

Application of nonlinear link in strut and tie model for joint planar expansion

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Abstract

An application of nonlinear link with strut and tie model of seismic retrofit technique for existing reinforced concrete beam-column connections using planar joint expansion is proposed. The model is constructed based on experimental results of one control specimen, three retrofitted specimens and FEM analysis of strengthening method proposed by authors. Truss model which represents behavior and flowing of force in joint and planar expansion is constructed based on strut and tie concept. Nonlinear link is added in critical elements to represent nonlinear behavior. By solving statically indeterminate truss and checking strut and tie limitation, the design region solution is received. The model is not only predicts close value of maximum column shear force, failure mode and joint shear force but also predict column capacity and displacement relation. Proposed simplify strut and tie model is helpful for practical engineer in designing planar joint expansion retrofitting beam-column connection.

1. Introduction

A joint retrofitting technique called “planar joint expansion” [1] is proposed by authors. In this method, the beam-column joint is two-dimensionally enlarged by cast in-situ concrete. The in-situ cast joint expansion can be independently installed in transverse and longitudinal directions (Figure 1).

The method is comparatively easy in application and cost-effective. According to test results [1], retrofitted specimens could change failure mechanism from joint shear failure to beam flexure failure mode. The strength, stiffness and energy dissipation of retrofitted specimens were greatly improved. The planar joint expansion was effective to reduce joint shear stress and improved anchorage bond of beam bar within the joint. The plastic hinge formation could be moved away from column face, thus preventing joint shear failure.

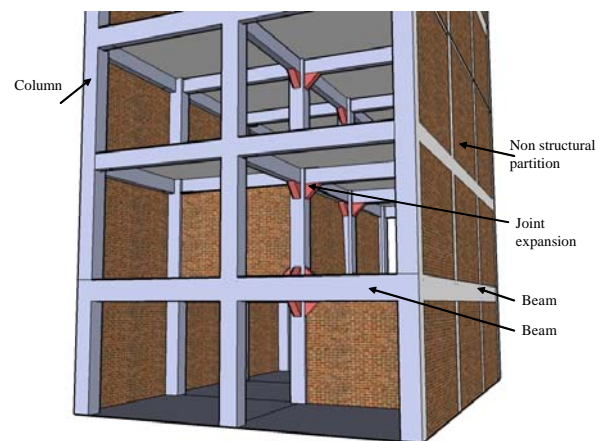


Figure 1 3D Beam-column frame system retrofitted with planar joint expansion

Strut-and-tie model for beam-column joint has been initially proposed by Park and Paulay [2]. Currently, there exist a variety of enhanced models are in literature [3-6]. However most of these works have focused on designing beam-column joints. In this paper, the application of strut and tie model is

proposed for capturing forces flowing and predicting joint shear capacity after retrofitted by planar joint expansion method. Finite Element Analysis and experimental results of control specimen and three retrofitted specimen are also played an important role for construct strut and tie model.

2. Summary of experimental studies and FEM analysis

2.1 Tested specimens and set-up

The experimental program consisted of one control specimen and three interior beam-column specimens, namely, J0, PJE1-PJE3. Specimen J0 was un-strengthened control specimen. Specimens PJE1-PJE3 were strengthened specimens. All specimens were half-scale with cruciform shape consisting of beams and columns extending from joint faces to mid-length and mid-height, respectively. The specimen size and reinforcement detail of control and retrofitted specimens were identical with each other except in the retrofitted parts. Figure 2 show size and reinforcement detail of control specimen, J0.

