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Original Article

Group-buying inventory policy with demand under Poisson process

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Abstract

The group-buying is the modern business of selling in the uncertain market. With an objective to minimize costs for sellers arising from ordering and reordering, we present in this paper the group buying inventory model, with the demand governed by a Poisson process and the product sale distributed as Binomial distribution. The inventory level is under continuous review, while the lead time is fixed. A numerical example is illustrated.

Keywords: demand, group-buying, inventory model, lead time, Poisson process

1. Introduction

The development of E-business has expanded to every corner of the world. In recent years, many modern business models for Internet-based selling have emerged with the advent of Social-Commerce with web-based group buying being gaining more popularity among both sellers and consumers.

Group-buying is one of the business strategies, which attracts buyers with similar interest, and allows them to obtain significant volume discount from the seller on the product they wish to purchase (Liu and Sutato, 2012). The seller, in turn, benefits from selling the product at higher volume.

In the group-buying process, the seller determines the discounted price, the starting and ending of the sale period, and the fixed number of units required by the deal. Participating buyers would win the deal only if the final number of purchasing unit meets the predetermined number at the end of time. For instance, the market price of one mobile phone is $199; one web-based group-buying seller may offer it for the price of $179 only when at least 10 units are purchased. To attract other buyers, the seller displays on the web the number of units having been so far ordered, in other words, how many more units still need to be ordered for the product to be sold at a reduced price. If the order exceeds 10 units at the end of the signified time of the deal, the buyers are committed to purchasing the mobile phone; however, the discount is called off if the total number of units ordered is fewer than 10.

Group-buying models have been widely proposed from many researchers. Cheney (2010) claimed that the group-buying strategy attracts the cheapskates, those who will not pay the full price of the products but wait a few days for a lower price. Anand and Aron (2003) have proven that the group-buying strategy is more effective than the posted-pricing when the demand is uncertain. Only if the distribution of the buyers' demand is known, the sellers are almost always better off by running the posted-price strategy. Chen *et al.* (2004) compared between the group-buying and the posted-pricing strategies when the buyers' arrival is of a Poisson process. They found that the group-buying outmatched the posted price mechanisms. Chen *et al.* (2007) studied the seller's pricing strategy under group-buying in

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three situations, the seller's expected profit, economies of scale, and risk-seeking seller by comparing the group-buying with the fixed-price mechanism. They found that the group-buying outperformed the fixed-price mechanism in the latter two situations, while the former saw the equivalent effect for both mechanisms. Chen *et al.* (2010) pointed out that group-buying under demand uncertainty was likely more effective where there is larger low valuation than high valuation demand. Jing and Xie (2011) found that the group-buying presided individual selling and another social interaction strategies when interpersonal communication is very efficient and also when the product valuation of the less-informed consumer segment is high. Chen *et al.* (2012) obtained the optimal rationing threshold policy under group-buying for a retailer facing multiple demands.

In this work, we propose a new group-buying inventory model that would aid sellers in planning their strategies under the unknown demand of customers. It is organized as follows. The group-buying inventory model with a restriction on the holding cost and the lead time demand governed by a Poisson process of the buyer is presented in Section 2. A numerical example for minimizing the total cost is illustrated in Section 3. Finally, Section 4 contains conclusions.

2. The Group-Buying Inventory Model

In this section, we illustrate an inventory model corresponding to the group-buying process for group-buying websites. There are three variables, which could be defined as follows.

P : Price of product
N : Number of buyers required
T : Time for each deal

We assume that the customer arrival process is a Poisson process with stationary and independent increments. The deal begins at time zero and ends when either of the following two happens.

(a) The number of buyers equals to N, then the deal is success. The products will be delivered to the buyers.

(b) The number of buyers cannot meet N until the time T, the process ends without any products delivered to the buyers.

We now assume the following seven assumptions for developing the model.

1. The demand is a random variable with known distribution.
2. The next deal restarts immediately after a deal ends.
3. An inventory level is under continuous review.
4. The lead time of the inventory replenishment is fixed.
5. An order quantity of size Q per cycle is placed every time.
6. The stock level reaches a certain re-order point r.
7. The shortage cost is not allowed.

The following notations are adopted for developing our model:

Q : The replenishment quantity
R : The re-order point
L : The lead time
SS : The safety stock
A, B : Positive integer
X : A random variable distributed as Binomial distribution.

Figure 1 illustrates the group-buying inventory model, if the number of buyers meets N within time T or "deal yes" then the goods are sold at volume N and the inventory level will decrease to Q-N. In contrast, if the deal is not a success or "deal no", goods are not sold and the inventory level remains unchanged. The inventory level decreases to zero when the time reaches BT and at this time, the volume is replenished.

