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Contributed Paper

Mean-Field Analysis of the Ising Hysteresis Relaxation Time

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ABSTRACT

In this work, the mean-field analysis was used to model Ising spins under the influence of time-oscillating magnetic field. The magnetic hysteresis was extracted by solving the non-linear differential equation of the magnetization in response to the field. The dynamic order parameter and its relaxation time were investigated as a function of temperature and characteristic of the field. From the results, it was found that the dynamic order parameter decayed in time and converged to zero or nonzero values depending on temperature, the magnetic field frequency and amplitude. The relaxation time was found to be small at high temperatures and fields, but increasing on approaching the dynamic phase boundaries. Furthermore, the relaxation time increased with increasing field frequencies due to the shift of the dynamic phase boundary as varying the frequencies. The detailed description to these phenomena can be explained by considering the dynamic phase diagram on the temperature and field amplitude plane for various field frequencies.

Keywords: ising model, mean-field, hysteresis, relaxation time.

1. INTRODUCTION

The dynamic response of the magnetic system in the presence of an oscillating magnetic field has been of great interest due to its fascinating phenomena in terms of dynamic hysteresis behavior and the nonequilibrium phase transition [1]. Of particular interest, spin models under the framework of statistical mechanics were often used to observe the magnetic hysteresis phenomena since they were able to reveal the origin of the magnetic phase transitions and critical phenomena in nature [2-4].

However, among various spin models, the Ising spin has gained a great deal of attention due to its success in predicting magnetic properties in some systems. For example, it was shown to be appropriate for describing both static and dynamic magnetic behavior in highly anisotropic single-domain nanoparticles and uniaxial thin films [5-13]. Therefore, incorporating with statistical mechanics techniques (e.g. the mean-field analysis [1,14-22] or the Monte Carlo simulations [1,18-25]), it is then possible to extract the dynamic magnetization information of the magnetic system in the presence of an oscillating magnetic field to investigate hysteresis properties. Nevertheless, since the hysteresis is actually a dynamic property, the retrieved results may be ambiguity and less useful if the results are measured before hysteresis has arrived at its steady state. This could be due to the bias arisen from initial setting up conditions. Therefore, the criterion in terms of steady state condition should be firstly drawn. This can be done by investigating the dynamic order parameter as a function of time.

The dynamic order parameter was defined as the time averaged magnetization over a full period of the oscillating magnetic field. On approaching the steady state, the dynamic order parameter converges to a time-independent value. In general, converging rate was found to depend on the field and the temperature [1]. Furthermore, the dynamic order parameter can also be used to define the dynamic phase transition (boundary) on the field amplitude and temperature plane [1]. At the phase transition boundary, the dynamic ferromagnetic ordered (finite dynamic order parameter) is separated from the dynamic paramagnetic disordered (zero dynamic order parameter) phases.

Generally, mean-field techniques and Monte Carlo simulations were often used to investigate hysteresis and this dynamic order parameter (e.g. references [1,14-25]). There are several advantages and weaknesses between these two techniques. For example, the mean-field is not truly dynamic in nature due to the ignorance of fluctuation. Therefore, the system may get struck in one local minima of the free energy and cannot proceed to the global minima. Consequently, if the field is far less than the required coercive field, the response magnetization may vary periodically but asymmetrically even at zero frequency (quasi-static limit). Nonetheless, the mean-field analysis is still very useful in extracting magnetic properties since its results reflect the behavior at thermodynamic limit. As a result, on some conditions, it provides a better performance upon other techniques e.g. the Monte Carlo simulation which is susceptible to errors when the finite size effect is not properly taken into account [26,27]. Moreover, only a few parameters are relevant in solving the mean-field equation, so one can study more complex systems with less mathematical or computational efforts. Therefore, the mean-field is undoubtedly of a great use where other methods become less tractable due to the problem complexity.

