GUIDELINES FOR APPLYING STATISTICAL PROCESS CONTROL METHOD TO MONITOR THE TEMPERATURE OF CERAMIC FURNACS

Karin Kandananond*

Received: August 22, 2012; Revised: January 11, 2013; Accepted: January 23, 2013

Abstract

The temperature control of a firing process is critical to the quality of any ceramic products. One of the temperature monitoring techniques is statistical process control or the utilization of control charts. However, the structure of observed data, autocorrelation, significantly affects the capability of the monitored processes. The selection of inappropriate types of control chart in this scenario might lead to an excessive number of false alarms or the loss of the ability to detect a special cause. As a result, a characterization of widely used control charts is performed in order to understand the characteristics of each control chart under autocorrelated situations. The results indicate that EWMA charts are more robust to false alarms than MR charts except when ϕ is between 0.6 and 1.0. Moreover, when there is an assignable cause, EWMA charts have also outperformed MR charts because of their sensitivity to a shift. To validate the empirical results, both charts are utilized to monitor a set of actual autocorrelated processes in the observed temperature of a ceramic furnace. In conclusion, the characterization has given practitioners opportunities to choose and utilize a suitable control chart to monitor the temperature of ceramic furnaces.

Keywords: Autocorrelation, autoregressive (AR), furnace, temperature

Introduction

Statistical process control (SPC) is a methodology used for monitoring and reducing variations in manufacturing processes and the main tools of SPC are control charts. Normally, SPC works under the assumption that the observed data is independent. However, because of advanced measurement technology, shortened sampling intervals, and the nature of processes, especially in continuous processes such as the temperature control of furnaces, the independence of each observation has been violated in many scenarios. The lack of independence for each sample always comes in the form of serial autocorrelation. This behavior of process outputs will significantly deteriorate the performance of control charts. The consequence is that a control chart will signal fault alarms more often or does not signal when there is a shift. Likewise, the monitoring process of

Faculty of Industrial Technology, Rajabhat University Valaya-Alongkorn, 1 Moo 20 Paholyothin Rd., Klong-Luang District, Prathumthani, 13180, Thailand.

* *Corresponding author*

ceramic furnaces also has a problem regarding the autocorrelation issue, as indicated in Bisgaard and Kulachi (2005). Hence, choosing the appropriate type of charts is critical for effectively monitoring the temperature of furnaces.

Literature Review

According to the literature, several authors have suggested different approaches to solve the autocorrelation issues of SPC. These options include non-standard SPC charts and some sophisticated techniques which are considered difficult for practitioners to implement in real-life situations. The objective of this study focuses on selecting the available quality tools that most practitioners are familiar with and which are simple for them to use. For this reason, the characterization of these tools under an autocorrelation situation should be fully understood. The first step leading to the performance characterization of standard charts is the capability to simulate the different types of autocorrelation. Under normal, uncorrelated conditions, process models have a fixed mean (μ), and the fluctuation around a mean is the result of white noise (a_t). However, when observations are correlated, the correlation structure and drift in the mean can be characterized by disturbances. If process observations vary around a fixed mean and have a constant variance, this type of variability is called stationary behaviour. Other wise, the behaviour is non-stationary. MacGregor (1998) indicated that there are 2 types of disturbances, deterministic and stochastic disturbances. Stochastic disturbances are random and might be stationary or non-stationary, so they are the main source of autocorrelation in the data. On the other hand, deterministic disturbances are a step shift or ramp in the process mean. Box *et al.* (1976) introduced a stochastic difference equation that can model stochastic disturbances. This equation is used to forecast 1-step ahead disturbances, according to whether the data characteristics are stationary or non-stationary. It is expressed in the form of an autoregressive integrated moving average model (ARIMA) as shown in Equation (1).

$$\Delta_d Y_t = \mu + \phi_1 \Delta_d Y_{t-1} + \phi_2 \Delta_d Y_{t-2} + \dots + \phi_p \Delta_d Y_{t-p} + a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q} \quad (1)$$

The ARIMA (p, d, q) model indicates p as the order of the autoregressive part, d as the amount of difference, and q as the order of the moving average part. As recommended by Box *et al.* (1976), ARIMA (1, 0, 0) and ARIMA (1, 1, 0) are likely to be the most suitable models to represent the stationary processes, while ARIMA (0, 1, 1) is the appropriate choice for the non-stationary processes. Therefore, several authors (Jiang *et al.*, 2000; MacCarthy and Wasusri, 2001; Noorossana *et al.*, 2003; Hwang *et al.*, 2008; Apley, 2012) utilized the ARIMA models to simulate both stationary and non-stationary processes.