Consider the customer's arrival process as a Poisson process with arrival rate λ, the process $Y(t)$ represents the number of a customer arriving during time $(0,t)$. Then $Y(t)$ has

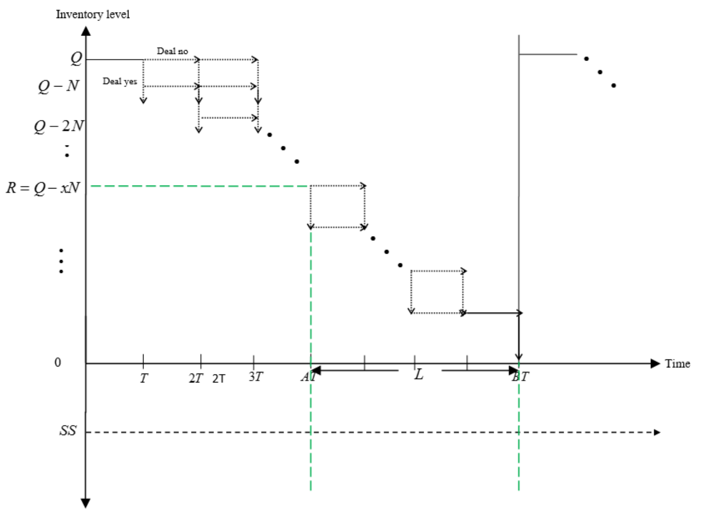

Figure 1. The inventory level of a replenishment cycle

a Poisson distribution with a mean λt from Narayan (1972) given as

$$P[Y(t)=N]=e^{-\lambda t}\left(\lambda t\right)^{N}/N! \quad (1)$$

The cumulative distribution function of $Y(t)$ is given by

$$P[Y(t)\le N]=\sum_{n=0}^{N}e^{-\lambda t}\left(\lambda t\right)^{n}/n! \quad (2)$$

In our model, let p_T be the probability of the deal that ends before the time T, in other words, the probability that the products are sold is

$$p_T=P[Y(T)\ge N]=1-\sum_{n=0}^{N-1}e^{-\lambda T}\left(\lambda t\right)^{n}/n! \quad (3)$$

In Figure 1, let the demand X be a random variable from Binomial distribution, which represents the number of times the products are sold for the interval of the replenishment cycle, where each time an event is either a success, the products are sold, or a cancellation of the deal, the products are not sold. Therefore, at the lead time (L), we then assume X_L to be the number of times where the products are sold during lead time (L) of B-A times. Therefore, X_L has a binomial distribution with parameters B-A and P_T. The probability mass function of X_L is

$$P(X_L=x)=\binom{B-A}{x}p_T^{x}\left(1-p_T\right)^{B-A-x} \quad x=0,1,\ldots,B-A. \quad (4)$$

Therefore, if the number of sales during the lead time is multiplied by the number of items sold each, it will be total number of items sold during the lead time (NX_L) (say, demand during the lead time).

The optimal value R^* is formed as follows:

$$P(NX_L\le R)=1-\alpha$$

$$P(X_L\le R/N)=1-\alpha$$

$$\sum_{x=0}^{R/N}\binom{B-A}{x}p_T^{x}\left(1-p_T\right)^{B-A-x}=1-\alpha\,. \quad (5)$$

where α is the probability that stock-out will occur between the time an order quantity is placed and the order quantity is received.

When considering the relationship between the lead time (L) and the time for each deal (T), we then obtain

$$L\approx(B-A)T \quad (6)$$

and

$$B-A\approx L/T\,. \quad (7)$$

Therefore, B-A could be approximated by L/T. So, Eq. (5) can be improved as given

$$\sum_{x=0}^{R/N}\binom{L/T}{x}p_T^{x}\left(1-p_T\right)^{(L/T)-x}=1-\alpha\,. \quad (8)$$

Hence, we can calculate the re-order point R* by Eq. (8) and find the amount of safety stock (SS) as provided by

$$SS=R^*-E(NX_L)=R^*-NLp_T/T\,. \quad (9)$$

The optimal value of Q^* is found by minimizing the function of total cost (TC), as shown in Eq. (10). The function TC of the group-buying inventory model could be expressed as

$$TC = \text{Setup cost} + \text{Unit cost} + \text{Holding cost} \quad (10)$$

The time between consecutive replenishments of inventory is referred to as a cycle. In Figure 1, the cycle length is BT. The TC per unit time is obtained from the following components.

The setup cost per cycle is K.

The unit cost for producing per cycle is cQ where represents the unit price paid.

The average inventory level during a cycle is $BT((Q+SS+SS)/2)=BT(Q/2+SS)$ units, and then the hold cost per cycle is $hBT((Q/2)+SS)$.

Therefore, the TC per cycle is

$$TC=K+cQ+hBT((Q/2)+SS) \quad (11)$$

Then, the TC per unit time is

$$TC=(K+cQ+hBT((Q/2)+SS))/BT$$

$$=K/BT+cQ/BT+h((Q/2)+SS) \quad (12)$$

where $B=(L/T)+(Q-R)/(p_t/N)$.

The optimal value Q^* can be obtained by minimizing TC of Eq. (13). Setting the first derivative of TC with respect to Q equal to zero and solving yields the solution

$$Q^*=((G^{1/2}-Lp_tN)/T)+R \quad (13)$$

where $G=2(kp_tNT+(RTcp_tN)-Lcp_t^{2}N^{2})/h$.

