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*Review Article*

# Central composite design within strip-strip-plot structure for three-stage industrial processes

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**Abstract**

In manufacturing processes, some process factors are involved with two or three previous processes conditions. Current research has shown that strip-plot structure is an interestingly proposed for the experimental design in multi-stage processes. Thus, this research is addressed to utilize the strip-plot structure in three multi-stage processes. The experimental design involves first order and second-order, Central Composite Design (CCD), within strip-strip-plot structure. D-optimal criteria are calculated to estimate the efficiency of the design. The expected response model is composed of main, quadratic, within-stage interaction and cross-stage interaction effects. Two food manufacturing processes are chosen to prove the multi-stage design of experiments. Their multiple regression models are calculated and optimized by standard software. The practical confirmation results are in a favor of the approach.

**Keywords:** Central Composite Design, strip-strip-plot structure, multi-stage processes, cross-stage interaction effects, D-optimal criteria

## 1. Introduction

Nowadays, design of experiment (DOE) is widely utilized in all industries to improve quality in process. However, there are some processes that their qualities depend on not only parameters of final-stage process but also parameters of a few previous-stage processes. Recent researches for DOE in multi-stage processes are much more interested than in the past ten years. The industrial applications of split-plot and strip-plot designs have been performed for multi-stage processes in which their main purposes to reduce the expense of experiments. In order to investigate the cross-stage effects, such as interaction effects between stage-1 and stage-2, stage-1 and stage-3, etc., strip-plot design is more appropriated than split-plot design. Vivacqua (2003) performed strip-plot experimental design on two-stage processes in battery factory. A strip-strip-plot design was proposed in investigating the effects of factors and their interactions in three-stage processes of a wafer factory by Paniagua-Quinones and Box (2008). Full factorial designs had been applied for all three processes. In the year 2009, both of them proposed half fractional designs for the final third-stage process in order to reduce experimental cost. From that time, many researchers had published literatures involving strip-plot design for multi-stage processes. Recently, Arnouts *et al.* (2010) was proposed D-optimal strip-plot structure with full and half fractional factorial design to reduce the large numbers of row crossing with column of strip-plot structure. In the year 2013, they had performed D-optimal strip-plot experiments with 2-level factors and a 4-level categorical factor for three-multi-stage processes in which some cells, stage-3, are missing experimental data.

The research is aimed to create experimental designs that can provide quadratic terms and multi-stage interaction effects for three-multi-stage processes. Standard designs, such as factorial design and Central Composite Design (CCD),

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are utilized within strip-strip-plot structure. The predicted response model at final-stage should compose of all main, quadratic, within-stage interaction and cross-stage interaction effects.

## 2. Materials and Methods

### 2.1 Strip-Strip-plot structures for three-stage processes

In a strip-strip-plot structure, the two-ways classification random model with balanced data is utilized. Searle *et al.* (2006) introduced in matrix notation as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon} \tag{1}$$

with $$\begin{bmatrix}\mathbf{u}_1' & \mathbf{u}_2' & \cdots & \mathbf{u}_m'\end{bmatrix}' ; \begin{bmatrix}\mathbf{Z}_1' & \mathbf{Z}_2' & \cdots & \mathbf{Z}_m'\end{bmatrix} \tag{2}$$

where,

$\mathbf{Y}$ is an $N\times 1$ vector of experimental data,

$\boldsymbol{\beta}$ is an $p\times 1$ vector of fixed effect parameters,

$\mathbf{X}$ is a $N\times p$ coefficient matrix,

$\mathbf{u}$ are random effect vectors that occur in the data,

$\mathbf{Z}$ are corresponding incidence matrices of u, and

$\boldsymbol{\varepsilon}$ is an error of $N\times 1$ vector.

For three-multi-stage processes, m is equal to 2 in that stage-1 and stage-2 random effect vectors are $\mathbf{u}_1 = \boldsymbol{\delta}$ and $\mathbf{u}_2 = \boldsymbol{\gamma}$ with their corresponding matrices $\mathbf{Z}_\delta$ and $\mathbf{Z}_\gamma$, respectively. Stage-3 random effects will occur within unexplained errors $\boldsymbol{\varepsilon}$. The three-multi-stage model will be

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}_\delta\boldsymbol{\delta} + \mathbf{Z}_\gamma\boldsymbol{\gamma} + \boldsymbol{\varepsilon} \tag{3}$$

where rows (r), columns (c) and cells (k) are represented for stage-1, stage-2 and stage-3 treatments, respectively. The total sample size (N) is equal to rck runs. The crossing structure among stages of experimental treatments (**T**) is shown in Table 1.