Nevertheless, even with mean-field picture, there are still considerable points needed to be taken into account in completing dynamic magnetic hysteresis pictures especially close to the dynamic phase transition. For instance, the precise dynamic phase boundary is not carefully investigated in details e.g. how the phase boundary on the filed-temperature plane depends on the frequency of the field and how the magnetic systems relax to their steady states close to the boundary. Therefore, the nature of the hysteresis changes near the dynamic transition point as a function of field frequency is still left to identify. The interesting in this issue comes from the fact that the dynamic of the hysteresis behavior strongly depends on the field frequency. At high frequencies, the system cannot follow the field promptly resulting in asymmetric hysteresis loop. On the other hand, at low frequencies, the system has more chances to catch up with the field which results in a lower phase lag and hence symmetric hysteresis loop becomes more possible. Consequently, the phase diagram (on the field amplitude and temperature plane) specifying the dynamic phase boundary should also depend on the frequency magnitude. As a result, even in the dynamic paramagnetic phase where zero dynamic order parameter is guaranteed, it should take more hysteresis loops from its initial state to relax and reach symmetric steady hysteresis loop at higher frequencies. On the other hand, the relaxation time (in terms of number of field oscillating loops) in approaching the symmetric hysteresis loop has still not been investigated comprehensively. Subsequently, how the relaxation time corresponds to the field parameters and temperature are still left in questions.

Therefore, there comes the main motivation of this study in finding some answers to these proposed questions numerically. The nonequilibrium dynamic phase transition of the Ising model in the presence of an oscillating magnetic field was investigated by the mean-field analysis. The relaxation behavior of the dynamic order parameter especially near the dynamic transition point was extracted and confirmed with the priori works [8,19] to show the accuracy of our calculations via temperature-amplitude diagrams. After that, based on the accuracy confidence, we further elaborated the study in details by extracting results from many temperatures, frequencies and amplitudes. This is for obtaining the relaxation time for the scaling analysis purpose which is not yet reported in literatures. To outline, section 2 gives the detailed analysis of the mean-field technique in investigating the relaxation behavior of the dynamic order parameter. In section 3, the temperature, the field frequency and the field amplitude variations of the relaxation time are reported. Then, a brief summary of all the results is given in section 4, the conclusion section.

2. METHODOLOGIES

In this study, we considered the two-dimensional Ising Hamiltonian under the influence of time varying external magnetic field i.e.

$$H = -J\sum_{<ij>} s_i s_j - h'(t)\sum_i s_i, \tag{1}$$

where the spin s_i takes on the values ± 1, $<ij>$ indicates that the sum includes only first nearest-neighbor, and $h'(t)$ is the time t dependent magnetic field. J is the exchange interaction where in this picture s_i is dimensionless, so both J and $h'(t)$ has a unit of energy. Another way in writing equation (1) is to write J in terms of the critical temperature T_C. This is due to that, for isotropic nearest neighbor only interaction systems, the mean-field critical temperature for nearest neighbor interaction Ising model is $k_B T_C = ZJ$ [28] which gives $J = k_B T_C / Z$, where Z is the coordination number and k_B is the Boltzmann's constant respectively. In addition, J was set to 1 so it becomes the unit of energy used in this study.

Considering the Ising Hamiltonian equation (1), in a nonequilibrium state, the time evolution of the average magnetization in response to oscillating external field under mean-field approximation can be described by [1,17]

$$\begin{aligned}\tau\frac{dm}{dt} &= -m + \tanh\left(\frac{ZJm(t)+h'(t)}{k_B T}\right) \\ &= -m + \tanh\left(\frac{m(t)+h(t)}{T}\right)\end{aligned}, \tag{2}$$

since $ZJ = k_B T_C$. In equation (2), $m(t)$ is the instantaneous magnetization (the average of all spins) as a function of time t, $h(t) \equiv h'(t)/ZJ = h_0 \sin(\omega t)$ is a sinusoidal magnetic field with frequency ω and amplitude h_0, and T is the temperature. The units of h_0 and T are $k_B T_C$ and T_C respectively. By defining units of h and T in this way, the study can be applied to any spatially isotropic systems ranging from square lattice where $Z = 4$ to the cubic lattices (i.e. simple cubic, body centered cubic and face centered cubic which have $Z = 6,8$ and 12 respectively) with ease. In addition, the time τ was set to 1, so it becomes the unit of time used in this study. Then, equation (2) was solved numerically using the fourth order Runge-Kutta method [29]. The time-step used in this numerical solving was set to the field period divided by 10000 allowing 10000 data points in each field loop to evaluate the period average magnetization (the results are not significantly different with more data points). Therefore, one can see that the time-step is not fixed but depends on the frequency (or period) of the field, and this makes the computing time to increase with increasing frequencies. This is from the fact that to increase the frequency means to reduce the field period. Hence, if the time-step is fixed, there becomes less and less data point for the analysis with increasing the frequency, which makes the extraction of hysteresis properties becomes less useful e.g. too few data points to accurately calculate the properties. A way out to this problem is to allow the time-step to scale with the frequency. Nevertheless, to pay its cost, at high frequencies, the hysteresis takes more loops to reach steady state, so there will be more numerical operations required. As a result, since the number of numerical operations is proportional to the real human time so the required computer resource (time) is very lengthy at high frequencies.