The minimum total cost is given by

$$TC_{\min}=kp_tN/(Q^*-R)+Lp_tN+h\left(Q^*/2+R^*-NLp_t/T\right)$$

$$+CQ^*PN/((Q^*-R)T+Lp_tN)\,. \quad (14)$$

3. Numerical Example

In this section, we illustrate our numerical results to investigate the impact of the group-buying inventory control on the seller's total cost. There are two variables taken into consideration: 1) the lead time, and 2) the probability that stock-out will occur during the lead-time. We adjust the data from Chen *et al.* (2012) for this numerical study. We set the number of buyers required at 31 people, the time for each deal at about 100 days. The buyer's arrival rate λ = 0.5 per day. The setup cost is 5,000 per cycle, the holding cost is 0.1 per unit time, the ordering cost is 10 per unit time, the lead time is L = 100, 200, 300, 400 and 500 days and that the probability that stock-out will occur during lead time α = 0.1, 0.2, 0.3, 0.4 and 0.5. The effect of lead time and probabilities of stock out on Q^*, R^* and $TC_{\min}$ are shown in Tables 1 and 2, respectively.

Table 1. The effect of lead time and the probability that stock-out on Q^* and R^*.

α	R^*									
	100		200		300		400		500	
	Q^*	R^*	Q^*	R^*	Q^*	R^*	Q^*	R^*	Q^*	R^*
0.1	176	31	176	62	176	93	176	124	176	155
0.2	176	31	176	62	176	93	176	124	176	155
0.3	176	31	176	62	176	93	176	124	176	155
0.4	176	31	176	62	176	93	176	124	176	155
0.5	176	31	176	62	176	93	176	124	176	155

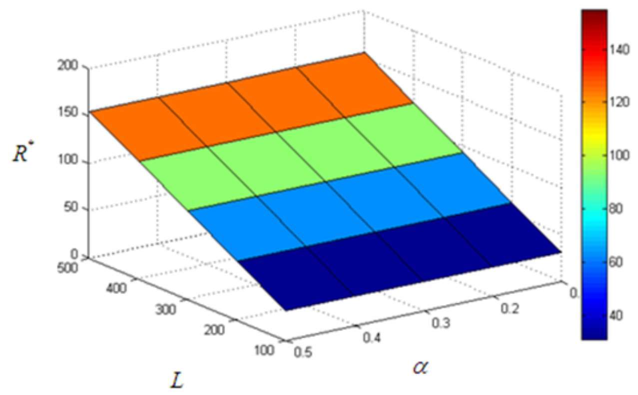

Figure 2. The effect of L and α on R^*

From Table 1, we plot the re-order point with various values of each lead time and the probability that stock-out occurs as shown in Figure 2,

As shown in Table 2 and Figure 2, the re-order point increases with increasing lead time for every probability of the stock-out during the lead time, however the replenishment quantity remains uneffected.

From Table 2, we plot the minimum total cost with the selected values of each lead time and the probability of stock-out as shown in Figure 3,

From Table 2 and Figure 3, the minimum total cost increases with increasing lead time for every probability of the stock-out during the lead time.

Table 2. The effect of lead time and the probability that stock-out on the TC_{min}

α	L				
	100	200	300	400	500
0.1	20.6961	20.7044	20.7127	20.721	20.7293
0.2	20.6961	20.7044	20.7127	20.721	20.7293
0.3	20.6961	20.7044	20.7127	20.721	20.7293
0.4	20.6961	20.7044	20.7127	20.721	20.7293
0.5	20.6961	20.7044	20.7127	20.721	20.7293

4. Conclusions

This paper has investigated the group-buying inventory system with uncertain demand where the customers arrival is of a Poisson process under the holding cost restriction. We can calculate optimal solution of Q^*, R^* and TC_{min} from Eqs. (8), (13) and (14), respectively. The results from the numerical example suggest that the lead time has an effect on the seller's total cost. When the lead time is increasing, the re-order point and the minimum total cost will increase but the replenishment quantity remains constant. This study could be used for the case where demand is a Poisson process and the product sale is binomially distributed. Furthermore, the group-buying seller can organize the optimal order size and the re-order point by using the model presented here. Another possible extension of this study is the inventory system of the group-buying auction model subjected to a price curve. Other variables including product shortage, backorder and varying cost could also be added into the model to improve the sellers strategy in order/reordering the products.

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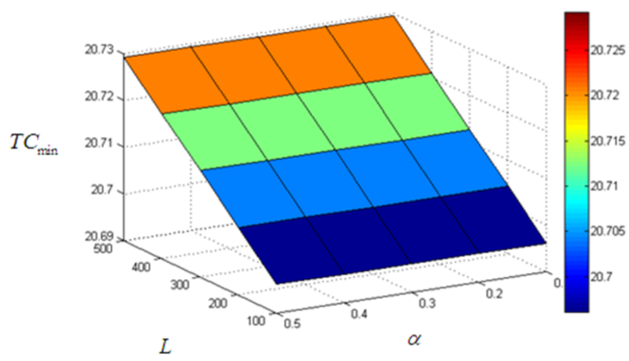

Figure 3. The effect of L and α on TC_{min}

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