It is assumed that $\boldsymbol{\delta} \sim N\left(\mathbf{0}_r, \sigma_\delta^2\mathbf{I}_r\right)$, $\boldsymbol{\gamma} \sim N\left(\mathbf{0}_c, \sigma_\gamma^2\mathbf{I}_c\right)$ and $\boldsymbol{\varepsilon} \sim N\left(\mathbf{0}_N, \sigma_\varepsilon^2\mathbf{I}_N\right)$ are all random effects where $\mathbf{0}$ and $\mathbf{I}$ are zero vector and identity matrix and normally distributed around their zero means. Their variances $\sigma_\delta^2$, $\sigma_\gamma^2$ and $\sigma_\varepsilon^2$ are referred as of stage-1, stage-2, stage-3, respectively.

Their covariance, $\mathrm{cov}(\boldsymbol{\delta},\boldsymbol{\gamma}) = \mathbf{0}_{r\times c}$, $\mathrm{cov}(\boldsymbol{\delta},\boldsymbol{\varepsilon}) = \mathbf{0}_{r\times N}$ and $\mathrm{cov}(\boldsymbol{\gamma},\boldsymbol{\varepsilon}) = \mathbf{0}_{c\times N}$ are zero because each of them is independent to others. Thus the model vector Y contains variance-covariance matrix $\mathbf{V} = \mathrm{var}(\mathbf{Y})$ as

$$\mathbf{V} = \sigma_\delta^2\mathbf{Z}_\delta\mathbf{Z}_\delta' + \sigma_\gamma^2\mathbf{Z}_\gamma\mathbf{Z}_\gamma' + \sigma_\varepsilon^2\mathbf{Z}_N \tag{4}$$

$$\mathbf{I}_N = \mathbf{I}_r \otimes \mathbf{I}_c \otimes \mathbf{I}_k ; \ \mathbf{Z}_\delta = \mathbf{I}_r \otimes \mathbf{1}_c \otimes \mathbf{1}_k \text{ and } \mathbf{Z}_\gamma = \mathbf{1}_r \otimes \mathbf{I}_c \otimes \mathbf{1}_k \tag{5}$$

where **1** and **I** are as

$$\mathbf{1}_r = \begin{bmatrix}1\\1\\\vdots\\1_r\end{bmatrix} ; \ \mathbf{1}_c = \begin{bmatrix}1\\1\\\vdots\\1_c\end{bmatrix} ; \ \mathbf{1}_k = \begin{bmatrix}1\\1\\\vdots\\1_k\end{bmatrix} \text{ and} \tag{6}$$

$$\mathbf{I}_r = \begin{bmatrix}1 & 0 & \cdots & 0\\0 & 1 & \cdots & 0\\\vdots & \vdots & \ddots & \vdots\\0 & 0 & \cdots & 1_r\end{bmatrix} ; \mathbf{I}_c = \begin{bmatrix}1 & 0 & \cdots & 0\\0 & 1 & \cdots & 0\\\vdots & \vdots & \ddots & \vdots\\0 & 0 & \cdots & 1_c\end{bmatrix} ; \ \mathbf{I}_k = \begin{bmatrix}1 & 0 & \cdots & 0\\0 & 1 & \cdots & 0\\\vdots & \vdots & \ddots & \vdots\\0 & 0 & \cdots & 1_k\end{bmatrix} \tag{7}$$

with the introducing of variance ratio, $\eta_\delta = \dfrac{\sigma_\delta^2}{\sigma_\varepsilon^2}$ and $\eta_\gamma = \dfrac{\sigma_\gamma^2}{\sigma_\varepsilon^2}$

$$\mathbf{V} = \sigma_\varepsilon^2\left(\eta_\delta\mathbf{Z}_\delta\mathbf{Z}_\delta' + \eta_\gamma\mathbf{Z}_\gamma\mathbf{Z}_\gamma' + \mathbf{I}_N\right) \tag{8}$$

According to Arnouts *et al.* (2013), under the assumption of normality, the maximum likelihood estimator of $\boldsymbol{\beta}$ is generalized least-squares estimator as

$$\hat{\boldsymbol{\beta}}_{GLS} = \left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y} \tag{9}$$

with its variance-covariance matrix

$$\mathrm{cov}\left(\hat{\boldsymbol{\beta}}\right) = \left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1} \tag{10}$$

The optimal strip-plot design utilizes the information matrix $\left(\mathbf{M} = \mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)$ to form D-optimality criteria. The more value of the determinant of information matrix, $|\mathbf{M}|$, will provide the better experimental design. This research use Borkowski (2003)'s formula as

$$\mathbf{D}_{\mathrm{eff}} = 100\left(\frac{\left|\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right|^{1/p}}{N}\right) \tag{11}$$

where p is the number of parameters in **X** matrix and N is the total runs.

