ and $y_{i,n}$ are an internal state variable of the feedback inputs from the constituent neurons in the network and a refractoriness of the i^{th} chaotic neuron, respectively. N is the number of neurons in the network. k is the parameters of the feedback inputs. I_i is an external output to the CNN. w_{ij} is the synaptic weight to the i^{th} constituent neuron from the j^{th} constituent neuron. It should be noted that a neuron has no self-synaptic connection, i.e. $w_{ij} = 0$ for $i = j$. In this paper, the weights are determined by the symmetric auto-associative matrix of n binary patterns designated by

$$w_{ij} = \frac{1}{n} \sum_{p=1}^{n} (2x_i^p - 1)(2x_j^p - 1) \tag{15}$$

where n is the total number of stored memory patterns and p is the order of stored binary patterns. As for demonstrations, Figure 7 illustrates three simple binary patterns employed for demonstration CNN restoration performances; each pattern comprises five binary pixels, corresponding to the network constructed with 25 neurons. The parameters k, α_1 and α_2 in the simulations were 1, 10, -3, respectively. The communication was achieved by 1-neighboring neurons. The initial conditions of all chaotic neurons described in Equation 10 were equally set at 0.5. Figure 8 shows the restoration of the binary patterns obtained from the partially stored patterns where the recall patterns were achieved at 20, 9 and 18 iterations. In addition, Figure 9 shows the restoration of the binary patterns obtained from the noisy patterns where the recalled patterns were was achieved at 6, 20, and 14 iterations. It is apparent that the CNN has successfully restored all the memorized patterns. Furthermore, Table 1 also exhibits the comparison of operation time of a discrete-time Hopfield, a symmetric map (Tao *et al*., 2011) and proposed chaotic neuron model, which each patterns are obtained by injecting the different ratio of noise. It is indicated that the average of successful recall iterations of the trigonometric chaotic neuron model is lower than for another models and 20% of noise is the maximum of the recognition ability. The processing time depends on computer performance. According to the Intel (R) Xenon (R) CPU E5-1603@2.8 GHz with 64-bit operating system, the processing time to recover the original alphabet was approximately 3-5 seconds. It is really fast process due to the chaotic maps operate in iteration rather than other techniques where system are solve by differential equation, which really takes time for execution. In terms of limitation of this method, the proposed technique may not suitable for a long phase or sentences. This is very common feature similar to other method. The future work will focus on the recognition of a word or phrase in order to apply for real applications.

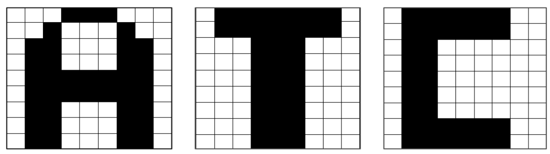

Figure 7. Three memorized binary patterns employed for demonstration CNN restoration performances.

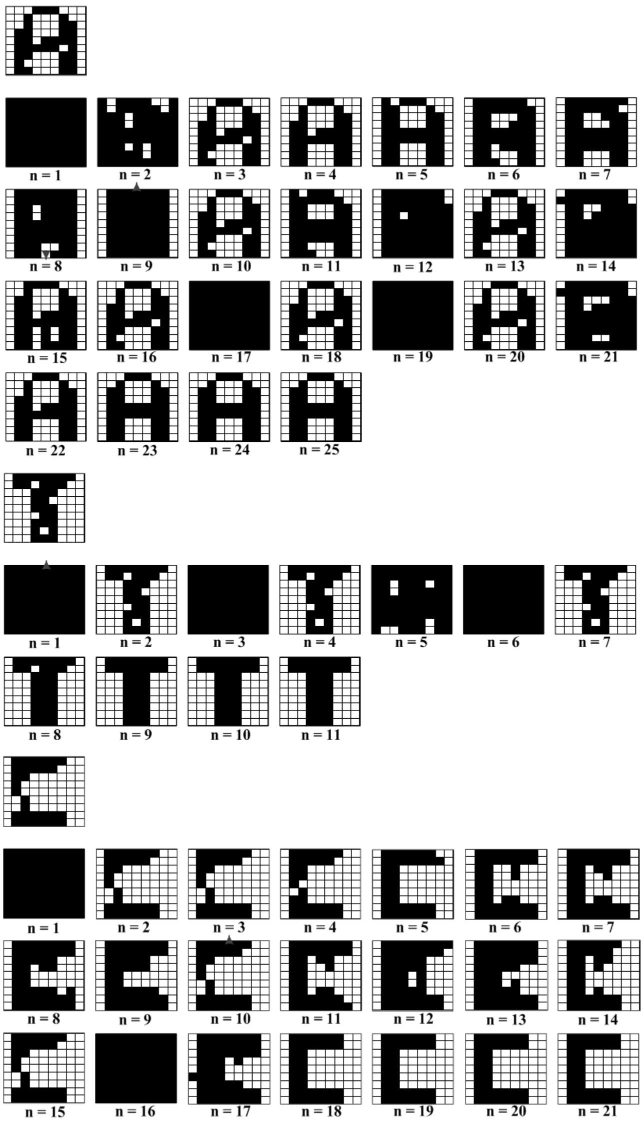

Figure 8. The output of associative memory showing the restoration of the memorized binary patterns when the input patterns are partially distorted.

