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Original Article

The study on χ^3 sequence spaces

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Abstract

Let χ^3 denote the space of all triple gai sequences and Λ^3 the space of all triple analytic sequences. This paper is devoted to the general properties of χ^3.

Keywords: gai sequence, analytic sequence, triple sequence, dual, monotone metric

1. Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex sequences (x_{mnk}), where $m,n,k \in \mathbb{N}$, the set of positive integers.Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double sequence spaces is found in N. Subramanian *et al.* (2008, 2009), Hardy (1917) and many others. Later on investigated by some initial work on triple sequence spaces is found in Esi *et al.* (2015,2014), Sahiner *et al.* (2007) and many others. The initial work on modulus function or Orlicz functions and some other sequence spaces is found in Y. Altin *et al.* (2003,2006,2009).

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq}\,(m,n,k = 1,2,3,...)$$

A sequence $x = (x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0 \text{ as } m,n,k \to \infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^2 is a metric space with the metric

$$d(x,y) = sup_{m,n,k}\left\{|x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m,n,k : 1,2,3,...\right\}, \tag{1}$$

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for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in Γ^3. Let $\phi = \{finite\ sequences\}$.

Consider a double sequence $x = (x_{mnk})$. The $(m,n,k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \Sigma_{i,j,q=0}^{m,n,k} x_{ijq}\delta_{ijq}$ for all $m,n,k \in \mathbb{N}$,

$$\delta_{mnk} = \begin{bmatrix} 0 & 0 & ...0 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & ...1 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \end{bmatrix}$$

with 1 in the $(m,n,k)^{th}$ position and zero other wise.

A sequence $x = (x_{mnk})$ is called triple gai sequence if $\left((m+n+k)!|x_{mnk}|\right)^{\frac{1}{m+n+k}} \to 0$ as $m,n,k \to \infty$. The triple gai sequences will be denoted by χ^3.

Consider a triple sequence $x = (x_{mnk})$. The $(m,n,k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \Sigma_{i,j,q=0}^{m,n,k} x_{ijq}\Im_{ijq}$ for all $m,n,k \in \mathbb{N}$; where $\Im_{ijq}$ denotes the triple sequence whose only non zero term is a $\frac{1}{(i+j+k)!}$ in the $(i,j,k)^{th}$ place for each $i,j,k \in \mathbb{N}$.

An FK-space(or a metric space) X is said to have AK property if $(\Im_{mnk})$ is a Schauder basis for X, Or equivalently $x^{[m,n,k]} \to x$.

An FDK-space is a triple sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings are continuous.

Let X be a BK space. Then X is said to have monotone metric if $d\left(x^m, y^m\right) \geq d\left(x^n, y^n\right)$ for $m > n$ and

$$d(x,y) = sup_{m,n,k}\left\{\left((m+n+k)!|x_{mnk} - y_{mnk}|\right)^{\frac{1}{m+n+k}} : m,n,k : 1,2,3,...\right\}.$$

If X is a sequence space, we give the following definitions:

(i) X' is continuous dual of X;

(ii) $X^{\alpha} = \left\{a = (a_{mnk}) : \Sigma_{m,n,k=1}^{\infty} |a_{mnk}x_{mnk}| < \infty \text{ for each } x \in X\right\}$;

(iii) $X^{\beta} = \left\{a = (a_{mnk}) : \Sigma_{m,n,k=1}^{\infty} a_{mnk}x_{mnk} \text{ is convergent, for each } x \in X\right\}$;

(iv) $X^{\gamma} = \left\{a = (a_{mn}) : sup_{m,n\geq 1}\left|\sum_{m,n,k=1}^{M,N,K} a_{mnk}x_{mnk}\right| < \infty, \text{ for each } x \in X\right\}$;

(v) Let X be an FK-space $\supset \phi$; then $X^f = \{f(\Im_{mnk}) : f \in X'\}$;

(vi) $X^{\delta} = \left\{a = (a_{mnk}) : sup_{m,n,k} |a_{mnk}x_{mnk}|^{1/m+n+k} < \infty, \text{ for each } x \in X\right\}$;