Wardell *et al.* (1992) assessed the performance of traditional charts when the tested data was modeled by ARIMA (1, 1, 0). According to their work, the average run length (ARL) was used to measure the robustness of a designated chart. Moreover, the performance of each standard chart was also assessed by benchmarking with each other. One of these works is the performance comparison of $\overline{X}$ and exponentially weighted moving average (EWMA) charts by English *et al.* (2000).

Regarding the choice of control charts, MacCarthy and Wasusri (2002) made an interesting remark that the traditional chart (Shewhart) and the advanced charts, EWMA and cumulative sum charts, might have a better performance than non-standard charts when data was autocorrelated. Therefore, a number of works have been carried out to implement standard charts against correlated processes based on the ARIMA model. Their common objective is to assess the capability of each chart to detect a special cause when data is autocorrelated.

For their application, SPC charts are widely utilized in different industries, including ceramic manufacturing. In the ceramic industry, the SPC method is popularly applied to monitor the firing process, i.e., the temperature of ceramic furnaces. However, the application problem is that usually the temperature of ceramic furnaces is autocorrelated. As a result, the temperature control will be effective only if the right kind of control charts is correctly chosen (Bisgaard and Kulachi, 2005).

A number of studies have been introduced to solve this problem but the downside of these approaches is that they are complicated or are not user friendly for people on the shop floor. For this reason, the utilization of these techniques might not be suitable to ceramic factory practitioners who are familiar with the traditional standard charts. The objective of this study is the use of empirical study for characterizing the performance of standard charts under an autocorrelated situation so that practitioners will have guidelines for deploying the most appropriate charts from those available.

Materials and Methods

The autocorrelated temperature of furnaces is simulated by following the ARIMA model and the level of autocorrelation is controlled by adjusting the autoregressive (AR) parameter. The special cause is also added to the process and the moving range (MR) and EWMA charts are utilized to monitor the individual measurement of a process mean to detect a shift. The quality characteristic, the simulated temperature, is represented by Y. The evaluation of the control chart performance is measured by considering the ARL which is the average number of points plotted before a point indicated an out-of-control state. The observation of a process is considered from period 1 to 550 (t = 1, 2, 3,…, 550) and the process output (Y_{t+1}) is equal to

$$Y_{t+1} = T + N_{t+1}. \quad (2)$$

where T is the target temperature.

The source of autocorrelation is process disturbances, characterized by the AR moving average model, AR (1), as shown in Equation (3):

$$N_{t+1} = \phi N_t + a_{t+1}\,;\ -1 < \phi < 1, \quad (3)$$

where N_{t+1} and N_t are disturbances at time t+1 and t, respectively, a_{t+1} is a random error at time t+1, and ϕ is the AR parameter.

After an assignable cause occurs in the process, a shift of size δ_0 in the form of a step function is injected into a process at time t = 50 as:

$$\delta(t) = \begin{cases} 0; t < t_{50} \\ \delta_0; t \geq t_{50} \end{cases} \quad (4)$$

where δ_0 is the magnitude of a shift. The observations are monitored by a Shewhart MR chart and an EWMA chart. The control limits for a MR chart are

$$\left.\begin{aligned} UCL &= \overline{Y} + \overline{MR}/d_2 \\ CL &= \overline{Y} \\ LCL &= \overline{Y} - \overline{MR}/d_2 \end{aligned}\right\} \quad (5)$$

where $\overline{Y}$ is the process mean and equals $(\sum_{t=1}^{n} Y_t)/n$, $MR = |Y_t - Y_{t-1}|$, $\overline{MR}$ is the average of the moving average, and d_2 = 1.128 (the moving range of n = 2 observations).

For an EWMA chart, the control limits are expressed as:

$$\left.\begin{aligned} UCL &= \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} \\ CL &= \mu_0 \\ LCL &= \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} \end{aligned}\right\} \quad (6)$$

where μ_0 is the average of preliminary data, L is the width of control limits, and λ is the weight assigned to the observation. The values of L and λ used were recommended by Lucas and Saccucci (1990).

Performance Characterization

The experiment is based on the simulation of an ARIMA model, AR (1), processes. Regarding the simulation, each treatment is composed of 10000 iterations which are accomplished by using Palisade's @Risk® version 5.5. The random errors (a_t) from each period are simulated by following a normal distribution with a zero mean and a constant variance as: $a_t \sim N(0,1)$. The range of all the studied factors is shown in Table 1.

According to Table 1, the AR parameter gradually increases by 0.2 from -1 to 1 while the shift size is varied from 0 to 4 (0, 1, 2, 3 and 4).

Results

The temperature data is simulated and monitored by the SPC charts. The ARLs which indicate the characterization of the MR charts are shown in Table 2.

On the other hand, Table 3 illustrates the ARL results for the EWMA charts.