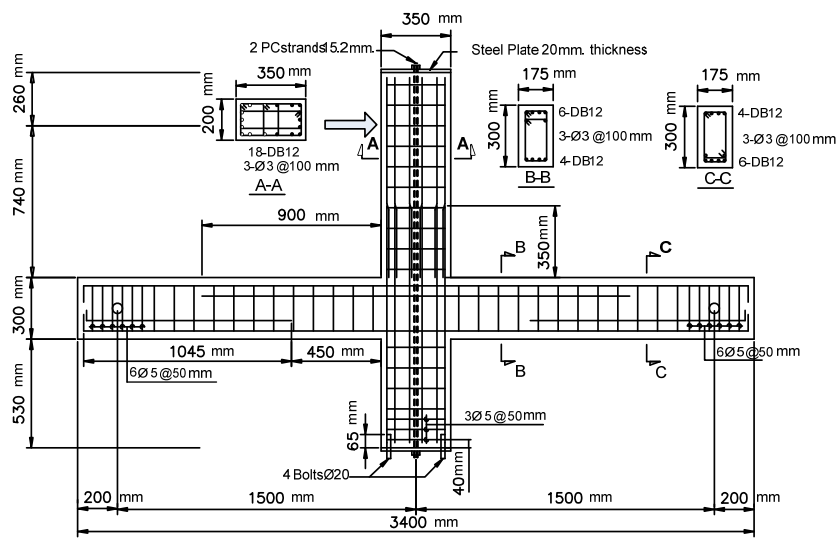


Figure 2 Dimension and reinforcing detail of control specimen, J0

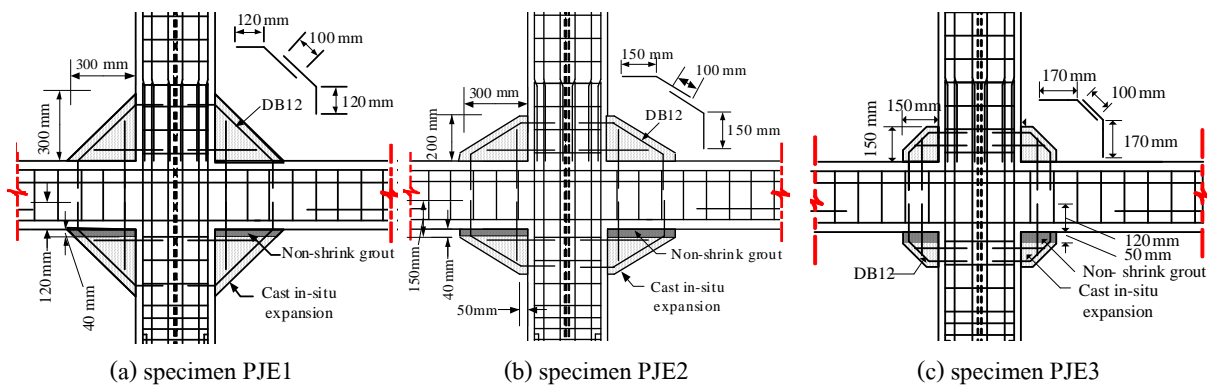


Figure 3 Dimension and reinforcing detail of strengthened specimens.

Each retrofit specimen was strengthened by different size of planar joint expansion as shown in Figure 3. Cast in-situ RC joint planar expansion with 300 x 300 mm triangular shape

was fabricated around four corners of specimens PJE1. As for PJE2, the size of expansion was reduced to 200 mm along column to reduce the increased shear due to column

shortening, while it was kept at 300 mm along beam. In order to compare the size influence of planar joint expansion, the size of planar joint expansion was reduced to 150 x 150 mm for specimen PJE3.

2.2 Experimental result and FEM analysis

Regarding to experimental result, control specimen failed by joint shear failure. The cracks were mainly concentrated in the joint. The concrete in the joint spalled off, exposing column longitudinal bars as shown in Figure 4a. For retrofit specimen, specimen PJE1 and PJE2 failed by beam flexural failure. The severe crack occurred both sides of beam at the edge of expansion while little diagonal crack in joint as

shown in Figure 4b-4c. At the end of the test (4% drift ratio), concrete cover at the bottom face of the beam spalled off, exposing buckled beam bars.

The failure mode of specimen PJE3 is different from each other. As mentioned, specimen PJE3 was strengthened with 150 x 150 mm expansion. The failure occurs in the joint panel as well as in the expansion part. FEM also predicts severe damage in both joint region and the expansion, similar to the experimental result. As shown in Figure 4d, cracks are formed parallel to the edge of expansion, indicating compression strut in the diagonal direction. Specimen PJE3 which failed by joint shear failure.

