Chiang Mai J. Sci. 2015; 42(4) : 996-1004
http://epg.science.cmu.ac.th/ejournal/
Contributed Paper

Yrast States and B(E2) Values of Even $^{100-102}$Ru Isotopes Using Interacting Boson Model (IBM-1)

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Received: 12 May 2014
Accepted: 17 November 2014

ABSTRACT

The yrast-state bands and E2 transition rates for the even $^{100-102}$Ru isotopes are studied in the framework of the interacting boson model (IBM-1). The values of the parameters have been determined using the IBM-1 Hamiltonian which yield the best fit to the available experimental energy levels. The theoretical energy levels have been obtained by Matlap computer program for Ruthenium isotopes with neutron number N= 56 and 58 up to spin-parity 28^+ and 16^+, respectively. Furthermore, the B(E2) values are calculated and compared with the experimental data. The ratio of excitation energies of the first 4^+ and the first 2^+ excitated states, $R_{4/2}$, is also studied for the classification of symmetry of these nuclei. The moment of inertia and the potential energy surface of $^{100-102}$Ru isotopes also calculated. Back bending phenomena of those nuclei are studied. The calculated results of yrast energy band and B(E2) values are compared with the previous experimental results and the obtained theoretical calculations in IBM-1 are in good agreement with the experimental energy level. The results show that the $^{100-102}$Ru isotopes are vibrational deformed nuclei and they are dynamical symmetry U(5) in the interacting boson model IBM-1.

Keywords: interacting boson model-1, even-even isotopes, Ruthenium, energy level, B(E2), potential energy

1. INTRODUCTION

The interacting boson model (IBM-1) has been successfully describing collective nuclear characters in the medium mass nuclei [1]. This model treats pairs of valence nucleons (particles/holes) as bosons with angular momentum l = 0 (s bosons) and l = 2 (d bosons). There is no distinction of IBM-1 for proton and neutron degree of freedom. Naturally, the IBM has to take into account the fact that every nuclear state has a definite total nuclear angular momentum J or rather that the eigenvalue of the angular momentum operator J^2 is $J(J+1)$ cot $\hbar$. J is an integer. A boson interacts with the inert core of the nucleus (closed shells) from which results its single boson energy ε.

The even-even Ruthenium isotopes are part of an interesting region near to the closed proton shell at Z =50, while the number of neutrons in the open shells is much larger, as such these nuclei have been commonly considered to exhibit vibrational-like properties. Ruthenium isotopes has atomic number Z = 44 which is existed six less of protons close to the magic number Z= 50. Ruthenium isotopes with neutron N= 56 and 58 are very much of interest because they exist on the stability line and has been supposed as an anharmonic vibration-like nucleus of U(5) limit in the IBM-1. The microscopic anharmonic vibrator approach (MAVA) has been used in investigating the low-lying collective states in Ruthenium isotopes [2]. A lot of experimental and theoretical studies on the structure of yrast level and electromagnetic transition properties of doubly even Ru isotopes have been investigated [3-7].

Recently, the properties of the yrast states for $^{100-110}$Pd even-even nuclei had been established [8] . The electromagnetic reduced transition probabilities of even-even $^{104-112}$Cd isotopes were studied [9]. Electromagnetic reduced transition properties of ground state band of even-even $^{102-112}$Pd isotopes were studied by means of Interacting Boson Model-1[10, 11]. The investigations of low-lying states of ^{184}W and ^{184}Os nuclei were studied by Sharrad et al. [12].

Systematic previous studies have raised the possibilities to find the application of interacting boson model to predict the yrast states, electromagnetic transition and the potential energy surfaces to know the type of deformation exists in ^{100,102}Ru isotopes.

2- IBM-1 MODEL

2.1 Calculation of Energy Levels

Interacting Boson Model (IBM-1) [1] had widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. The vibrational model used geometric approach, the IBM employs a severely truncated model space and such as, calculations are possible for nuclei with N nucleons, providing a quantitative mechanism to compare experimental results and calculated values [13]. In the first approximation of IBM-1, only pairs with angular momentum L = 0 (called S-bosons) and L = 2 (called d-bosons) are considered.