Table 1. Range of parameters for AR (1)

Factor	Low	High
AR parameter; ϕ	-1	1
Types of charts	MR	EWMA
Shift size	0	4

Table 2. ARL results for MR charts

ϕ	Shift	ARL	ϕ	Shift	ARL
-1.0	0	**84.44**	1.0	0	**62.55**
	1	**82.24**		1	**29.13**
	2	**81.97**		2	**8.00**
	3	**81.71**		3	**4.76**
	4	**81.10**		4	**4.03**
-0.8	0	**92.51**	0.8	0	**81.40**
	1	**94.72**		1	**74.61**
	2	**91.06**		2	**56.72**
	3	**85.47**		3	**27.89**
	4	**79.18**		4	**8.02**
-0.6	0	**65.33**	0.6	0	**48.87**
	1	**64.32**		1	**45.41**
	2	**63.70**		2	**33.78**
	3	**56.21**		3	**16.82**
	4	**47.46**		4	**4.97**
-0.4	0	**54.5**	0.4	0	**43.45**
	1	**52.91**		1	**40.52**
	2	**48.73**		2	**32.13**
	3	**41.9**		3	**19.24**
	4	**31.93**		4	**7.55**
-0.2	0	**46.59**	0.2	0	**42.44**
	1	**45.95**		1	**40.68**
	2	**41.11**		2	**33.37**
	3	**33.51**		3	**22.72**
	4	**21.59**		4	**10.35**

Table 2. ARL results for MR charts (Continued)

ϕ	Shift	ARL	ϕ	Shift	ARL
0	0	**43.87**			
	1	**41.97**			
	2	**36.97**			
	3	**26.41**			
	4	**12.07**			

Table 3. ARL results for EWMA charts

ϕ	Shift	ARL	ϕ	Shift	ARL
-1.0	0	**104.36**	1.0	0	**5.13**
	1	**16.67**		1	**2.21**
	2	**7.07**		2	**1.85**
	3	**4.58**		3	**1.44**
	4	**3.43**		4	**1.03**
-0.8	0	**474.29**	0.8	0	**10.16**
	1	**24.54**		1	**4.03**
	2	**8.98**		2	**2.74**
	3	**5.58**		3	**2.28**
	4	**4.03**		4	**2.01**
-0.6	0	**456.34**	0.6	0	**25.03**
	1	**21.12**		1	**5.80**
	2	**8.09**		2	**3.41**
	3	**5.07**		3	**2.63**
	4	**3.71**		4	**2.22**
-0.4	0	**411**	0.4	0	**54.18**
	1	**17.53**		1	**7.47**
	2	**7.09**		2	**3.89**
	3	**4.51**		3	**2.85**
	4	**3.35**		4	**2.34**
-0.2	0	**334.89**	0.2	0	**115.8**
	1	**14.4**		1	**9.44**
	2	**6.12**		2	**4.49**
	3	**3.97**		3	**3.12**
	4	**2.99**		4	**2.48**
0	0	**219.69**			
	1	**11.67**			
	2	**5.24**			
	3	**3.50**			
	4	**2.69**			

According to Tables 2 and 3, when assignable causes exist in the process, the EWMA charts are able to detect shifts much faster than the MR charts (the ARL_1 of the EWMA charts is lower than those of the MR charts). Moreover, the empirical study also reveals that the EWMA charts are more robust to the autocorrelation structure of data (the ARL_0 of the MR charts being lower than those of the EWMA charts) than the MR charts when there is no shift in the process. However, this above conclusion might not be holistic since the ARL_0 of the MR charts is much higher than those of the EWMA charts when ϕ equals 0.6, 0.8 and 1.0.

Result Validation

The research results are examined by applying the different types of control charts. The sample set of autocorrelated processes is the actual temperature of a ceramic furnace (adopted from Bisgaard and Kulachi (2005)) as shown in the time series plot (Figure 1).

Due to the autocorrelation function (ACF) plot in Figure 2, the AR (1) process seems to be an appropriate model for the process. The coefficients of the model are calculated using a statistical package, Minitab, and the value of ϕ is equal to 0.7273. Therefore, the AR(1) model for this set of data is $Y_{t+1} = 430.805+0.7273*Y_t+a_{t+1}$. Due to the simulated results, when the AR (1) coefficient is between 0.6 and 0.8 and there is no shift, the MR charts are a suitable choice for the SPC method. After the MR and EWMA charts are deployed to monitor the temperature of furnaces, the number of points located outside the control limits for the MR charts is significantly lower than the EWMA charts as shown in Figures 3 and 4. Therefore, the simulated results are validated after applying each chart to the actual case study.