Table 1. Crossing structure of three multi-stage processes.

| | | Stage-2 | | | |
|---|---|---|---|---|---|
| | | $T_1$ | $T_2$ | ... | $T_c$ |
| | | Stage-3 | Stage-3 | Stage-3 | Stage-3 |
| Stage-1 | $T_1$ | $T_{111}$ ⋮ $T_{11k}$ | $T_{121}$ ⋮ $T_{12k}$ | ⋮ | $T_{1c1}$ ⋮ $T_{1ck}$ |
| | $T_2$ | $T_{211}$ ⋮ $T_{21k}$ | $T_{221}$ ⋮ $T_{22k}$ | ⋮ | $T_{2c1}$ ⋮ $T_{2ck}$ |
| | ⋮ | ⋮ | ⋮ | ⋱ | ⋮ |
| | $T_r$ | $T_{r11}$ ⋮ $T_{r1k}$ | $T_{r21}$ ⋮ $T_{r2k}$ | ⋮ | $T_{rc1}$ ⋮ $T_{rck}$ |

### 2.2 Central Composite Design within strip-strip-plot structure

Supposed some interested processes obtain second-order properties, thus the model for the strip-strip-plot for three-stage processes is as

$$f\left(X_i^R, X_j^C, X_{ijk}^{Cell}\right) = X\beta \tag{12}$$

For example, letting each stage contains two vital few factors. The expansion of second-order model of six factors will be

$$
\begin{aligned}
X\beta &= \beta_0 \\
&+\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 \quad \ldots \text{ stage-1 effects} \\
&+\beta_3 x_3 + \beta_4 x_4 + \beta_{34} x_3 x_4 + \beta_{33} x_3^2 + \beta_{44} x_4^2 \quad \ldots \text{ stage-2 effects} \\
&+\beta_5 x_5 + \beta_6 x_6 + \beta_{56} x_5 x_6 + \beta_{55} x_5^2 + \beta_{66} x_6^2 \quad \ldots \text{ stage-3 effects} \\
&+\beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 \\
&\qquad \ldots \text{ stage-1} \times \text{stage-2 effects} \\
&+\beta_{15} x_1 x_5 + \beta_{16} x_1 x_6 + \beta_{25} x_2 x_5 + \beta_{26} x_2 x_6 \\
&\qquad \ldots \text{ stage-1} \times \text{stage-3 effects} \\
&+\beta_{35} x_3 x_5 + \beta_{36} x_3 x_6 + \beta_{45} x_4 x_5 + \beta_{46} x_4 x_6 \\
&\qquad \ldots \text{ stage-2} \times \text{stage-3 effects}
\end{aligned}
$$

### 2.2 Constructing experimental design

We utilize the role of equivalence of ordinary least square and generalized least square (OLS-GLS) to construct designs which estimated parameters are not involved with variance-covariance matrix **V**.

$$\hat{\beta}_{GLS} = \left(X'V^{-1}X\right)^{-1} X'V^{-1}Y = \hat{\beta}_{OLS} = \left(X'X\right)^{-1} X'Y \tag{13}$$

The equivalence of OLS-GLS brings the benefit that the estimated coefficient $\hat{\beta}$ can be calculated from standard software.

In order to construct the OLS-GLS equivalence experimental design, Vining *et al.* (2005) and Parker *et al.* (2007) has provided necessary conditions as balance and/or orthogonal properties that each design should contain. The guideline will be

1. Balance design: all main effects, within-stage interaction effects and cross-stage interaction effects should have balance property, $\sum_{i=1}^{r,c} \pm 1 = 0$.
2. Balance design: within row or stage-1 and within column or stage-2 should contain balance property, $\sum_{i=1}^{r,c} \pm 1 = 0$.
3. Each cell will contain equal number of k runs.
4. Each cell should conform balance property, $\sum_{i=1}^{k} \pm 1 = 0$.
5. Cell allocation method is randomization.

## 3. Experiment, Results, and Discussion

### 3.1 Factory experiment

To illustrate the CCD design construction within strip-strip-plot structure, two industrial processes are chosen to confirm its design efficiencies. There are macaroni ready meal and pork ham processing processes at a food company.