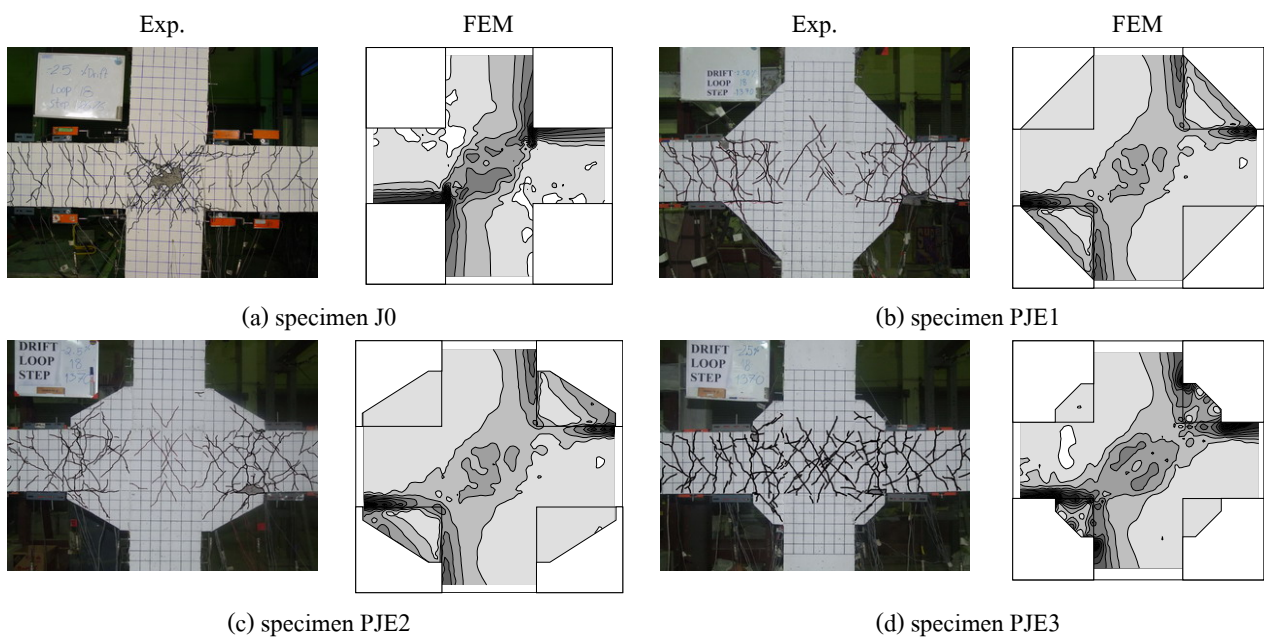


Figure 4 Observed crack pattern and principal compressive stresses at Drift 2.5%

To understand how the force is transmitted in strengthened specimens, the development of principal compressive stresses is investigated. As shown in Figure 4, the diagonal compressive struts clearly form in the joint panel of all specimens. The diagonal strut functions as the main load bearing mechanism. The horizontal component of this strut must be equilibrated with the horizontal joint shear force

caused by unbalanced moment from the adjoining beams. For specimens PJE1 and PJE2, the compressive strut also develops in joint panel and gradually expands. At 1.75% drift ratio, the diagonal strut remains intact. It is noted that the intensity of principal compressive stress varies with the size of expansion. The principal stress is highly intensified in PJE3 and least intensified in PJE1. As seen, principal

compressive stresses also form diagonally along the edge of two opposite expansions. This additional compressive strut provides the additional secondary load bearing mechanism that supports the force transfer between beam and column via expansion. As a result, the principal stress in the primary strut is reduced, thus protecting the joint from failure. For specimen PJE3, however, the size of expansion is inadequate to support the secondary load bearing mechanism. The compressive stress is thus concentrated in both joint panel and expansion. The failure of the specimen is caused by the collapse of the primary and secondary struts. The shear strength of the joint is the sum of these two mechanisms.

3. Strut and tie model

3.1 Geometries of strut and tie

It can be seen that the stress field show apparently diagonal strut in joint region and sub strut in planar expansion for all specimen (Figure 4). For joint region, one is primary diagonal strut mechanism which equilibrates concrete compression forces in beams and columns and some bond forces in the compression zones. Thus in the model, main strut is assumed diagonally coinciding with the direction of the principal compressive stress of the concrete (Figure 5a). The concept is similar to Park and Paulay model[2]. The others strut mechanisms which consist in the model are two sets of steep and flat struts. Steep strut represent vertical mechanism includes of one vertical tie and two steep struts (Figure 5b). The vertical tie is made up of the intermediate column bars as Hwang and Lee model[3]. Flat strut regard as truss mechanism sustains the forces transferred from the column and beam main bars by the bond mechanism[6, 7] (Figure 5c).