In solving equation (2), the initial value for the magnetization is $m(t = 0) = 1.0$. Then, from the instantaneous magnetization $m(t)$, the dynamic order parameter is defined as

$$Q(n) = \left(\frac{\omega}{2\pi}\right)\oint_n m(t)\,dt = \frac{\omega}{2\pi}\int_{t_{n-1}}^{t_n} m(t)\,dt, \quad (3)$$

where n refers to the considered n^{th} cycle and $t_n = 2\pi n/\omega$. In this study, $Q(n)$ was extracted using the closed Newton-Cotes formulas (Trapezoidal rule) [29]. Then, the decaying in time of Q at the n^{th} cycle was assumed to take an exponential form which is supported by good fits i.e. [19]

$$Q \equiv Q(n) = Q_0 \exp\left(-\frac{n}{\Gamma}\right) + Q(\infty), \quad (4)$$

where Q_0 is an n independent constant, and $Q(\infty)$ is the static background behavior of the Q. The above form shows that Q relaxes exponentially with the number of loops n of the oscillating field. The relaxation time factor Γ is therefore equal the n that gives $(Q(n)$-$Q(\infty))=Q_0 / e$. However, since the measured n is always an integer, to extract Γ from n in this ways may lead to ambiguity and unreliable results. A better way to obtain a more accurate Γ is to use nonlinear fit. Nevertheless, due to an increase of degree of freedoms (more parameters), the fit may not straightforwardly converge if too few input data points are available. Therefore, a more simple form of the fit where there is a small number of input data, e.g. a linear fit, may be more appropriate.

In general, $Q(\infty)$ is finite for dynamic ferromagnetic phase ($Q \neq 0$), but zero for dynamic paramagnetic phase ($Q = 0$) where there occurs symmetric hysteresis shape. Hence, if we choose a set of conditions that makes the system to lie in the dynamic paramagnetic phase, where $Q(\infty) = 0$, equation (4) reduces to

$$Q(n) = Q_0 \exp\left(-\frac{n}{\Gamma}\right). \quad (5)$$

As a result, it is possible to extract Γ via the least square linear fit of the double logarithmic function of equation (5) by considering

$$\log Q = \log Q_0 - \frac{n}{\Gamma}, \quad (6)$$

where Γ can be obtained from inverse of the slope obtained from the linear fit. Nevertheless, in the dynamic ferromagnetic phase, the use of equation (6) is not useful due to the existence of $Q(\infty)$ so $Q(n)$ does not varnish even n tends to infinity. Consequently, to believe if the extracted Γ from equation (6) is reliable, the R^2 (R-square) from the linear fit between $\log Q$ and n should be very close to 1. In this study, only $R^2 \geq 0.95$ are considered, or else Γ is denoted as undefined since the fit is no longer with good confidence ($R^2 < 0.95$) which means that the system may be in dynamic ferromagnetic phase as equations (5,6) are not valid.

3. Results and Discussion

From the results, the hysteresis profiles as a function of temperature and external field were found. For instance, Figure 1 shows the hysteresis loop calculated at frequency $\omega = 1\tau^{-1}$, amplitude $h_0 = 0.4\ k_BT_C$, and temperatures $T = 0.3$, 0.6 and 0.8 T_C. In the figure, the asymmetric hysteresis loops occur at $T = 0.3$ and 0.6 T_C whereas the symmetric hysteresis loop occurs at $T = 0.8\ T_C$. This implies that with increasing temperature, the asymmetric behavior tends to decay into the symmetric behavior. This is since the increase of thermal energy tends to compensate the exchange interaction among magnetic spins. Therefore, the system becomes more capable in climbing up local minima and traveling to other phase space (configuration) which finally results in symmetric pattern of the hysteresis loop (where the magnetization equally resides in the up and down direction of the applied field over one full period of the field). Consequently, the decay of dynamic order parameter Q was found to be temperature dependent.