### 3.3.1 Macaroni ready meal experiment

**1. Constructing experimental design**

Considering historical data and our team's experience, three multi-stages are selected to perform the experiment. Based on macaroni process diagram in Figure 1, all raw materials are prepared in parallel. The stage-1 critical process is the mixing process between macaroni and tomato sauce with its critical factor mixing time, defined as factor A. Then the mixed staff is on waiting for loading into ready meal boxes. The waiting process is the stage-2 process with its critical factor waiting time, defined as factor B. The final stage-3 process is the loading machine which it's setting pressure pushes the setting volume of mixed staff into ready meal boxes. Its critical factors are blow time, volume amount and spray time, which are defined as factors C, D and E respectively. Finally the mixing staff in ready meal boxes are weighed and defined as experimental response, macaroni weight.

All key cooking parameters are sensitive and involved to others that causes CCD is chosen for all three stages. The total numbers of key cooking factors are five. Stage-1 factor A and stage-2 factor B contain 3 levels, -1, 0, 1. Stage-3 factor C and D, CCD is provided with $\alpha = \pm 1$ whereas factor E contain only 2 levels, -1, 1. Utilizing balance property to all main effects within their columns, the OLS-GLS CCD within strip-strip-plot design is shown in Table 2. The strip-strip-plot design is started with all treatments in stage-1 are stripped

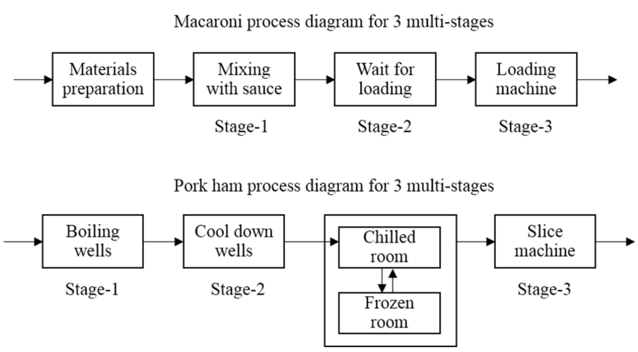


Figure 1. Three multi-stages process diagram for macaroni and pork ham.

Table 2. Three-stage processes DOE for macaroni process.

| | | | Stage-2 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | B | | | | | | | | |
| | | | -1 | | | 0 | | | 1 | | |
| Stage-3 | | | C | D | E | C | D | E | C | D | E |
| Stage-1 | A | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 |
| | | | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 |
| | | | -1 | 0 | 1 | 1 | -1 | 1 | -1 | 0 | -1 |
| | | | 0 | -1 | -1 | 1 | 1 | -1 | 0 | 1 | 1 |
| | | | 1 | 0 | 1 | 0 | -1 | 1 | 1 | 0 | -1 |
| | | | 0 | 0 | 1 | 0 | 1 | -1 | 0 | 0 | -1 |
| | | 0 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 |
| | | | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 |
| | | | -1 | 0 | -1 | -1 | 0 | 1 | 1 | -1 | 1 |
| | | | 0 | 1 | 1 | 0 | -1 | -1 | 1 | 1 | -1 |
| | | | 1 | 0 | -1 | 1 | 0 | 1 | 0 | -1 | 1 |
| | | | 0 | 0 | -1 | 0 | 0 | 1 | 0 | 1 | -1 |
| | | 1 | -1 | 0 | -1 | -1 | -1 | 1 | -1 | -1 | -1 |
| | | | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 0 | 1 |
| | | | 0 | 0 | -1 | 0 | -1 | 1 | 0 | -1 | -1 |
| | | | 0 | 1 | 1 | 0 | 1 | -1 | 0 | 0 | 1 |
| | | | 1 | 0 | -1 | 1 | -1 | 1 | 1 | -1 | -1 |
| | | | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 0 | 1 |

with all treatments in stage-2. The total cell numbers is equal to 3×3 = 9 cells. Stage-3 treatments will be randomly placed within all 9 cells for balanced data case. Each cell contains 6 treatments that causes the total run (rck) is 54 runs.