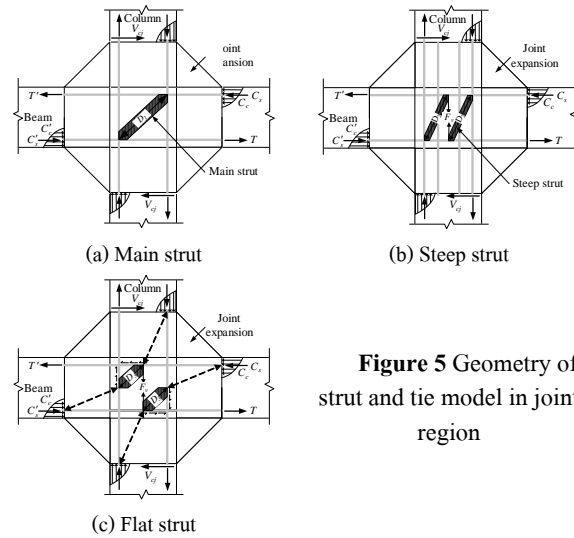


Figure 5 Geometry of strut and tie model in joint region

For planar expansion, the additional compressive strut providing the additional secondary load bearing mechanism that supports the force transfer between beam and column via expansion is model by two struts and two ties diagonally acting through planar expansion in the opposite side. The diagonal strut is constructed according with dominant principal compressive strut within planar expansion. The tie is to stabilize planar truss stability. Figure 6 show global strut and design region of all specimens.

3.1.1 Longitudinal element

The longitudinal element, which is located at the boundary of a cross section for beam and column, is assumed to be positioned at the centroid of the longitudinal reinforcing bars (Figure 7a). The properties of member are concrete for strut and steel for tie member. The areas of the concrete beam and column elements at boundaries of the cross section are calculated as $A_c = a_b \times b_b$ and $a_c \times b_c$, respectively, where b_b , b_c = width of beam and column section, a_b and a_c = depth of beam and column element, respectively. The depth of beam and column element is assumed as effective depth of flexural compression zone

which can be approximated as shown in equation (1) and equation(2).

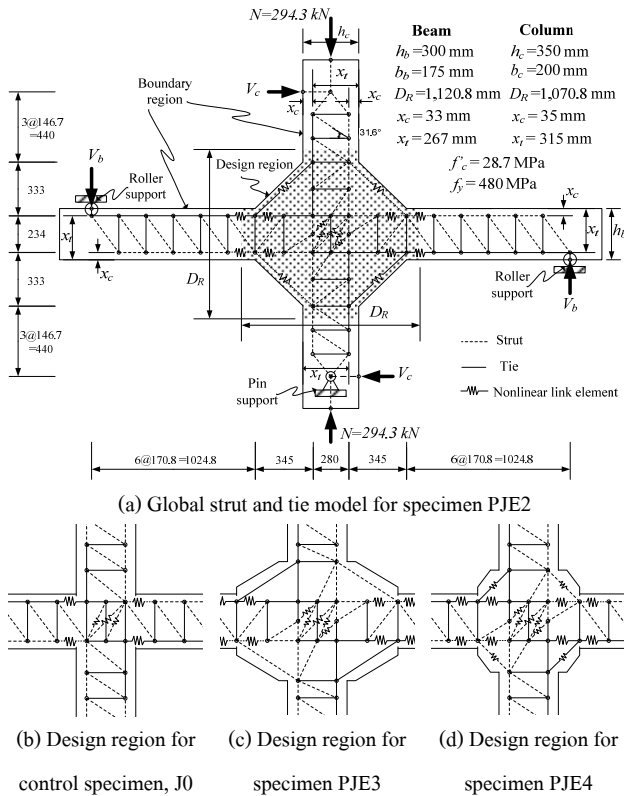


Figure 6 Proposed strut and tie model

$$a_b = \frac{A_s f_y}{0.85 b_b f'_c} \quad (1)$$

$$a_c = \left(0.25 + 0.85 \frac{N}{A_g f'_c} \right) h_c \quad (2)$$

Where A_s is the area of tensile reinforcement of the beam; b_b is the beam width; N is the axial force acting on the column; A_g is the gross area of the column section; and h_c is the thickness of the column in the direction of loading. The areas of the tie element at the boundaries are set to the areas of the corresponding longitudinal reinforcing bars : A_{sL} (Figure7a).

3.1.2 Transverse element

Since the transverse elements in beam and column remain in tension during the loading, they are modeled only with

the transverse reinforcing bars (Figure 7b). The cross-sectional area of the transverse element is defined as $A_{sT} = \rho_t s b_b$ or $A_{sT} = \rho_t s b_c$ for beam and column, respectively. Where ρ_t is ratio of the transverse shear reinforcement and s is spacing of the transverse element. The proposed strut and tie model is applicable only to RC members satisfying shear resistance provided by reinforcing bars equal or greater than applied shear force.