The Hamiltonian of the interacting bosons in IBM-1 is given by ref ([14] Scholten et al., 1978).

$$H = \sum_{i-1}^{N} \varepsilon_i + \sum_{i<j}^{N} V_{ij} \qquad (1)$$

where ε is the intrinsic boson energy and Vij is the interaction between bosons i and j.

The multi-pole form of the IBM-1 Hamiltonian is given by ref. [15]

$$H = \varepsilon \hat{n}_d + a_0(\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4)$$

where $\hat{n}_d = (d^{\dagger}.\tilde{d}),\ \hat{P} = \frac{1}{2}(\tilde{d}.\tilde{d}) - \frac{1}{2}(\tilde{s}.\tilde{s})$

$$(2)$$

$$\hat{L} = \sqrt{10}\ [d^{\dagger} \times \tilde{d}]^{(1)}$$

$$\hat{Q} = [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]^{(2)} - \frac{1}{2}\sqrt{7}\ [d^{\dagger} \times \tilde{d}]^{(2)}$$

$$\hat{T}_3 = [d^{\dagger} \times \tilde{d}]^{(3)} \text{ and } \hat{T}_4 = [d^{\dagger} \times \tilde{d}]^{(4)}$$

Here $\hat{n}_d$ is the number of d boson, $\hat{p}$ is the pairing operator for the S and d bosons, $\hat{L}$ is the angular momentum operator, $\hat{Q}$ is the quadrupole operator, $\hat{T}_3$ and $\hat{T}_4$ are the octupole and hexadecapole operators, respectively.

The Hamiltonian as given in Eq.(2) tends to reduces to three limits, the vibration U(5), γ-soft O(6) and the rotational SU(3), starting with the unitary group U(6) and finishing with group O(2) [16]. In U(5) limit, the effective parameter is ε, in the γ-soft limit, O(6), the effective parameter is the pairing a_0 and in the SU(3) limit, the effective parameter is the quadrupole a_2.

The eigenvalues for the three limits are given as follows [8]:

$$U(5): E(n_d, v, L) = \varepsilon n_d + K_1 n_d (n_d + 4) + K_4 v (v + 3) + K_5 L (L + 1) \quad (3)$$

$$O(6): E(\sigma, \tau, L) = K_3 [N(N + 4) - \sigma(\sigma + 4)] + K_4 \tau(\tau + 3) + K_5 L (L + 1) \quad (4)$$

$$SU(3): E(\lambda, \mu, L) = K_2 (\lambda^2 + \mu^2 + 3(\lambda + \mu) + \lambda\mu) + K_5 L (L + 1) \quad (5)$$

K_1, K_2, K_3, K_4 *and* K_5 are other forms of strength parameters.

2.2 Moment of inertia (ϑ) and gamma energy E_γ

The relation between the moment of inertia (ϑ) and gamma energy E_γ is given by

$$2\vartheta/\hbar^2 = \frac{2(2I-1)}{E(I)-E(I-2)} = \frac{4I-2}{E_\gamma} \quad (6)$$

And the relation between E_γ and $\hbar\omega$ is given by

$$\hbar\omega = \frac{E(I)-E(I-2)}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} = \frac{E_\gamma}{\sqrt{I(I+1)}-\sqrt{(I-2)(I-1)}} \quad (7)$$

2.3 Reduced Transition Probabilities B(E2)

To calculate the *B(E2)* value, the reduced matrix elements of the E2 transition operator (T^{E2}) has the form

$$T^{E2} = \alpha_2[d^+s + s^+d]^{(2)} + \beta_2[d^+d]^{(2)} \quad (8)$$

where α_2 is the role of effective boson charge and β_2 is a parameter related to α_2. The low-lying levels of even-even nuclei (L_i =2,4,6,8……) usually decay by *E2* transition to the lower-lying yrast level with $L_f = L_i$-2. The reduced transition probabilities in *IBM-1* are given for the limit *U(5)* [14].

$$B(E2); L+2 \rightarrow L) \downarrow = \tfrac{1}{4}\alpha_2^2(L+2)(2N-L) \quad (9)$$

where L is the state that nucleus transition and N is the boson number, which is equal to half the number of valence nucleons (proton and neutrons). From the given experimental value of transition ($2^+ \rightarrow 0^+$), one can calculate the value of the parameter α_2^2 for each isotope, where indicates square of effective charge. This value is used to calculate the transition 10^+ to 8^+, 8^+ to 6^+, 6^+ to 4^+, 4^+ to 2^+ and 2^+ to 0^+.