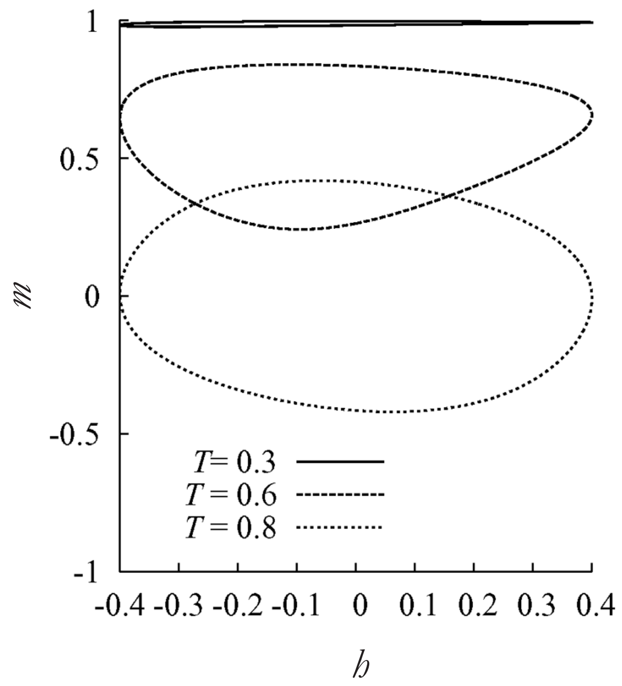

Figure 1. Examples of hysteresis loops presenting asymmetric hysteresis loops for which Q are finite ($T = 0.3$ and 0.6 T_C) and the symmetric hysteresis loop where Q vanishes ($T = 0.8\ T_C$) at $\omega = 1\ \tau^{-1}$ and $h_0 = 0.4\ k_BT_C$.

In following, Figure 2 shows the time (number of cycles of the oscillating field, number of hysteresis loops) n in relating to Q calculated at $\omega = 1\ \tau^{-1}$, $h_0 = 0.4\ k_B T_C$, and $T = 0.3$, 0.6 and 0.8 T_C. As can be seen, Q decays from an initial value and become time-independent at large n. At some temperatures i.e. $T = 0.3$ and 0.6 T_C, in Figure 2, Q's converge to finite values, but at a higher temperature i.e. $T = 0.8\ T_C$, it converges to zero. This is due to that the higher temperature T brings higher thermal energy to the system. Therefore, at high enough temperature, e.g. at $T = 0.8\ ZJ/k_B$, the magnetization has enough energy to sweep symmetrically leading to $Q = 0$, or else Q becomes finite at lower temperatures.

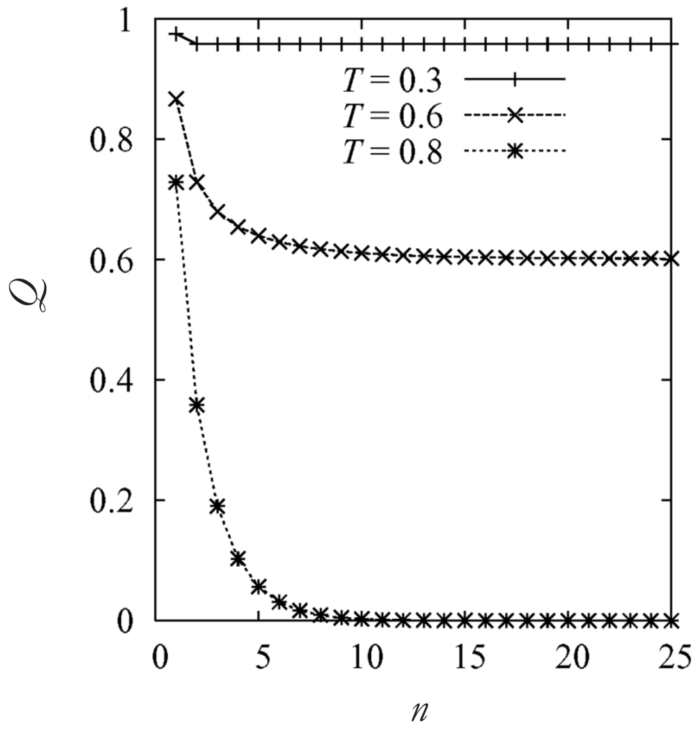

Figure 2. The Q dependence of the number of evolution loops (cycles) n performed at $\omega = 1\ \tau^{-1}$and $h_0 = 0.4\ k_B T_C$ for temperatures $T = 0.3$, 0.6 and 0.8 T_C.