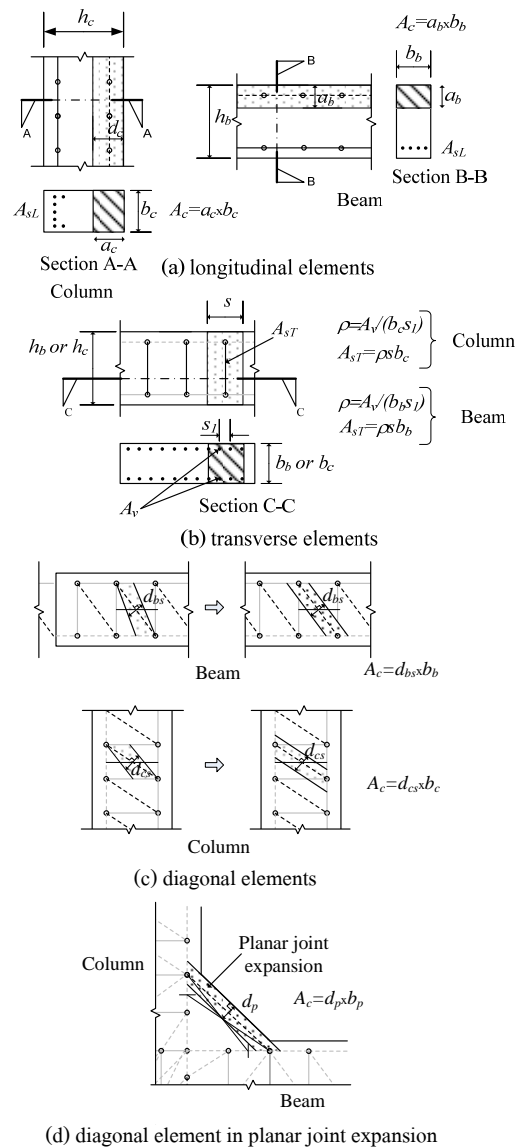


Figure 7 Modeling of strut and tie elements

3.1.3 Diagonal element

The diagonal concrete elements are divided in equal length in beam and column with the angle of the diagonal

concrete element range from 31° to 59° [8]. The depths of diagonal strut in beam (d_{db}), column (d_{dc}), diagonal strut in joint region, d_{j1} , d_{j2} , d_{j3} and planar joint expansion (d_p) are determined based on the geometry of the truss model [9], as shown in Figure 7c-d. To calculate the idealized effective cross-sectional area of struts in D-region (A_c), the perpendicular distance, (d_{db} , d_{dc} , d_{j1} , d_{j2} and d_p), between the diagonal strut, taken at the mid-depth of the joint and quarter depth in d_{j3} , are multiplied by the sectional width, b_b for beam, b_c for column and b_p for joint.

The area of the diagonal reinforcing bar element is set to the cross-sectional area of the diagonal reinforcement. Regarding to reinforcement design in planar joint expansion, the tie force in this element is useful for designer to select optimum reinforcement in joint expansion.

3.1.4 Nonlinear element

To capture nonlinear behavior of beam column joint retrofitted by planar joint expansion, nonlinear link elements are added in critical member that can go beyond yielding under seismic load. In proposed model, there are 4 nonlinear tie links on both sides of beam at the outer and inner joint expansion. The tri-linear model of reinforcing bar [10, 11] is adopted in this study. The relation between stress and strain is shown in Figure 8. As for compression zone, there are 5 and 4 nonlinear links element in joint region and both sides of beam, respectively. Combined tension- compression model of concrete for normal stress orthogonal and parallel to a crack [12] which used in WCOMD[13], nonlinear finite element program, is used in this study. The stress-strain relation is shown in Figure 9.