3. RESULTS AND DISCUSSION

3.1 Boson Number and Prediction of Symmetry

A boson represents the pair of valence nucleons, and boson number is counted as number of collective pairs of valence nucleons. A simple correlation exists between nuclei showing identical spectra and their valence proton number (N_p) and neutron number (N_n). The number of valence proton N_p and neutron N_n has a total N = ($N_p + N_n$) = $n_p + n_n$ bosons. At present ^{100}Sn doubly-magic nucleus is taken as an inert core to find boson number. In the framework of IBM-1, the nuclei of ^{100}Ru and ^{102}Ru with N= 56 and 58 have proton boson hole numbers 3 and neutron boson particle numbers 3 and 4, respectively. Therefore total number of boson number of ^{100}Ru and ^{102}Ru are 6 and 7, respectively.

Symmetry shape of a nucleus can be predicted from the energy ratio R = $E4_1^+/E2_1^+$, where $E4_1^+$ is the energy level at 4_1^+ and $E2_1^+$ is the energy level at 2_1^+. The R has a limit value of 2 for the vibration nuclei U(5), 2.5 for γ-unstable nuclei O(6) and finally, 3.33 for rotational nuclei SU(3). The R values of low-lying energy levels of ^{100}Ru and ^{102}Ru isotopes are 2.27 and 2.33, respectively. Figure 1 shows experimental values of R = $E4_1^+/E2_1^+$, U(5), O(6) and SU(3) limits. Their results are consistent with a U(5) symmetry, as shown in ref. [17] for even-even ^{100}Ru and ^{102}Ru isotopes.

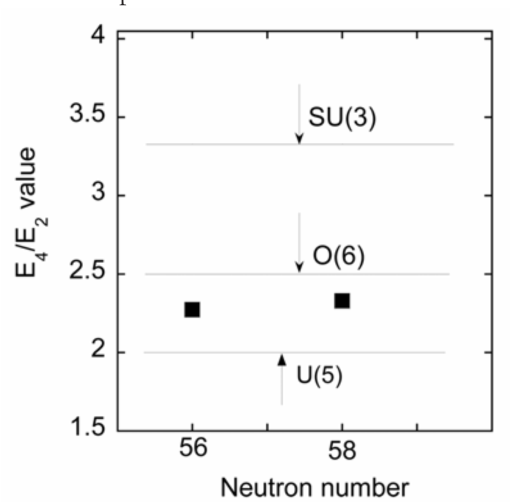

Figure 1. $E(4_1^+)/E(2_1^+)$ in experimental values [21] , U(5), O(6) and SU(3) limit of ^{100,102}Ru isotopes.

3.2 Yrast States

The IBM-1 has calculated the energy of different states (i.e. 0^+, 2^+, 4^+,.... 28^+) for doubly even ^{100}Ru and ^{102}Ru isotopes. The IBM-1 calculations have been performed with no distinction made between the neutron and proton bosons. For the analysis of the yrast states in the ^{100}Ru and ^{102}Ru nuclei up to 28^+ states and 16^+ states, we tried to keep the number of free parameters in the Hamiltonian to a minimum. Overall best fit was achieved for the yrast state bands of doubly even isotopes ^{100}Ru and ^{102}Ru. Each nucleus at the evolving states is determined using Eq. (3). Table 1 shows the values of these parameters those were used to calculate the energy of the yrast- states for the isotopes Z = 44 with N = 56 and 58 under this study. The energy level fits with IBM-1 are presented in table 2. Figure 2 shows yrast states of ^{100}Ru and ^{102}Ru isotopes as a function of angular momentum. The agreement between calculated theory and experiment is good and reproduced well. Furthermore, the calculated results are in good agreements with the new experimental data [17, 18] for ^{102}Ru isotope.