As suggested in Figure 2, in order to observe hysteresis properties in its steady state, several hysteresis loops need to be discarded until Q becomes n independent. However, the n required to reach the steady state is also a function of frequency of the field. For example, at a lower frequency, there is more time supplied to the system in each field-sweeping loop. Thus, the system has more tendencies in catching the characteristic change in the field signal and results in small number of loops required for the steady state. Conversely, if the frequency is high, the system will then require more loops in approaching steady state. Hence, to specify the number of loops required to discard in order to obtain the hysteresis behavior at its steady state at various frequencies, the investigation of how the number of loops n in relating to ω is needed. Note that one of the outcomes from performing this investigation can be regarded in terms of optimizing calculation resource.

If the relationship between n and ω is formulated, it could then be a suggestive information on how possible to obtain the hysteresis at the steady state within some time (loops) limitation instead of using the trial and error method.

Figure 3 shows an example of the n required to bring Q to zero, as a function of frequency ω at T = 1.0 T_C for various amplitude h_0. Note that at this Q = 0 means symmetric hysteresis loop is obtained, and the system is currently in dynamic paramagnetic phase. Remark that Q may not actually decay to zero but a finite value if temperature and the field amplitude are not high enough which causes system to lie in the dynamic ferromagnetic phase. Therefore, close to dynamic phase boundary, it becomes very difficult to judge if Q = 0 (still in dynamic paramagnetic phase) or very small but finite (already in dynamic ferromagnetic phase but close to phase boundary). Consequently, in this study the $Q = 10^{-5}$ was chosen to be the representative of the 'zero Q'. This is because to wait until Q becomes zero is not practical in the case that $Q(\infty)$ does not cease but is very small. As a result, $Q = 10^{-5}$ was chosen and denoted as a cutoff limit since it was found to compromise with rounding errors and computing time.

In addition, from Figure 3, the linear relation between n and ω is found. Hence, it is possible to assume the relation that $n = c_1\omega + c_2 = \frac{2\pi c_1}{P} + c_2$ where c_1 and c_2 are proportional constants and the P is the field period i.e. $P = 2\pi/\omega$. As can be seen, this suggests the hyperbola-like relation between the n and the

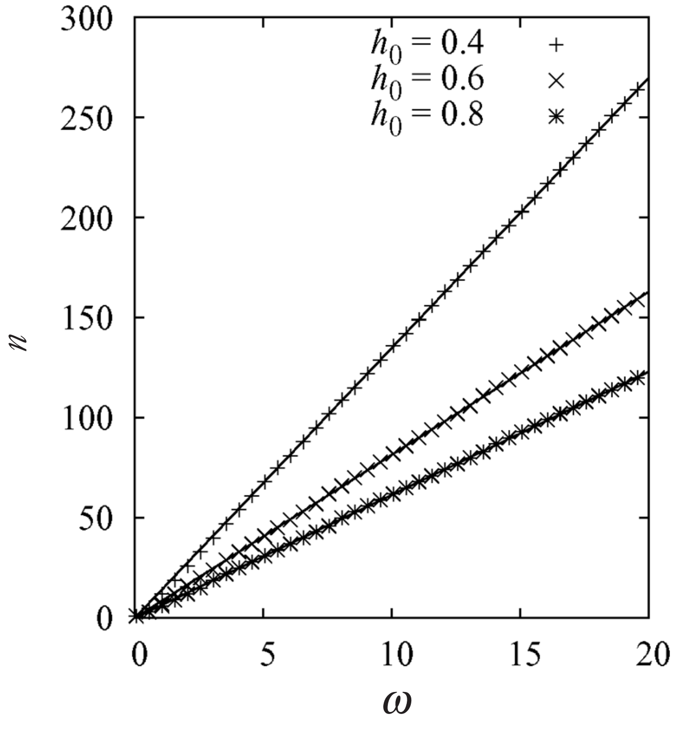

Figure 3. The value n as a function of ω at h_0 = 0.4, 0.6, and 0.8 k_BT_C at temperature $T = T_C$. Note that, in this figure, n refers the number of loops required to obtain 'zero Q'. Also, notice that, n sharply increase with decreasing h_0. All linear fits have R^2 = 0.9999.

period P i.e. $(n-c_2)P = 2\pi c_1$ in obtaining the symmetric hysteresis loop at fixed T and h_0. This implies that the total time $(n-c_2)P$ to reach this symmetric steady state is constant. In another word, the real human time to achieve the symmetric steady state from an initial state does not depend on the field frequency (period). On contrary, this is not true for the computing time because the computation process depends on the number of calculations (operations). Since at high ω the system requires more n to reach the symmetric $Q = 0$ phase and number of data points in each loop (cycle) is fixed to 10000, then the number of calculations (operations) increases with increasing ω. Thus at high ω more calculations is required and more computing time is needed. This is the goal of this study which aims to extracts the relaxation time factor Γ (representing the time to obtain the symmetric steady hysteresis state) and find out how it qualitatively relates to the set up conditions i.e. ω, T and h_0.