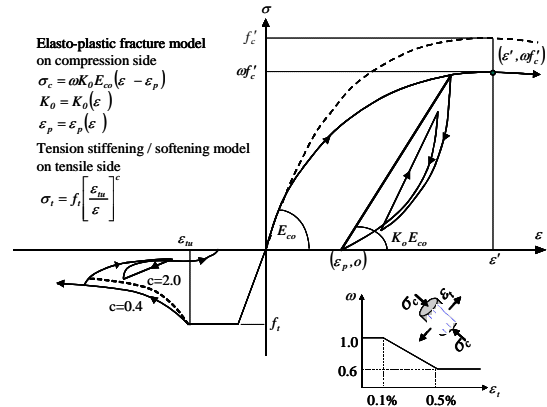


Figure 8 Combined compression-tension model of concrete

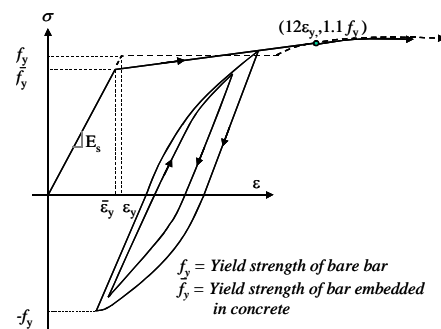


Figure 9 Reinforcing bar model

3.2 Strut and tie model verification and discussion

A powerful nonlinear SAP2000 program[14] has been used to analysis this proposed model. By increasing applied column shear force, V_C and solving statically indeterminate planar truss in strut and tie model as shown in Figure 6, the force in member strut forces and tie forces are received. Tie members are compared with limitation, 1.25 time of yield strength of reinforcement and verified with the forces converted from measuring strain attached with in beam and column bar. 1.25 is ACI code[15] recommendation for reserved strength of reinforcement after yielding. The strut members are compared with stress limit, f_{cu} . The strut limitations used in the model is calculated as

$$f_{cu} = \nu f'_c \tag{3}$$

where f_{cu} is the effective compressive strength of concrete in strut, MPa

f'_c is the concrete cylinder strength in MPa

ν is the effectiveness factor per ACI code[15].

$\nu = 0.64$ is used with strut that located along longitudinal reinforcement such as longitudinal strut in beam and column and diagonal strut in planar joint expansion.

$\nu = 0.51$ is used for strut in joint region and diagonal strut in beam and column.

$\nu = 0.68$ is used for nodal type which represents support of design region such as compression strut in beam and column at the edge of joint expansion.

3.2.1 Failure modes

The column shear force is gradually applied monotonic load until any member reached its maximum capacity. The failure mode is also predicted with failure mode of truss member. Results of planar truss are shown in Figure 10. As shown, the model predicts crushing failure ($0.68f'_c$) of concrete beam and beyond yielding point of tie member at the edge of joint expansion for specimen PJE1 and PJE2. Thus specimen PJE1 and PJE2 are defined as beam flexural failure same as the experimental result. Both models also show imminent failure of beam bar fracturing at the edge of joint expansion, $1.24f_y$ and $1.10f_y$ for specimen PJE1 and PJE2, respectively, according to test result that the beam bottom bar was cut off in 4.0 % drift ratio. For specimen PJE3, the model show diagonal flat

strut in joint region reaching stress limitation ($0.51f'_c$) and beam at the edge of joint expansion almost reaching it stress limitation ($0.67f'_c$). Thus specimen PJE3 define as joint failure according to experimental result that specimen PJE3 failed by joint shear failure with severe damage of beam at the edge of joint expansion.

3.2.2 Column shear force and drift ratio

The comparison between strut and tie model and experimental load versus drift ratio relation is shown in Figure 11. A reasonable match is obtained for envelope load. The yield load predicted by model is 83.7, 96.3, and 80.4 kN at drift ratio 0.98%, 1.13% and 1.27% while the experimental yield load is 85.9, 98.9 and 81.4 at the corresponding drift ratio 1.00% for specimen PJE1-PJE2 and 1.25% for specimen PJE3. With this result, the ratio of column shear force between the prediction and experiment at yield is 0.97 for specimen PJE1-PJE2 and 0.99 for specimen PJE3. For maximum load, the model show 92.6, 105.8 and 86.3 kN while the experimental maximum load is 97.6, 112.0 and 94.7 kN for specimen PJE1-PJE3, respectively. Thus, the ratio of maximum column shear force predicted by proposed model and experiment is 0.95, 0.94 and 0.91, for specimen PJE1-PJE3, respectively. As seen, the proposed strut and tie model is underestimates the load-carrying capacity because the disadvantage of transverse element which is ignored the shear resistance of concrete. Modeling techniques that can address the shear resistance of concrete have to be developed.