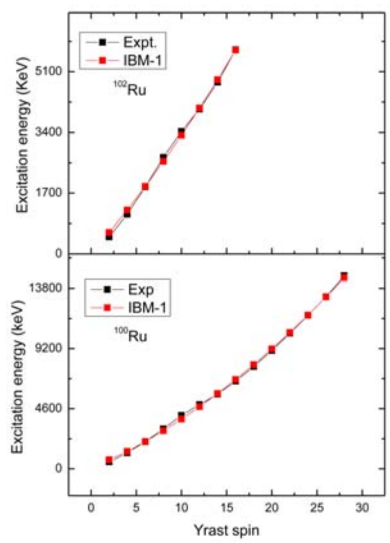

Figure 2. Excitation energy as a function of yrast spin for a) ^{100}Ru and b) ^{102}Ru isotopes.

3.3 Evaluation of Nuclear Collectivity and Moment of Inertia

To measure the evolution of nuclear collectivity, Figure 3 gives the comparisons of the ratios $R_L = E(L^+)/E(2)$ as a function of angular momentum (L) in the ground-state band for ^{100}Ru and ^{102}Ru isotopes. We present energies of the yrast sequences using IBM-1 (normalized to the energy of their respective 2_1^+ levels) in both nuclei and have compared them with previous experimental values [19].

The figure 3(a) and 3(b) show that IBM -1 calculation fit the U(5) predictions generally. However, the R_L values between theory and experiments are increases towards higher spin.

Table 1. Boson number and calculated Parameters in (KeV) for even $^{100-102}$Ru isotopes.

Isotopes	N	$\varepsilon(keV)$	$K_1(keV)$	$K_4(keV)$	$K_5(keV)$
^{100}Ru	6	524.155	11.258	9.977	2.529
^{102}Ru	7	507.968	12.317	8.959	-1.215

Table 2. Yrast spin (I), excitation levels E(I), transition energies (E_γ) , moment of inertia ($2\vartheta/\hbar^2$) and square of rotational energy $(\hbar\omega)^2$ for even ^{100,102}Ru isotopes [17, 18, 21].

Nucl.	I	E(I) MeV		Transition	E_γ MeV		$2\vartheta/\hbar^2$ MeV^{-1}		$(\hbar\omega)^2$ MeV^2	
		Exp	Cal		Exp	Cal	Exp	Cal	Exp	Cal
^{100}Ru	2	0.540	0.696	$2^+\rightarrow0^+$	0.540	0.696	11.11	8.62	0.220	0.284
	4	1.227	1.334	$4^+\rightarrow2^+$	0.687	0.638	20.38	21.94	0.115	0.099
	6	2.077	2.095	$6^+\rightarrow4^+$	0.850	0.861	25.88	22.89	0.179	0.184
	8	3.063	2.918	$8^+\rightarrow6^+$	0.986	0.823	30.43	36.45	0.242	0.169
	10	4.086	3.805	$10^+\rightarrow8^+$	1.023	0.887	37.15	42.84	0.260	0.196
	12	4.921	4.754	$12^+\rightarrow10^+$	0.835	0.949	55.10	48.47	0.173	0.225
	14	5.717	5.765	$14^+\rightarrow12^+$	0.796	1.011	67.84	53.41	0.158	0.255
	16	6.718	6.840	$16^+\rightarrow14^+$	1.001	1.075	61.94	54.53	0.250	0.289
	18	7.829	7.977	$18^+\rightarrow16^+$	1.112	1.137	62.95	61.57	0.309	0.323
	20	9.059	9.177	$20^+\rightarrow18^+$	1.230	1.200	63.42	65.00	0.378	0.360
	22	10.380	10.439	$22^+\rightarrow20^+$	1.321	1.262	65.10	68.15	1.745	1.592
	24	11.742	11.765	$24^+\rightarrow20^+$	1.362	1.326	69.02	70.89	0.206	0.195
	26	13.172	13.153	$26^+\rightarrow24^+$	1.407	1.411	72.49	72.29	0.495	0.496
	28	14.739	14.603	$28^+\rightarrow26^+$	1.567	1.450	70.20	75.86	0.614	0.525
^{102}Ru	2	0.475	0.598	$2^+\rightarrow0^+$	0.475	0.598	12.63	10.03	0.038	0.059
	4	1.106	1.229	$4^+\rightarrow2^+$	0.631	0.631	22.19	22.19	0.097	0.097
	6	1.873	1.893	$6^+\rightarrow4^+$	0.767	0.664	28.68	33.13	0.146	0.109
	8	2.703	2.589	$8^+\rightarrow6^+$	0.830	0.697	36.14	43.04	0.171	0.121
	10	3.431	3.319	$10^+\rightarrow8^+$	0.728	0.730	52.20	52.05	0.132	0.133
	12	4.052	4.081	$12^+\rightarrow10^+$	0.621	0.762	74.07	60.37	0.096	0.145
	14	4.803	4.876	$14^+\rightarrow12^+$	0.751	0.795	71.90	67.92	0.141	0.158
	16	5.717	5.704	$16^+\rightarrow14^+$	0.914	0.828	67.83	74.88	0.209	0.171