$X^{\alpha}.X^{\beta}, X^{\gamma}$ are called α-(or Köthe -Toeplitz) dual of X, β-(or generalized- Köthe -Toeplitz) dual of X, γ- dual of X, δ - dual of X respectively. X^{α} is defined by Gupta and Kamptan . It is clear that $X^{\alpha} \subset X^{\beta}$ and $X^{\alpha} \subset X^{\gamma}$, but $X^{\alpha} \subset X^{\gamma}$ does not hold.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [1981] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c, c_0$ and ℓ_{∞}, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}.$. Here w, c, c_0 and ℓ_{∞} denote the classes of all, convergent, null and bounded scalar valued single sequences respectively. The above spaces are Banach spaces normed by

$$x = |x_1| + sup_{k \geq 1} |\Delta x_k|$$

Later on the notion was further investigated by many others. We now introduce the following difference triple sequence spaces defined by

$$Z(\Delta) = \{x = (x_{mnk}) \in w^3 : (\Delta x_{mnk}) \in Z\}$$

where $Z = \Lambda^3, \chi^3$ and $\Delta x_{mn} = (x_{mn} - x_{mn+1} - x_{mn+2}) - (x_{m+1n} - x_{m+1n+1} - x_{m+1,n+2}) - (x_{m+2n} - x_{m+2n+1} - x_{m+2,n+2})$ for all $m, n, k \in \mathbb{N}.$

2. Definitions and Preliminaries

A sequence $x = (x_{mnk})$ is said to be triple analytic if $sup_{mnk} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$ The vector space of all triple analytic sequences is usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}} \rightarrow 0$ as $m, n, k \rightarrow \infty.$ The vector space of triple entire sequences is usually denoted by Γ^3. A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m+n+k)!|x_{mnk}|)^{1/m+n+k} \rightarrow 0$ as $m, n, k \rightarrow \infty.$ The vector space of triple gai sequences is usually denoted by χ^3. The space χ^3 is a metric space with the metric

$$d(x, y) = sup_{m,n,k} \left\{ ((m+n+k)!|x_{mnk} - y_{mnk}|)^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, ... \right\} \quad (2)$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in χ^3.

3. Main Results

3.1 Proposition

χ^3 has monotone metric.

Proof: We know that

$$d(x, y) = sup_{m,n,k} \left\{ ((m+n+k)!|x_{mnk} - y_{mnk}|)^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, ... \right\}$$

$$d(x^n, y^n) = sup_n \left\{ ((3n)!|x_{nnn} - y_{nnn}|)^{\frac{1}{3n}} \right\}$$

$$d(x^m, y^m) = sup_m \left\{ ((3m)!|x_{mmm} - y_{mmm}|)^{\frac{1}{3m}} \right\}$$

Let $m > n$. Then $sup_m \left\{ ((3m)!|x_{mmm} - y_{mmm}|)^{\frac{1}{3m}} \right\} \geq sup_n \left\{ ((3n)!|x_{nnn} - y_{nnn}|)^{\frac{1}{3n}} \right\}$

$$d(x^m, y^m) \geq d(x^n, y^n), m > n \tag{3}$$

Also $\left\{d(x^n, y^n) : n = 1,2,3,...\right\}$ is monotonically increasing bounded by $d(x,y)$.

For such a sequence

$$sup_n \left\{\left((3n!)\left|x^{nnn} - y^{nnn}\right|\right)^{\frac{1}{3n}}\right\} = n \overset{lim}{\rightarrow} \infty\, d(x^n, y^n) = d(x,y) \tag{4}$$

From (3) and (4) it follows that $d(x,y) = sup_{m,n,k}\left\{\left((m+n)!\left|x_{mnk} - y_{mnk}\right|\right)^{\frac{1}{m+n+k}}\right\}$ is a monotone metric for χ^3. This completes the proof.

3.2 Proposition

The dual space of χ^3 is Λ^3. In other words $(\chi^3)^* = \Lambda^3$.