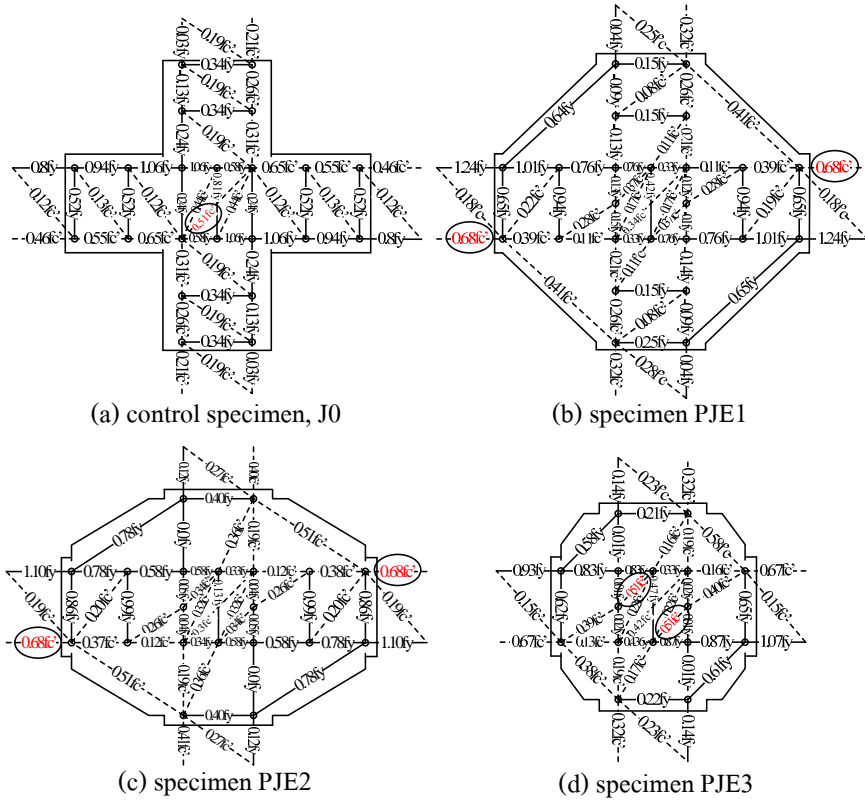


Figure 10 Result of strut and tie model

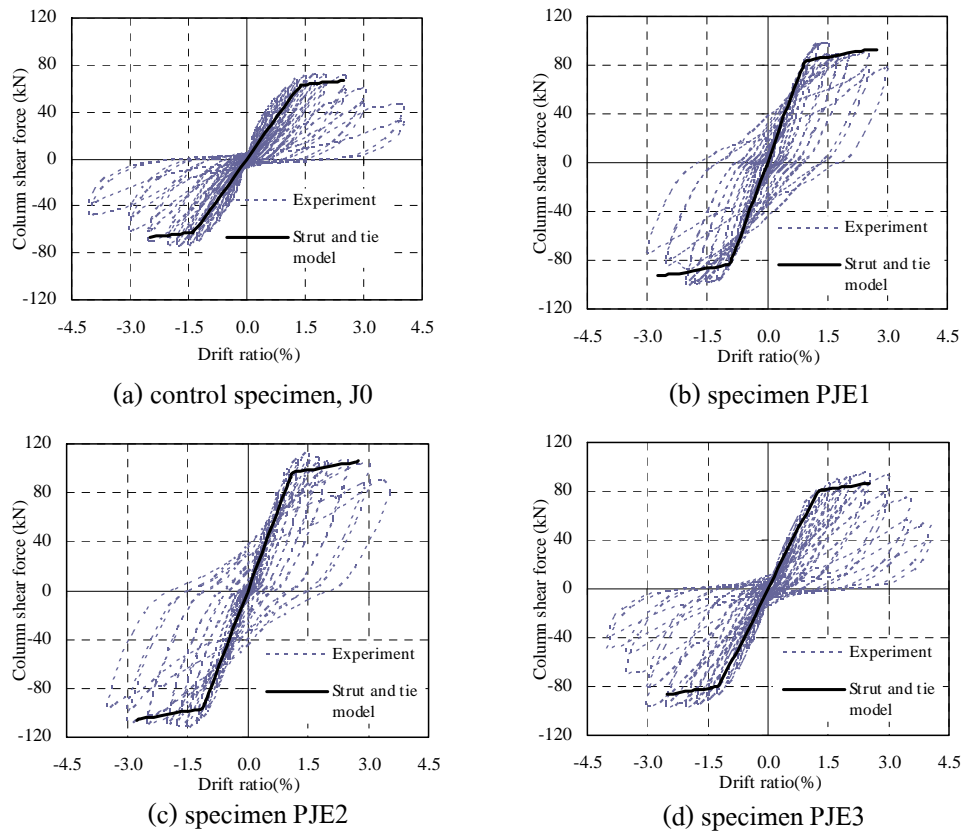


Figure 11 Relationship between column shear force and drift ratio for strut and tie model and experiment