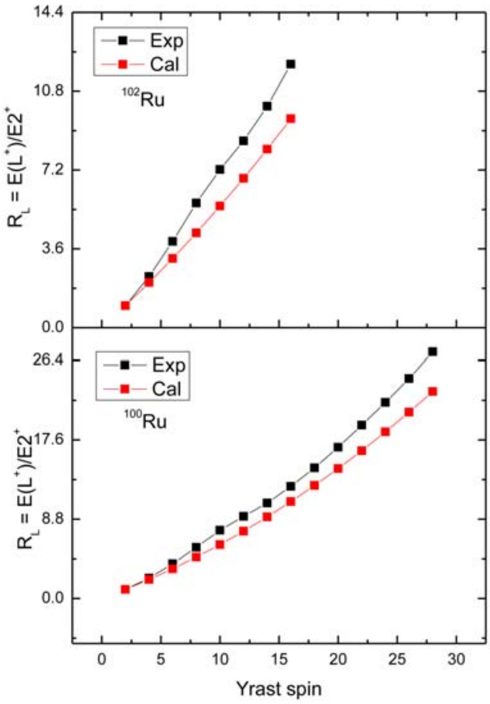

Figure 3. Energy ratio $R_L = E(L^+)/E(2_1^+)$ as a function of yrast spin for a) ^{100}Ru and b) ^{102}Ru isotopes. L = 2, 4, 6,…., 28.

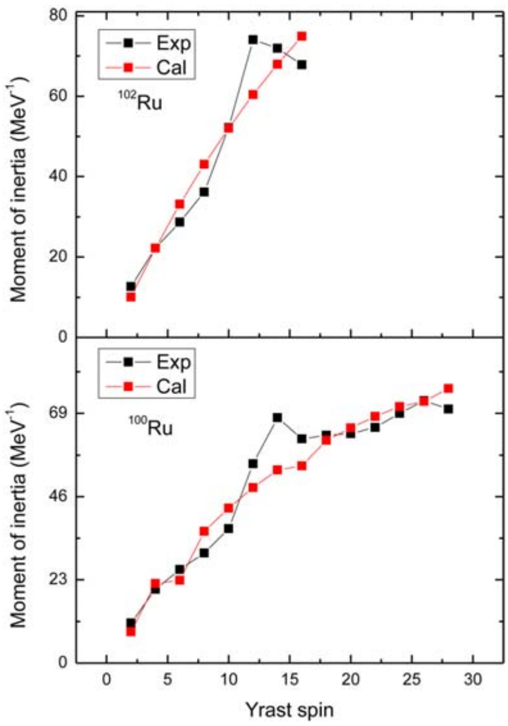

Figure 4. Moment of inertia as a function of yrast spin for a) ^{100}Ru and b) ^{102}Ru isotopes.

The positive parity yrast levels are connected by a sequence of stretched E2 transition with energies which increase smoothly except around the backbends. The transition energy $\Delta E_{I,I-2}$ should increase linearly with I for the constant rotor as $\Delta E_{I,\,I-2} = I/2\vartheta(4I-2)$ does not increase, but decrease for certain I values. The moment of inertia and rotational energy have been calculated from Eq. (6) and (7), respectively. The ground state bands up to 28 and 16 units of angular momentum are investigated for moment of inertia in even ^{100}Ru and ^{102}Ru isotopes. The moments of inertia are plotted versus spin in Figure 4. It is shown that moment of inertia as a function of spin has good agreement theoretically as well as experimentally.