Next, by performing the linear fits on Q using equation (6), Γ are extracted which is found to increase with increasing ω e.g. see Figure 4. This is because of that at high frequency, the spins have less time to follow the external field signal. Therefore the reduction in Q requires more external field loops which leads to the increase of Γ. Moreover, from Figure 4, Γ tends to linearly relate to ω in a form $\Gamma = c_3\omega + c_4$, where c_3 and c_4 are proportional constants, with R^2 very close to 1. This suggests that Γ also relates to the period P in the form $\Gamma = \dfrac{2\pi c_3}{P} + c_4$, or the decay of Q, in equation (5) at fixed T and h_0, to the value Q_0/e at any frequencies requires number of the field loops equal to $(\Gamma - c_4)P = 2\pi c_3$. Also notice that, at a fixed ω, Γ increases with reducing the field amplitude. This is since

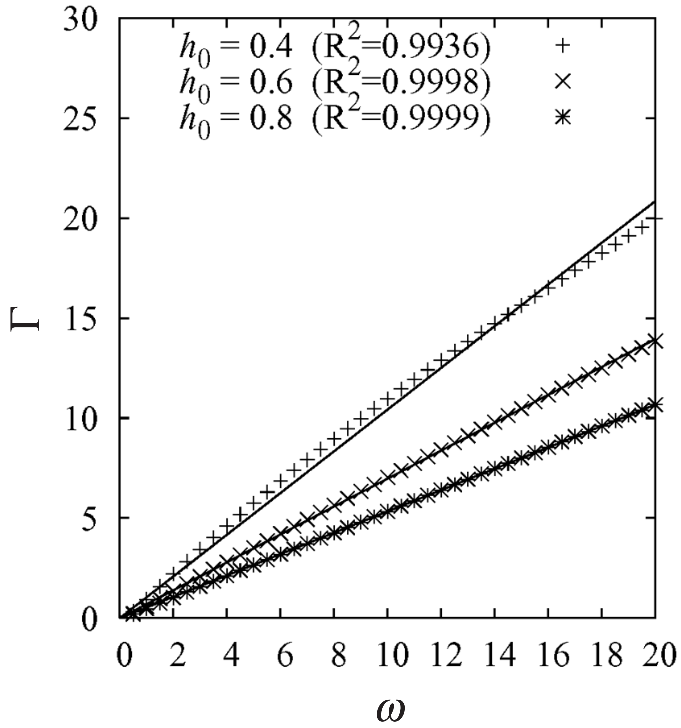

Figure 4. The relaxation exponent Γ as a function of ω for h_0 = 0.4, 0.6 and 0.8 $k_B T_C$ at $T = T_C$. The linear lines are from the least square linear fits to the data.

the higher amplitude provides higher magnetic energy to the system and this higher energy provides more magnetic force in causing the spins to follow the field. As a result, the magnetic system tends to reach steady state easier with higher amplitude. On the other hand, with the decrease of oscillating amplitude, the relaxation time increases because the external field-amplitude approaches the critical value $h_0 = 0$ (at $T = T_C$), the relaxation diverts at this critical point.

After that, with the extraction of Q, the conditions i.e. h_0, T, and ω, which give 'zero Q' are known. Therefore, the dynamic phase boundary separating the dynamic ferromagnetic and paramagnetic phases can be drawn e.g. see Figure 5. To explain in steps, at a fixed ω, we firstly choose h_0 and search for the minimum T that brings Q down to 10^{-5}. Then the next h_0 is chosen and the process gets repeated. The whole procedure is redone for all considered frequencies. In the figure, one may notice that each line of data separates the dynamic ferromagnetic region where $Q \neq 0$ (below line) out of the dynamic paramagnetic region where $Q = 0$ (above line). The results on this phase boundary diagram agree well with previous investigations e.g. reference [8], where in this study the more descriptive details on frequencies is provided.