Proof: We recall that

$$\Im_{mnk} = \begin{bmatrix} 0, & 0, & ...0, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \\ . \\ . \\ . \\ 0, & 0, & ...\frac{1}{(m+n+k)!}, & 0, & ... \\ 0, & 0, & ...0, & 0, & ... \end{bmatrix}$$

with $\frac{1}{(m+n+k)!}$ in the $(m,n,k)th$ position and zero's else where. With

$$x = \Im_{mnk}, \left(\left|x_{mnk}\right|\right)^{\frac{1}{m+n+k}} = \begin{bmatrix} 0^{\frac{1}{3}},. & . & . & 0^{\frac{1}{m+n+k}} \\ . \\ . \\ . \\ 0^{\frac{1}{m+n+k}}, & \left(\frac{1}{(m+n+k)!}\right)^{\frac{1}{m+n+k}}, & . & 0^{\frac{1}{m+n+k}} \\ & (m,n)^{th} \\ 0^{\frac{1}{m+2+k}}, & . & . & 0^{\frac{1}{m+n+2+k}} \end{bmatrix} = \begin{bmatrix} 0,. & . & . & 0 \\ . \\ . \\ . \\ 0, & \left(\frac{1}{(m+n+k)!}\right)^{\frac{1}{m+n+k}}, & . & 0 \\ & (m,n,k)^{th} \\ 0, & . & . & 0 \end{bmatrix}$$

which is a triple gai sequence. Hence $\Im_{mnk} \in \chi^3$. We have $f(x) = \sum_{m,n,k=1}^{\infty} x_{mnk} y_{mnk}$. With $x \in \chi^3$ and $f \in (\chi^3)^*$ the dual space of χ^3. Take $x = (x_{mnk}) = \Im_{mnk} \in \chi^3$. Then

$$|y_{mnk}| \leq fd(\Im_{mnk}, 0) < \infty \quad \forall m, n, k \tag{5}$$

Thus (y_{mnk}) is a bounded sequence and hence an triple analytic sequence. In other words $y \in \Lambda^3$. Therefore $(\chi^3)^* = \Lambda^3$. This completes the proof.

3.3 Proposition

χ^3 is separable.

Proof: It is routine verification. Therefore omit the proof.

3.4 Proposition:

Λ^3 is not separable.

Proof: Since $|x_{mnk}|^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$, so it may so happen that first row or column may not be convergent, even may not be bounded. Let S be the set that has triple sequences such that the first row is built up of sequences of zeros and ones. Then S will be uncountable. Consider open balls of radius 3^{-1} units. Then these open balls will not cover Λ^3. Hence Λ^3 is not separable. This completes the proof.

3.5 Proposition

χ^3 is not reflexive.

Proof: χ^3 is separable by Proposition 3.3. But $(\chi^3)^* = \Lambda^3$, by Proposition 3.2. Since Λ^3 is not separable, by Proposition 3.4. Therefore χ^3 is not reflexive. This completes the proof.

3.6 Proposition

χ^3 is not an inner product space as such not a Hilbert space.

Proof: Let us take

$$x = x_{mn} = \begin{bmatrix} 1/3!, & 1/5!, & 0, & 0, & \dots \\ 0, & 0, & 0, & 0, & \dots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix} \quad \text{and} \quad y = y_{mn} = \begin{bmatrix} 1/3!, & -1/5!, & 0, & 0, & \dots \\ 0, & 0, & 0, & 0, & \dots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}$$

$$d(x,0)=sup\begin{bmatrix}\left(3!\left|x_{111}-0\right|\right)^{1/3}, & \left(5!\left|x_{122}-0\right|\right)^{1/5}, & \ldots\\ \left(4!\left|x_{211}-0\right|\right)^{1/4}, & \left(6!\left|x_{222}-0\right|\right)^{1/6}, & \ldots\\ . & & \\ . & & \\ . & & \end{bmatrix}$$

$$=\sup\begin{bmatrix}\left(3!\left|1/3!-0\right|\right)^{1/3}, & \left(5!\left|1/5!-0\right|\right)^{1/5}, & \ldots\\ 0, & 0, & \ldots\\ . & & \\ . & & \\ . & & \end{bmatrix}\sup\begin{bmatrix}(1)^{1/3}, & (1)^{1/5}, & 0, & \ldots\\ 0, & 0, & 0, & \ldots\\ . & & & \\ . & & & \\ . & & & \end{bmatrix}$$