3.4 Back-bending phenomena

Moments of inertia as a function of square of rotational energy in even $^{100-102}$Ru nuclei are plotted in Figure 5. In the lowest order according to variable moment of inertia (VMI) model this should give a straight line in the plot of inertia $2\vartheta/\hbar^2$ as a function of $(\hbar\omega)^2$. It is seen that the back-bending behavior changes from one nucleus to another. First order backbend shows at 2^+ states in ^{100}Ru nucleus. First order back bend shows at 8^+ states in ^{102}Ru nucleus. Results are presented on collective $\Delta I = 2$ ground band level sequence for the variation of shapes for Ru isotopes with even neutron N=56 and 58. The back-bending phenomena appear clearly in the diagram $2\vartheta/\hbar^2$ vs $(\hbar\omega)^2$.

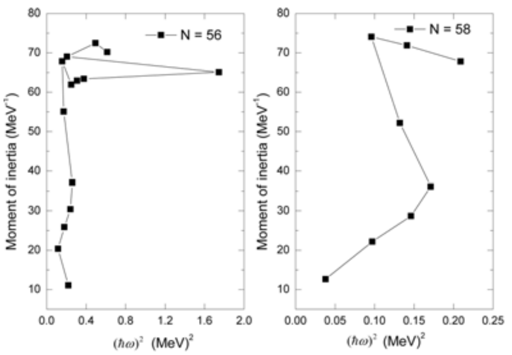

Figure 5. Moment of inertia as a function of square of rotational energy for a) ^{100}Ru and b) ^{102}Ru isotopes.

3.5 Potential Energy Surface

Potential energy surface (PES) by the Skyrme mean field method was mapped onto the PES of the IBM Hamiltonian [20, 21]. The expectation value of the IBM-1 Hamiltonian with the coherent state $|N, \beta, \gamma>$is used to create the IBM energy surface [22]. The state is a product of the boson creation operators b^{t}_{c} with

$$|N,\beta,\gamma\rangle = \frac{1}{\sqrt{N!}}\left(b_c^{\dagger}\right)^N |0\rangle, \tag{10}$$

$$b_c^{\dagger} = (1+\beta^2)^{-1/2}\left\{s^{\dagger} + \beta\left[\cos\gamma(d_0^{\dagger}) + \sqrt{1/2}\sin\gamma(d_2^{\dagger}+d_{-2}^{\dagger})\right]\right\}. \tag{11}$$

The energy surface as a function of β and γ, has been given [1].

$$E(N,\beta,\gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)+\frac{N(N-1)}{(1+\beta^2)^2(\alpha_1\beta^4+\alpha_2\beta^3\cos3\gamma+\alpha_3\beta^2+\alpha_4)}} \tag{12}$$

The calculated potential energy surfaces, $E(N, \beta, \gamma)$, of ^{100}Ru and ^{102}Ru are shown in Figure 6. It is shown that both are U(5) vibrational like nuclei.

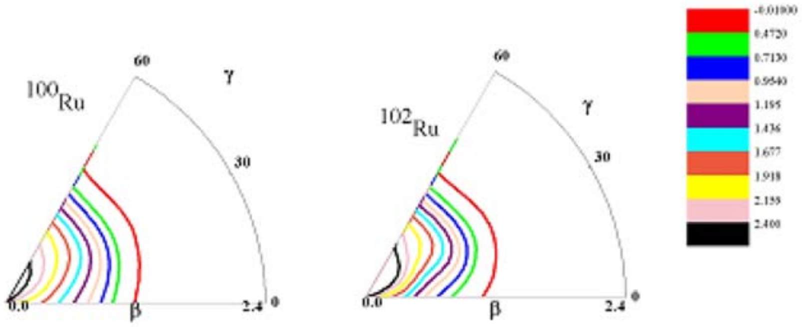

Figure 6. Potential energy surfaces for even a) ^{100}Ru and b) ^{102}Ru isotopes.