Next, the study investigates the effect of

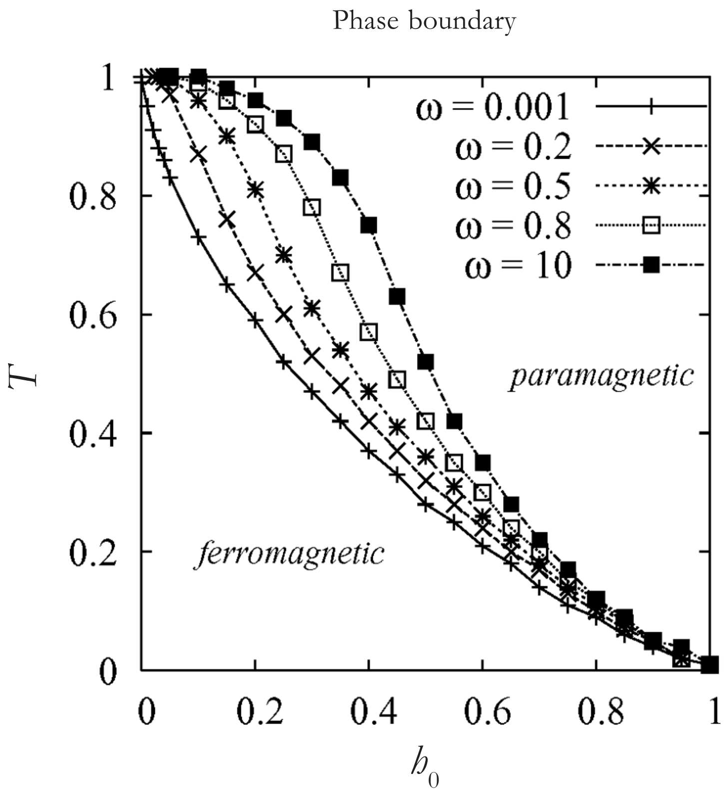

Figure 5. The h_0-T dynamic phase transition diagram for various frequencies ω. Sets of $\{h_0,T\}$ on a particular line (set of data performed at a same frequency) specifies phase boundary where the dynamic ferromagnetic phase ($Q \neq 0$) lies under the line and the dynamic paramagnetic phase ($Q = 0$) lies above the line.

temperatures and the external fields on Γ. From Figures 3 and 4 which show n-ω and Γ-ω relations at T_C, it can be seen both n and Γ sharply increase as h_0 approaches zero. In following, from Figure 6, Γ becomes greatest when h_0 tends to zero and T equals to T_C, where $(h_0,T) = (0,T_C)$ is known to lie on the phase boundary for any frequencies (see Figure 5). Therefore, this becomes an informative piece of evidence suggesting that the dynamic phase transition has a strong effect on the hysteresis properties e.g. Γ in this study. Consequently, to investigate this phenomena in details, the relaxation time factor Γ was extracted for various h_0, T and ω. The results for $\omega = 10$ and $0.5\ \tau^{-1}$ are presented in Figure 6. Note that some Γ cannot be extracted due to that the considered conditions (e.g. low T and low h_0) give the resultant hysteresis loop in asymmetric state and Q does not vanish. This case occurs if the system falls into the dynamic ferromagnetic phase, so Γ is undefined and is not displayed in the graph.

Note that this study stresses the use of equations (5,6) since we focused on the evolution of the hysteresis from an initial to its steady state only in the dynamic paramagnetic phase where the hysteresis has a symmetric shape and is useful in applications. Furthermore, in the dynamic paramagnetic phase, equations (5,6) can be used here because $Q(\infty)$ is zero so the relaxation time factor Γ can be defined and extracted, while the equations cannot be used to extract Γ from dynamic ferromagnetic phase because $Q(\infty)$ is not zero. Therefore, the word "undefined" we used here does not mean there is no relaxation time but to remind readers that to extract Γ by using equations (5,6) is not applicable in the dynamic ferromagnetic phase.

Also, as can be seen from Figure 6, on approaching the phase boundary lines on T-h_0 plane (in Figure 5), Γ seems to divert and finally become undefined if it crosses the phase boundary to the dynamic ferromagnetic region where the linear fit in the form of equation (6) is no longer useful (R^2 is small). However, with smaller magnitude of ω (compare Figures 6a and 6b), at fixed T and h_0, Γ significantly reduces. The reason is that the phase boundary lines for different ω are different. For instance, in Figure 5, the phase boundary line for lower ω tends to move downward. As a results, if a pair of (T,h_0) that is in the $\omega = 10\ \tau^{-1}$ dynamic paramagnetic phase, it will farther reside in the dynamic paramagnetic phase for the $\omega = 0.5\ \tau^{-1}$so Γ is even more smaller since Q decays faster. On the other hand, for a pairs of (T,h_0) in the dynamic ferromagnetic phase that is slightly below the $\omega = 10\ \tau^{-1}$ boundary line, the Γ cannot be extracted and becomes undefined (do not show in Figure 6a). However, if this pairs reside in the dynamic paramagnetic phase of the $\omega = 0.5\ \tau^{-1}$ (since the $\omega = 0.5\ \tau^{-1}$ curve locates lower), Γ is now possible to calculate due to the validity of equations (5,6). As being evident, all of these show that the dynamic phase diagram is very useful in predicting and understand the behavior of the relaxation time factor and perhaps the steady state hysteresis properties.