**2. Design efficiency analysis**

To calculate the D-optimal efficiency in Equation 11, the matrix **V** of Equation 8 is calculated starting from the matrices $\mathbf{Z}_\delta$, $\mathbf{Z}_\gamma$ and **X** which are in the form of

$$\mathbf{X} = \begin{array}{c} \text{Run} \\ 1 \\ \vdots \\ 54 \end{array} \begin{bmatrix} \mu & \mathbf{S1}\cdots\mathbf{S3} & \mathbf{S1}_{\text{int}}\cdots\mathbf{S3}_{\text{int}} & \mathbf{S1}_{\text{Q}}\cdots\mathbf{S3}_{\text{Q}} \\ 1 & \pm1,0 & \pm1,0 & 1,0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \pm1,0 & \pm1,0 & 1,0 \end{bmatrix};$$

$$\mathbf{Z}_\delta = \begin{bmatrix} 1_{r=1,c=1,k=1\sim6} & 0 & 0 \\ 1_{r=1,c=2,k=1\sim6} & 0 & 0 \\ 1_{r=1,c=3,k=1\sim6} & 0 & 0 \\ 0 & 1_{r=2,c=1,k=1\sim6} & 0 \\ 0 & 1_{r=2,c=2,k=1\sim6} & 0 \\ 0 & 1_{r=2,c=3,k=1\sim6} & 0 \\ 0 & 0 & 1_{r=3,c=1,k=1\sim6} \\ 0 & 0 & 1_{r=3,c=2,k=1\sim6} \\ 0 & 0 & 1_{r=3,c=3,k=1\sim6} \end{bmatrix} \text{ and}$$

$$\mathbf{Z}_\gamma = \begin{bmatrix} 1_{r=1,c=1,k=1\sim6} & 0 & 0 \\ 0 & 1_{r=1,c=2,k=1\sim6} & 0 \\ 0 & 0 & 1_{r=1,c=3,k=1\sim6} \\ 1_{r=2,c=1,k=1\sim6} & 0 & 0 \\ 0 & 1_{r=2,c=2,k=1\sim6} & 0 \\ 0 & 0 & 1_{r=2,c=3,k=1\sim6} \\ 1_{r=3,c=1,k=1\sim6} & 0 & 0 \\ 0 & 1_{r=3,c=2,k=1\sim6} & 0 \\ 0 & 0 & 1_{r=3,c=3,k=1\sim6} \end{bmatrix}$$

where,

$\mathbf{S1}\cdots\mathbf{S3}$: represents main effects A, B and C, D, E belonged to stage-1, stage-2 and stage-3, respectively,

$\mathbf{S1}_{\text{int}}\cdots\mathbf{S3}_{\text{int}}$: represents within-stage and cross-stage interaction effects. CD, DE, CE, AB, AC, BC, AD, BD, AE and BE, and

$\mathbf{S1}_{\text{Q}}\cdots\mathbf{S3}_{\text{Q}}$: represent quadratic effects $A^2$, $B^2$ and $C^2$, $D^2$ belonged to stage-1, stage-2 and stage-3, respectively.

By historical process data, the value of $\sigma_\varepsilon^2$, $\eta_\delta$ and $\eta_\gamma$ are selected as 0.3, 0.5 and 0.5 respectively. Together with the number of p is 20 and N is 54, the design efficiency $(\mathbf{D}_{\text{eff}})$ of macaroni process is 82.08% according to Equation 11.

## 3. Multiple regression analysis

The macaroni process is able to provide us a lot of experiment data with 15 replications of macaroni weight. The distribution of all data is presented in histogram diagram and normality test as shown in Figure 2. The experiment design pattern is conformed to OLS-GLS equivalence which its model estimation $(\hat{\beta})$ is straightforward calculated with any statistical software package because of the independency of variance-covariance, V. The multiple regression coefficients $(\hat{\beta})$ of both GLS and OLS are manually confirmed to be equal. The estimation analysis from a software package is shown in Table 3 and its ANOVA table as well.

From Table 3, essential information of main effects, A, B, C, D, E, quadratic effects, AA, CC, within-stage interaction effects, CD, and cross-stage interaction effects, AC, are able to be extracted out. The interaction effects, CD and AC, are graphically presented in Figure 3. The cross-stage interaction between stage-1 mix temp and stage-3 volume can provide information that the volume setting at loading machine should be set up in accordance with its mixing temperature. These significant terms are able to provide more accuracy into the final response model. The optimized solution will be more accurate because the previous stages' parameters can be suitably adjusted.