3.2.3 Horizontal joint shear stress

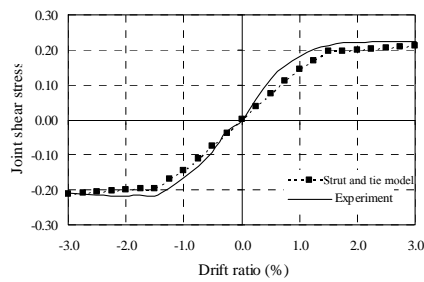
The horizontal joint shear stress (v_{jh}) is calculated from the horizontal joint shear force (V_{jh}) divided by the column core. The horizontal joint shear forces (V_{jh}) from strut and tie model is calculated as the summation of diagonal strut force in horizontal direction as shown in equation (4)

$$V_{jh} = D_1 \cos \theta_1 + D_2 \cos \theta_2 + D_3 \cos \theta_3 \quad (4)$$

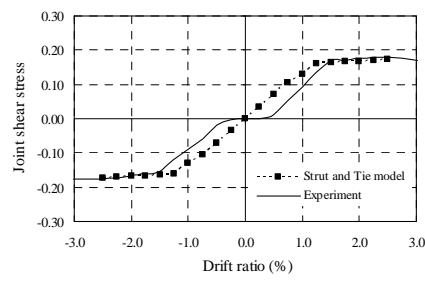
where $D_1 - D_3$ are main, steep and flat diagonal strut force, respectively. $\theta_1 - \theta_3$ are angle between diagonal strut and horizontal line for main, steep and flat strut, respectively. The experimental joint shear force is calculated using equation (5), where tension forces T and T' are calculated from measured strains of steel bars at opposite column faces via Kato's cyclic stress-strain model [16]. V_{cj} is column shear force

$$V_{jh} = T + T' - V_{cj} \quad (5)$$

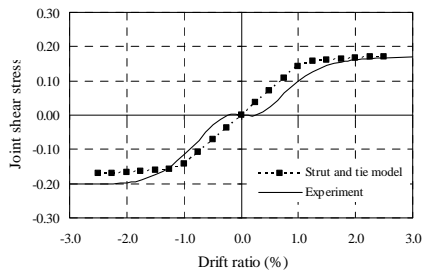
Figure12 shows horizontal joint shear stress (v_{jh}) versus drift ratio. The comparison is shown that strut and tie also predict close value as experiment. The maximum joint shear stress from model and experiment is $0.18f'_c$ for specimen PJE1 and PJE2 while it is $0.25f'_c$ for specimen PJE3. These values are according to the New Zealand Standard [17] that the joint shear stress should be below $0.2f'_c$ to avoid diagonal compression failure in joint. It can be observed that in the beginning of drift ratio for specimen PJE1-PJE3 the model predict a litter bit higher value of joint shear stress but in the final drift ratio it predicts as the same. Because; in the beginning, the effective of uncrack planar concretes exhibit good bond mechanism and share some part of tension forces from steel bars. But in the end after concrete crack most of tension force is mainly came from steel. Therefore, bond slip mechanism has to be developed in nonlinear link element.



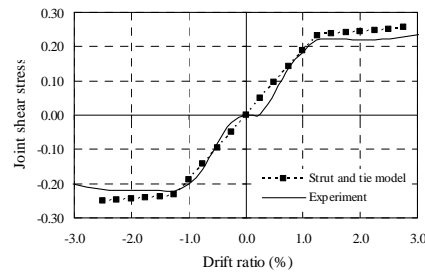
(a) control specimen, J0



(b) specimen PJE1



(c) specimen PJE2



(d) specimen PJE3

Figure 12 Relationship between joint shear stress and drift ratio

4. Conclusions

An application of nonlinear link with strut and tie model of seismic retrofit technique for existing reinforced concrete beam-column connections using planar joint expansion is proposed. Based on the verification of model, the following conclusions are drawn;

1. The proposed strut and tie model is constructed based on experimental results of three retrofitted specimens and FEM analysis which show flowing of stress field in joint region.
2. The non linear link elements are added in critical element to represent nonlinear mechanism of beam column joint retrofitted by planar joint expansion method.
3. By solving statically indeterminate planar truss model and check strut and tie limitation, the solution is received.
4. The model verify the same failure mode as experimental result. It also verify very close value in term of column shear force, envelop load and displacement relation, joint shear force and steel strain in beam reinforcement.

5. Recommendation

Although the proposed the proposed model predict the same failure mode and verify very close value with test result but it is a litter bit underestimate the load-carrying capacity. Because of the disadvantage of transverse element which is excluded the concrete shear resistant and bond slip effect in joint region is not included in the model. Modeling techniques that can address the shear resistance of concrete and bond slip have to be developed in further study.

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