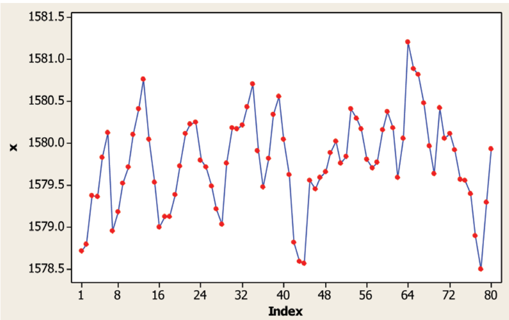

Figure 1. Time series plot of furnace temperature

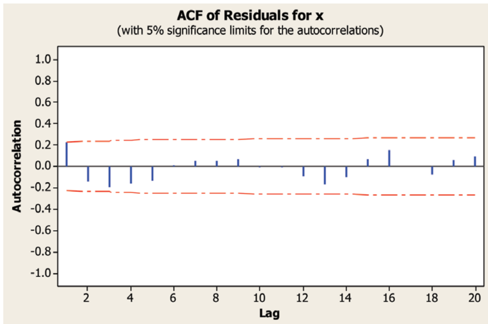

Figure 2. ACF plot of residual

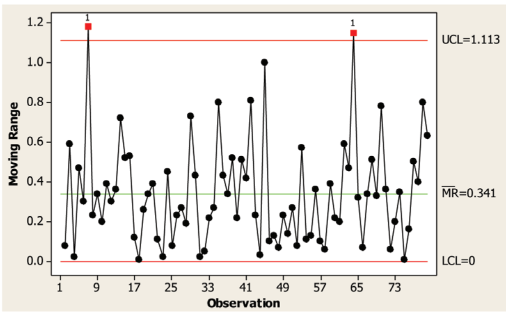

Figure 3. MR chart for the temperature

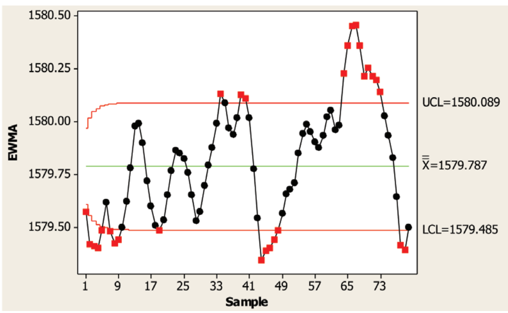

Figure 4. EWMA chart for the temperature

Conclusions

The monitoring of furnace temperature is a critical problem because of its autocorrelation nature. In this study, the available control charts (MR and EWMA charts) are assessed to characterize the performance of each chart under the autocorrelated situation. According to the empirical results, when assignable causes exist in the processes, the EWMA charts are able to detect shifts much faster than the MR charts. Another finding is that the EWMA charts are also more robust to the autocorrelation (in case that there is no shift) than the MR charts, but only when ϕ is between -1 and 0.6.

References

Apley, D.W. (2012). Design of exponentially weighted moving average control charts for autocorrelated processes with model uncertainty. Tehcnometrics, 45(3):187-198.

Bisgaard, S. and Kulachi, M. (2005). Quality quandaries: the effect of autocorrelation on statistical process control procedures. Quality Engineering, 17(3):481-489.

Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1976). Time Series Analysis: Forecasting and Control. John Wiley & Sons, NY, USA, 784p.

English, J.R., Lee, S., Martin, T.W., and Tilmon, C. (2000) Detecting changes in autoregressive processes with $\overline{X}$ and EWMA charts. IIE Transactions, 32(12):1103-1113.

Hwang, S., Chen, H., and Chang, C. (2008). An exponentially weighted moving average method for identification and monitoring of stochastic systems. Industrial Engineering and Chemical Research, 47(21):8239-8249.

Jiang, W., Tsui, K.L., and Woodall, W.H. (2000). A new SPC monitoring method: the ARMA chart. Technometrics, 42(4):399-410.

Lucas, J.M. and Saccucci, M.S. (1990). Exponentially weighted moving average control schemes: properties and enhancements. Technometrics, 32(1):1-12.

MacCarthy, B.L. and Wasusri, T. (2001). Statistical process control for monitoring scheduling performance-addressing the problem of correlated data. J. Oper. Res. Soc., 52(7):810-820.

MacCarthy, B.L. and Wasusri, T. (2002). A review of non-standard applications of statistical process control (SPC) charts. Int. J. Qual. Reliab. Manage., 19(3):295-320.

MacGregor, J.F. (1998). Online statistical process control. Chem. Eng. Prog., 84(10):21-31.

Noorossana, R., Farrokhi, M., and Saghaei, A. (2003). Using neural network to detect and classify out-of-control signals in autocorrelated processes. Qual. Reliab. Eng. Int., 19(6):493-504.

Wardell, D.G., Moskowitz, H., and Plante, R.D. (1992). Control charts in the presence of data correlation. Manage. Sci., 38(8): 1084-1105.