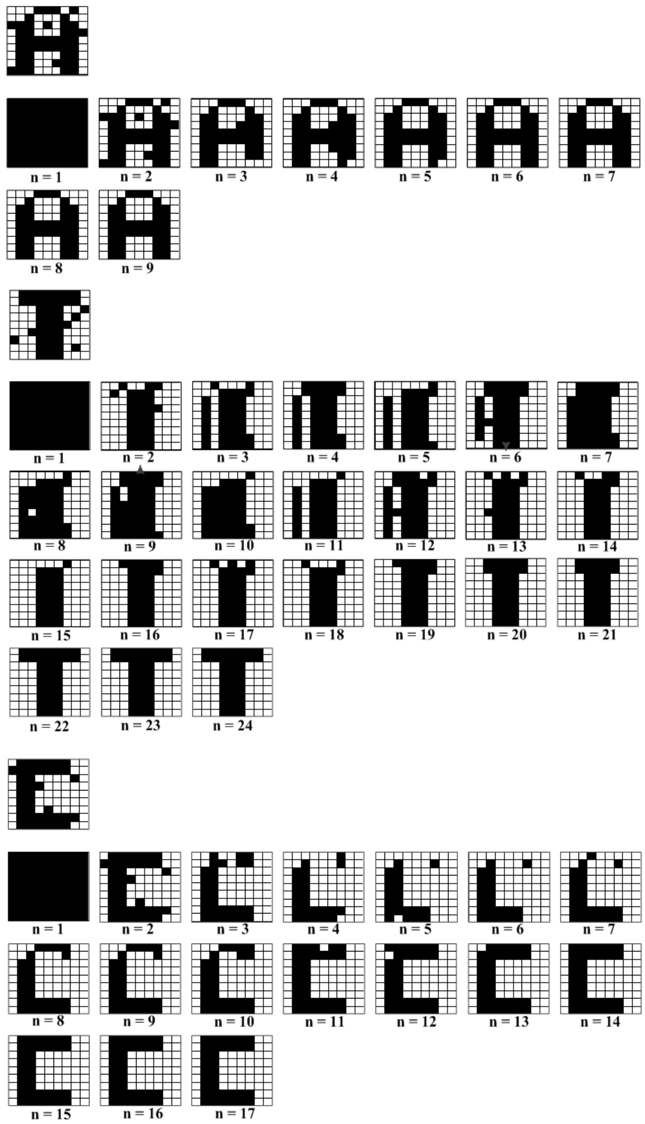

Figure 9. Output of associative memory showing the restoration of the memorized binary patterns when the input binary pattern are noisy.

Table 1. Comparison of success recall iterations.

Noise (%)	Hopfield ANN			Symmetric Map [Tao *et al.*]			Trigonometric Map		
	"A"	"T"	"C"	"A"	"T"	"C"	"A"	"T"	"C"
10	32	30	34	25	22	24	10	10	15
15	37	33	39	27	23	27	12	16	21
20	42	39	45	33	27	30	20	25	26

4. Conclusions

Since the nonlinear functions in most existing chaotic neurons are relatively complicated and a number of system parameters involved, the setting of appropriate parameters or additional parameter control techniques is necessarily required in order to sustain desirable performances of ANNs. This paper has therefore presented a robust cellular associative memory for pattern recognitions using composite trigonometric chaotic neuron models. Robust chaotic neurons were found through an exhaustive scan of positive Lyapunov Exponent bifurcation structures, which indicate the quantitative measure of chaoticity for a one-dimensional discrete-time dynamical system. The proposed chaotic neuron model is a composite of sine and cosine chaotic maps, which are independent from the output activation function. Dynamics behaviors are demonstrated through bifurcation diagrams and LE-based bifurcation structures. An application to associative memories of binary patterns in CNN topology is demonstrated using a signum output activation function. Examples of English alphabets are stored using symmetric auto-associative matrix of n-binary patterns. Simulation results have demonstrated that the CNN can quickly and effectively restore the distorted pattern to expected information.

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