$d(x,0)=1.$

Similarly $d(x,0)=1$. Hence $d(x,0)=d(y,0)=1$

$$x+y=\begin{bmatrix}1/3!, & 1/5!, & 0 & ,0 & \ldots\\ 0, & 0, & 0, & 0, & \ldots\\ . & & & & \\ . & & & & \\ . & & & & \\ 0, & 0, & 0, & 0, & \ldots\end{bmatrix}+\begin{bmatrix}1/3!, & -1/5!, & 0, & 0 & \ldots\\ 0, & 0, & 0, & 0, & \ldots\\ . & & & & \\ . & & & & \\ . & & & & \\ 0, & 0, & 0, & 0, & \ldots\end{bmatrix}=\begin{bmatrix}1/3, & 0, & 0, & 0 & \ldots\\ 0, & 0, & 0, & 0, & \ldots\\ . & & & & \\ . & & & & \\ . & & & & \\ 0, & 0, & 0, & 0, & \ldots\end{bmatrix}$$

$d(x+y,x+y)=$

$$sup\left\{\left((m+n+k)!\left(\left|x_{mnk}+y_{mnk}\right|-\left|x_{mnk}-y_{mnk}\right|\right)\right)^{\frac{1}{m+n+k}}:m,n,k=1,2,3,\ldots\right\}$$

$$d(x_{mnk}+y_{mnk},0)=sup\begin{bmatrix}\left(3!\left|x_{111}+y_{111}\right|\right)^{1/3}, & \left(5!\left|x_{122}+y_{122}\right|\right)^{1/5}, & \ldots\\ . & & \\ . & & \\ . & & \end{bmatrix}$$

$$\sup\begin{bmatrix} (3!|1/3!+1/3!|)^{1/3}, & (5!|1/5!-1/5!|)^{1/5}, & \ldots \\ . & & \\ . & & \\ . & & \end{bmatrix} = \sup\begin{bmatrix} (2)^{1/3}, & 0, & \ldots \\ 0, & 0, & \ldots \\ . & & \\ . & & \\ . & & \end{bmatrix} = \sup\begin{bmatrix} 1.2599, & 0, & \ldots \\ 0, & 0, & \ldots \\ . & & \\ . & & \\ . & & \end{bmatrix} = 1.2559$$

Therefore $d(x+y,0) = 1.2599$. Similarly $d(x-y,0) = 1.1487$

By parellogram law,

$$[d(x+y,0)]^2 + [d(x-y,0)]^2 = 2\left[(d(x,0))^2 + (d(0,y))^2\right]$$

$$\Rightarrow (1.2599)^2 + 1.1487^2 = 2\left[1^2 + 1^2\right] \Rightarrow 2.906 = 4.$$

Hence it is not satisfied by the law. Therefore χ^3 is not an inner product space. Assume that χ^3 is a Hilbert space. But then χ^3 would satisfy reflexivity condition. [Theorem 4.6.6 [Wilansky]] . Proposition 3.5, χ^3 is not reflexive. Thus χ^3 is not a Hilbert space. This completes the proof.

3.7 Proposition

χ^3 is rotund.

Proof: Let us take

$$x = x_{mnk} = \begin{bmatrix} 1/3!, & 0, & 0, & 0 & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix} \text{ and } y = y_{mnk} = \begin{bmatrix} 1/3!, & 0, & 0, & 0, & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . & & & & \\ . & & & & \\ . & & & & \end{bmatrix}$$

The $x = (x_{mnk})$ and $y = (y_{mnk})$ are in χ^3.

$$\text{Also } d(x,y) = \sup\begin{bmatrix} (3!|x_{111} - y_{111}|)^{1/3}, & \ldots((1+n+k)!|x_{1nk} - y_{1nk}|)^{\frac{1}{1+n+k}}, & 0, & \ldots. \\ . & & & \\ . & & & \\ ((m+1+k)!|x_{m1k} - y_{m1k}|)^{\frac{1}{m+n+k}}, & \ldots((m+n+k)!|x_{mnk} - y_{mnk}|)^{\frac{1}{m+n+k}}, & 0, & \ldots \\ 0, & \ldots & 0, & \ldots \end{bmatrix}$$

$$\text{Therefore } d(x,0) = sup\begin{bmatrix} 1, & 0, & 0, & 0 & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . \\ . \\ . \\ 0, & 0, & 0, & 0, & \ldots \end{bmatrix}$$