### 3.3.2 Ham processing meal experiment

Another example is the experiment in pork ham processing process. Based on pork ham process diagram in Figure 1, raw pork pieces are filled into ham rectangular metal blocks. The stage-1 critical process is the boiling processes, which all filled metal blocks are put into boiling wells with

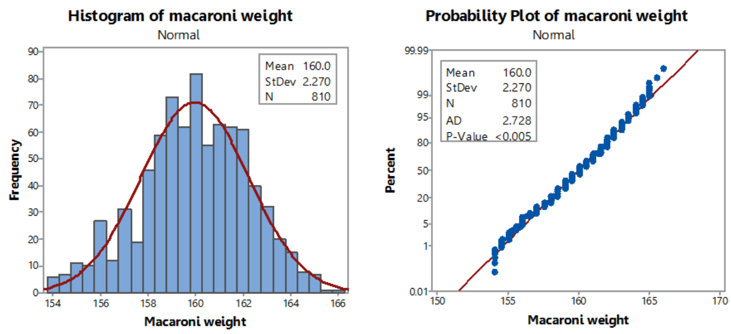


Figure 2. Macaroni experiment data with normal distribution properties.

Table 3. $\hat{\beta}_{OLS}$ estimation and ANOVA table for macaroni process.

$\hat{\beta}_{OLS}$ estimation

| Stage | Variable | Variable Name | Regression Coefficient | P-value |
|---|---|---|---|---|
| | Const | | 161.0699 | 0.0000 |
| 1 | A | Mix temp | -0.5385 | 0.0000 |
| 2 | B | Wait time | 0.1886 | 0.0185 |
| 3 | C | Volume | 1.2481 | 0.0000 |
| 3 | D | Blow time | 0.3213 | 0.0001 |
| 3 | E | Spray time | 0.1242 | 0.0564 |
| 1 | AA | Mix temp * Mix temp | -1.0159 | 0.0000 |
| 1 × 3 | AC | Mix temp * Volume | 0.2353 | 0.0161 |
| 3 | CC | Volume * Volume | -0.6537 | 0.0000 |
| 3 | CD | Volume * Blow time | -0.4300 | 0.0000 |

ANOVA for two way crossed classification

| Source | df | SS | MSS | F | P-value |
|---|---|---|---|---|---|
| Stage-1 | 2 | 342.38 | 171.19 | 36.1870 | 0.0000 |
| Stage-2 | 2 | 16.97 | 8.485 | 1.7936 | 0.1670 |
| Stage-3 (error) | 805 | 3808.22 | 4.7307 | | |
| Total | 809 | 4167.57 | | | |


Figure 3. Interaction effects of macaroni multi-stage processes.

its critical factor boil temperature and boil time, defined as factor A and B. Then the well-done pork in ham blocks is loaded into cool water of cool down wells. The cool down process is the stage-2 process with its critical factor cool down time, defined as factor C. After cool down, all ham blocks are moved into chilled room and frozen room waiting for slicing. The final stage-3 process is the slice machine which slices well-done pork into ham pieces. Their critical factors are knife sharpness, pork surface temperature and slicing speed which are defined as factors D, E and F respectively. During slicing, defective ham pieces, such as holes, tear-off and others, are segregated, weighed and defined as experimental response, loss slice weight.

The total numbers of key cooking factors are six. Stage-1 factor A, B and stage-2 factor C contain 2 levels, -1, 1. Stage-3 factor D and F, CCD is provided with $\alpha = \pm 1$ whereas factor D contain only 2 levels, -1, 1. The OLS-GLS CCD within strip-strip-plot design is shown in Table 4.

The strip-strip-plot design is started with all treatments in stage-1 are stripped with all treatments in stage-2. The total cell numbers is equal to $4 \times 2 = 8$ cells. Stage-3 treatments will be randomly placed within all 8 cells for balanced data case. Each cell contains seven treatments that cause the total run, rck is 56 runs.

To calculate the D-optimal efficiency in Equation 11, with historical process data, the value of $\sigma_\varepsilon^2$, $\eta_\delta$ and $\eta_\gamma$ are selected as 0.5, 0.5 and 0.5 respectively. Together with the number of p is 24 and N is 56, the design efficiency $(D_{eff})$ of pork ham process is 84.12%.

The collected loss slice weight data is widely ranged between closed to zero and closed to eight hundred gram. It is necessary to perform logarithm based ten transformation to raw data. The distribution of logarithm transformed data is presented in histogram diagram and normality test as shown in Figure 4. The $(\hat{\beta})$ estimation analysis from a software package is shown in Table 5 and its ANOVA table as well.