4. CONCLUSIONS

In this study, we investigated the dynamic order parameter as a function of temperature, the field frequency, and the field amplitude via the mean-field analysis. The study concentrated on how the dynamic order parameter and the number of loops required to obtain steady hysteresis loop relate to temperature and the field, where the relaxation time in dynamic paramagnetic phase was also obtained. The relaxation time was found to increase with decreasing both temperature and the field amplitude, and also frequency dependent, which can be described using the dynamic phase diagram. The increase of the

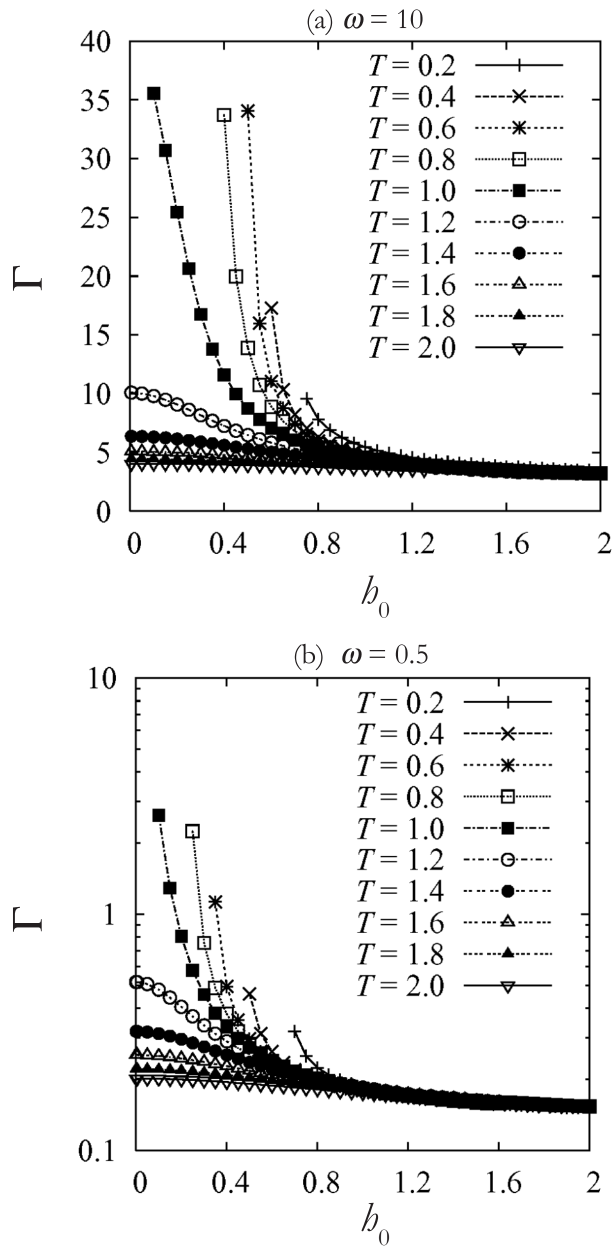

Figure 6. The relaxation time Γ as a function of the field amplitude h_0 for various temperatures T at (a) $\omega = 10\ \tau^{-1}$ and (b) $\omega = 0.5\ \tau^{-1}$.

relaxation time with reducing temperature and field is due to the transition from the dynamic paramagnetic to the dynamic ferromagnetic phase. This evidence suggests that the relaxation time dependence on the frequency is caused by the shift of the phase boundary. However, even the relation between the relaxation time and relevant parameters was formulated, there still some left and opened channel to further investigate other hysteresis properties. With some helps from the technique in estimating the relaxation time to achieve steady state reported in this study, it is expected that the full investigation on hysteresis properties will take less computer time consumption which could boost the fundamental and technological development on magnetic hysteresis topics in the future.

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