3.6 B(E2) Values

In order to calculate value of reduced transition probabilities, we have fitted the calculated absolute strength B(E2) of transitions within ground state band to experimental ones. The effective charge (α_2) of IBM-1 has been determined by normalizing experimental data $B(E2; 2^+_1\rightarrow 0^+_1)\downarrow$ of each isotopes using Eq. (9). From given experimental values of transition ($2^+_1 \rightarrow 0^+_1$), we have calculated value of parameter (α_2). Using known experimental $B(E2)\downarrow$ from $2^+_1\rightarrow 0^+_1$ transition, the reduced transition probabilities of $4^+_1 \rightarrow 2^+_1$, $6^+_1\rightarrow 2^+_1$and $8^+_1\rightarrow 2^+_1$ transitions of even-even ^{100,102}Ru isotopes are calculated using *IBM-1* and presented in Table 3. The calculated results are also compared with previous experimental results [23, 24] . It is shown that results of present work are in agreement within experimental error. Moreover U(5) limit would be confirmed by the expression for

B(E2) ratios as B(E2; $4_g \rightarrow 2_g$)/ B(E2; $2_g \rightarrow 0_g$) = 2(N-1)/N. The ratios of B(E2; $4_g \rightarrow 2_g$)/ B(E2; $2_g \rightarrow 0_g$) are 1.67(3) and 1.71(4) for ^{100}Ru and ^{102}Ru, respectively. The 2(N-1)/N value of ^{100}Ru and ^{102}Ru are 1.67 and 1.71, respectively. Therefore the present calculations are performed in the U(5) limit and therefore a good agreement between the calculated values and the experimental ones indicated that Ru isotopes with N=56 and 58 obey to this limit. The even-even $^{100-102}$Ru nuclei are nicely reproduced by the experimental data and their fits are satisfactory.

Table 3. B(E2)↓ in even $^{100-102}$Ru isotopes.

Nuclei	α_2^2 W.u. #	Transition level	B(E2) [23,24] W.u.	$B(E2)_{IBM-1}$ W.u.
^{100}Ru	5.93(07)	$2^+ \rightarrow 0^+$	35.6(4)	35.6(4)
		$4^+ \rightarrow 2^+$	51(4)	59.3(7)
		$6^+ \rightarrow 4^+$	<170	71.16(42)
		$8^+ \rightarrow 6^+$		71.16(42)
^{102}Ru	6.44(07)	$2^+ \rightarrow 0^+$	45.1(5)	45.1(5)
		$4^+ \rightarrow 2^+$	66(11)	77.28(84)
		$6^+ \rightarrow 4^+$	68(25)	96.6(11)
		$8^+ \rightarrow 6^+$	56(19)	103.04(112)
		$10^+ \rightarrow 8^+$		96.6(11)

4. Conclusion

Yrast states of even-even ^{100}Ru and ^{102}Ru isotopes have been calculated using interacting boson model-1. The energy levels up to 28^+ and 16^+ for ^{100}Ru and ^{102}Ru isotopes are obtained by the best fitted values of the parameters in the Hamiltonian of the IBM-1. The analyses of the calculated results for the low-lying positive parity energy spectra suggest a satisfactory agreement between IBM-1 and experimental data for the ground state band. Moment of inertia as a function of the square of the rotational angular energy for even neutrons N= 56 and 58 in Ru isotopes indicates the nature of back- bending properties. The potential energy surface and B(E2) values for ^{100}Ru and ^{102}Ru isotopes are calculated by IBM-1. The analytic IBM-1 calculation of yrast level, B(E2) values and potential energy surface of ^{100}Ru and ^{102}Ru isotopes established that they are as vibration deformed nuclei and close to U(5) symmetry.

Acknowledgments

Authors thanks to King Abdulaziz University to provide fund to complete the project.

References

[1] Arima A. and Iachello F., *The Interacting Boson Model*, Cambridge University Press, 1987.

[2] Kotila J., Suhonen J. and Delion D.S., *Phys. Rev. C*, 2003; **68**: 054322. DOI 10. 1103/physRevC68.054322.

[3] Che X.L., Zhu S.J. and Hamilton J.H., *Chin. Phys. Lett.,* 2004; **21**: 1904-1907. DOI 10. 1088/0256-307X/21/10/011.

[4] Che X.L., Zhu S.J. and Hamilton J.H., *Chin. Phys. Lett.,* 2006; **23**: 328-331. DOI http://cpl.iphy.ac.cn/ EN/Y2006/ V23/I2/328.