$d(0, y) = 1.$ Obviously $x = (x_{mnk}) \neq y = (y_{mnk})$

$$\text{but } (x_{mnk}) + (y_{mnk}) = \begin{bmatrix} 1/3!, & 0, & 0 & ,0 & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . \\ . \\ . \end{bmatrix} + \begin{bmatrix} 1/3!, & 0, & 0, & 0 & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} 1/3, & 0, & 0, & 0 & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . \\ . \\ . \end{bmatrix}$$

$$d(\frac{x_{mnk} + y_{mnk}}{2}, 0) = \sup\begin{bmatrix} \frac{\left(3!\left|x_{111} + y_{111}\right|\right)^{1/3}}{2}, \ldots, & \frac{\left((1+n+k)!\left|x_{1nk} + y_{1nk}\right|\right)^{\frac{1}{n+1+k}}}{2}, & 0\ldots \\ . \\ . \\ \frac{\left((m+1+k)!\left|x_{m1k} + y_{m1k}\right|\right)^{\frac{1}{m+1+k}}}{2}, \ldots, & \frac{\left((m+n)!\left|x_{mn} + y_{mn}\right|\right)^{\frac{1}{m+n+k}}}{2}, & 0, \ldots \end{bmatrix}$$

$$d(\frac{x_{mnk} + y_{mnk}}{2}, 0) = sup\begin{bmatrix} (2^{1/3})/2, & 0, & 0, & 0 & \ldots \\ 0, & 0, & 0, & 0, & \ldots \\ . \\ . \\ . \end{bmatrix} = 0.63.$$

Therefore χ^3 is rotund. This completes the proof.

3.8 Proposition

Weak convergence and strong convergence are equivalent in χ^3.

Proof:

Step 1: Always strong convergence implies weak convergence.

Step 2: So it is enough to show that weakly convergence implies strongly convergence in $\chi^3 . y^{(\eta)}$ tends to weakly in χ^3, where $(y^{(\eta)}_{mnk}) = y^{(\eta)}$ and $y = (y_{mnk})$. Take any $x = (x_{mnk}) \in \chi^3$ and

$$f(z) = \sum_{m,n,k=1}^{\infty} \left((m+n+k)! \left|z_{mnk} x_{mnk}\right|\right)^{\frac{1}{m+n+k}} \text{ for each } z = (z_{mnk}) \in \chi^3 \quad (6)$$

Then $f \in (\chi^3)^*$ by Proposition 3.2. By hypothesis $f(y^{\eta}) \to f(y)$ as $\eta \to \infty$.

$$f\left(y^{(\eta)} - y\right) \to 0 \quad \text{as} \quad \eta \to \infty \quad (7)$$

$\Rightarrow \sum_{m,n,k=1}^{\infty} \left(\left|y^{(\eta)}_{mnk} - y_{mnk}\right|^{\frac{1}{m+n+k}} \left((m+n+k)!\right)^{\frac{1}{m+n+k}} \left|x_{mnk}\right|^{1/m+n+k} \right) \to 0$ as $\eta \to \infty$. By using (6) and (7) we get since $x = (x_{mnk}) \in \Lambda^3$ we have $\sum_{m,n,k=1}^{\infty} \left|x_{mnk}\right|^{\frac{1}{m+n+k}} < \infty$ for all $x \in \Lambda^3$.

$$\Rightarrow \sum_{m,n,k=1}^{\infty} \left(m+n+k)! \left|y^{(\eta)}_{mnk} - y_{mnk}\right|\right)^{\frac{1}{m+n+k}} \to 0 \text{ as } \eta \to \infty .$$

$$\Rightarrow \sup_{mnk} \left((m+n+k)! \left|(y^{(\eta)}_{mnk} - y_{mnk}), 0\right|\right)^{\frac{1}{m+n+k}} \to 0 \text{ as } \eta \to \infty .$$

$$\Rightarrow \sup_{mnk} \left((m+n+k)! \left|y^{(\eta)}_{mnk} - y_{mnk}\right|\right)^{\frac{1}{m+n+k}} \to 0 \text{ as } \eta \to \infty .$$

$$\Rightarrow d\left(\left(y^{(\eta)} - y\right), 0\right) \to 0 \text{ as } \eta \to \infty .$$

$\Rightarrow d\left(y^{(\eta)} - y\right) \to 0$ as $\eta \to \infty$. This completes the proof.

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