Table 4. Three-stage processes DOE for pork ham process.

| | | | Stage-2 | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | C | | | | | |
| | | | -1 | | | 1 | | |
| Stage-3 | | | D | E | F | D | E | F |
| Stage-1 | A | B | | | | | | |
| | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 |
| | | | -1 | -1 | 1 | -1 | 0 | 0 |
| | | | -1 | 0 | 1 | -1 | 1 | -1 |
| | | | -1 | 1 | 1 | 1 | -1 | -1 |
| | | | 1 | -1 | 1 | 1 | 0 | -1 |
| | | | 1 | 0 | 0 | 1 | 1 | -1 |
| | | | 1 | 1 | 0 | 1 | 1 | 1 |
| | -1 | 1 | -1 | -1 | 0 | -1 | -1 | -1 |
| | | | -1 | 0 | 0 | -1 | -1 | 1 |
| | | | -1 | 1 | -1 | -1 | 0 | 1 |
| | | | 1 | -1 | -1 | -1 | 1 | 1 |
| | | | 1 | 0 | -1 | 1 | -1 | 1 |
| | | | 1 | 1 | -1 | 1 | 0 | 0 |
| | | | 1 | 1 | 1 | 1 | 1 | 0 |
| | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0 |
| | | | -1 | -1 | 1 | -1 | 0 | 0 |
| | | | -1 | 0 | 1 | -1 | 1 | -1 |
| | | | -1 | 1 | 1 | 1 | -1 | -1 |
| | | | 1 | -1 | 1 | 1 | 0 | -1 |
| | | | 1 | 0 | 0 | 1 | 1 | -1 |
| | | | 1 | 1 | 0 | 1 | 1 | 1 |
| | 1 | 1 | -1 | -1 | 0 | -1 | -1 | -1 |
| | | | -1 | 0 | 0 | -1 | -1 | 1 |
| | | | -1 | 1 | -1 | -1 | 0 | 1 |
| | | | 1 | -1 | -1 | -1 | 1 | 1 |
| | | | 1 | 0 | -1 | 1 | -1 | 1 |
| | | | 1 | 1 | -1 | 1 | 0 | 0 |
| | | | 1 | 1 | 1 | 1 | 1 | 0 |

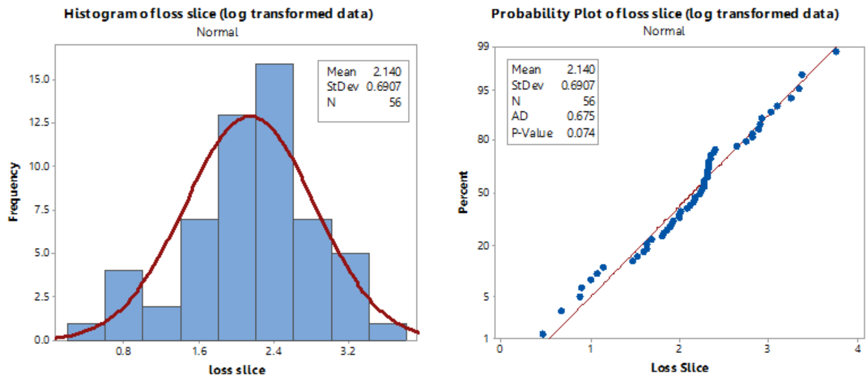


Figure 4. Pork ham experiment data with normal distribution properties.

Table 5. $\hat{\beta}_{OLS}$ estimation and ANOVA table for pork ham process.

$\hat{\beta}_{OLS}$ estimation

| Stage | Variable | Variable Name | Regression Coefficient | P-value |
|---|---|---|---|---|
| | Const | | 2.3528 | 0.0000 |
| 1 | A | Boil Temp | 0.3156 | 0.0001 |
| 1 | B | Boil Time | 0.0924 | 0.1968 |
| 2 | C | Cool Time | 0.1706 | 0.0214 |
| 3 | D | Sharpness | -0.0941 | 0.2020 |
| 3 | E | Surface Temp | -0.0412 | 0.6350 |
| 3 | F | Speed | 0.0700 | 0.4793 |
| 1 × 2 | BC | Boil Time * Cool Time | -0.2323 | 0.0079 |
| 1 × 3 | BE | Boil Time * Surface Temp | 0.2294 | 0.0095 |
| 3 | EF | Surface Temp * Speed | 0.1632 | 0.0871 |
| 3 | FF | Speed * Speed | -0.2986 | 0.0622 |

ANOVA for two way crossed classification

| Source | df | SS | MSS | F | P-value |
|---|---|---|---|---|---|
| Stage-1 | 3 | 6.6501 | 2.2167 | 6.1023 | 0.0013 |
| Stage-2 | 1 | 1.0643 | 1.0643 | 2.9300 | 0.0930 |
| Stage-3 (error) | 51 | 18.5260 | 0.3633 | | |
| Total | 55 | 26.2404 | | | |