[5] Zhu S.J., Luo Y.X. and Hamilton J.H., *Int. J. Mod. Phys. E*, 2009; **18**: 1717-1720.

DOI 10.1142/S0218301309013798.

[6] Frank A., Van Isacker P. and Warner D.D., *Phys. Lett. B*, 1987; **197**: 474-478. DOI 10.1016/0370-2693(87)91036-7.

[7] Troltenier D., Maruhm J.A., Greiner W., Velazquez V.A., Hess P.O. and Hamilton J.H., *Z. Phys. A*, 1991; **338**: 261-270. DOI 10.1007/BF01288188.

[8] Ahmed I.M., Abdullah H.Y., Ahmad S.T., Hossain I., Kasmin M.K., Saeed M.A. and Ibrahim N., *Int. J. Mod. Phys. E,* 2012; **21**: 1250101. DOI 10.1142/S02183013 12501017.

[9] Abdullah H.Y., Hossain I., Ahmed I.M., Ahmad S.T., Kasmin M.K., Saeed M.A. and Ibrahim N., *Indian J. Phys*., 2013; **87**: 571-575. DOI 10.1007/s12648-013-0257-9.

[10] Hossain I., Saeed M.A., Ghani N.N.A.M.B., Saadeh H., Hussein M. and Abdullah H.Y., *Indian J. Phys*., 2014; **88**: 5-9. DOI 10.1007/s12648-013-0374-5.

[11] Hossain I., Abdullah H.Y., Ahmed I.M. and Saeed M.A., *Chin. Phys. C,* 2014; **38**: 024104. DOI10.1088/1674-1137/38/2/024103.

[12] Sharrad F.I., Abdullah H.Y., Al-Dahan N., Umran N.M., Okhunov A.A. and Abu Kassim H., *Chin. Phys. C,* 2013; **37**: 034101. DOI 10.1088/1674-1137/37/3/034101.

[13] Green L.K., *Nuclear Structure of* ^{112}*Cd Through Studies of b Decay*, MSc Thesis, Guelph University, Canada, 2009.

[14] Scholten O., Iachello F. and Arima A., *Ann. Phys*., 1978; **115**: 325. DOI 10.1016/0003-4916(78)90159-8.

[15] Al-Maqtary K.A., *Jordan J. Phys*., 2013; **6**: 95-102. http//journals.yu.edu.jo/jjp/JJPIssues/Vol6No2pdf2013/7.pdf.

[16] Iachello F., *Nuclear Structure*, Plenum Press, New York, 1981.

[17] Duckwitz H., Pfeiffer M., Albers M., Bernards C., Fransen C., Jolie J., Petkov P., Radeck D., Thomas T. and Zell K.O., *Nuclear Phys. A,* 2013; **903**: 18-39. DOI 10.1016/j.nuclphysa. 2012.11.013.

[18] Lalkovski S., Rainovski G., Starosta K., Carpenter M.P., Fossan D.B., Finnigan *S*., Ilieva *S*., Joshi P. and Koike T., *Phys. Rev. C,* 2005; **71**: 034318. DOI 10.1103/Phys RevC.71.034318.

[19] Firestone R.B. and Baglin C.M., Table of Isotopes, 8th Edn., Wiley-VCH, Berlin, 1999.

[20] Robledo L.M., Rodriguez-Guzman R., and Sarriguren P., *J. Phys. G: Nucl. Part. Phys.,* 2009; **36**: 115104. DOI 10.1088/0954-3899/36/11/115104.

[21] Nomura K., Otsuka T., Shimizu N. and Guo L., *J. Phys. Conf. Ser.,* 2011; **267**: 012050. DOI 10.1088/1742-6596/267/1/012050.

[22] Casten R.F. and Zamfir N.V., *Phys. Rev. Lett.,* 2001; **87**: 052503. DOI 10.1103/PhysRev Lett.87.052503.

[23] Singh B., *Nucl. Data Sheets,* 2008; **109**: 297-516. DOI 10.1016/j.nds.2008.01.001.

[24] Frenne D., *Nucl. Data Sheets,* 2009; **110**: 1745-1915. DOI 10.1016/j.nds. 2009.06.002.