From Table 5, essential information of main effects, A, C, quadratic effects, EE, within-stage interaction effects, EF, and cross-stage interaction effects, BC, BE, are able to be extracted out. The interaction effects, EF, BC and BE, are graphically presented in Figure 5. The cross-stage interaction between stage-1 boil time and stage-2 cool down time can provide information that the lower setting of both factors will provide small amount of defective ham slice pieces. Another cross-stage interaction between boil time and ham surface temperature shows that both factors have proportional relationship. For example, longer boil time with low surface temperature will provide small amount of defective ham slice pieces. These significant added terms in the final response model provide previous stages' parameters can be adjusted to obtain better optimized solution.

## 4. Conclusions

In this paper, we discuss the second-order, CCD, within strip-strip-plot structure for three multi-stage processes. The cross pattern of strip-strip-plot design is expected to extract cross-stage interaction effects significantly including quadratic effects from processes. The pattern of design is strongly required the balance property in order to obtain the maximum information of matrix **M** and the GLS-OLS equivalence condition. For cell balance properties, all cells must be filled with

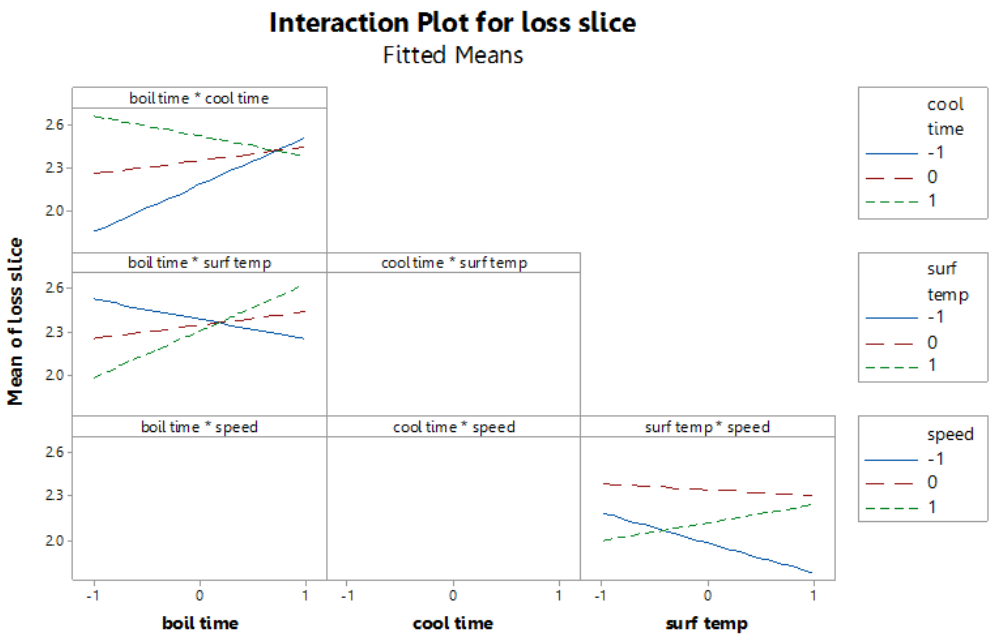


Figure 5. Interaction effects of pork ham multi-stage processes.

equal runs and allocation method will be randomization. The D-optimality criteria, $D_{eff}$, is utilized to judge the quality of design however it is still depended on historical data of $\sigma_\varepsilon^2$, $\eta_\delta$ and $\eta_\gamma$.

In practice, two GLS-OLS equivalence experiments have been performed in a food factory. The estimation of fixed effects, $\hat{\beta}$, which can be resulted from standard software have shown significantly all effects, main, quadratic, within-stage interaction and cross-stage interaction effect for both processes, macaroni and pork ham processes. This result can proved that strip-strip-plot design is suitable for cross-stage information. Based on the experimental results, for our next research, $D_{eff}$ level, 80%, with $\sigma_\varepsilon^2$, $\eta_\delta$ and $\eta_\gamma$, 0.5 for all, will be preferable level to generate the second order strip-strip-plot design patterns.

This paper deliberately provides the model and application of strip-strip-plot design for three multi-stage processes in a factory. Its applications are possible to all industries. In our next research, many OLS-GLS patterns for three multi-stages DOE will be generated to let the DOE will be widely utilized.

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