

A Four-quadrant Analog Multiplier Based on Switched-capacitor and Pulse-Width Amplitude Modulation Techniques

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Abstract

This article proposes a Four-Quadrant Analog Multiplier (4-QAM) applying switched-capacitor and pulse-width amplitude modulation (PWAM) principles. The features of the presented circuit are that it can function as analog multiplier with a wide dynamic range of input signal and no disturbing from deviation of carrier frequency of PWM signal. In addition, the circuit detail is simpler than that of the previously proposed circuits. It is then easy and applicable for employing it into Integrated Circuit (IC) realization to especially operate in low-frequency and low-power applications. The experimental results granted are in correspondence to the theoretical analysis.

1. Introduction

Analog multipliers are composed of the essential circuit elements of analog signal processing systems. For example, in waveform generators, power transducers, automatic gain controllers, including modulators, frequency doublers, and precision rectifiers. There are differently several methods for engendering the analog multiplying function, e.g., the analog multiplier based on translinear characteristics of BJTs [1], Quarter-square characteristics [2], logarithmic function [3], switched-capacitor [4-5] and PWAM or Pulse-width modulation (PWM) [6-9].

They have some different features and disadvantages among the previous approaches. The analog multipliers based on the bipolar translinear,

the quarter-square and the log-antilog function principles have some advantages in such they can operate with a high frequency response and simplicity of the circuit description. However, they can not allow a wide dynamic range of the input signal. In addition, temperature-dependence of semiconductor devices and matched transistors are involved. While the analog multiplier using the switched-capacitor principle, although it can support a relatively wider dynamic range of the input signal and the temperature-sensitivity of circuit performance is less than the first three principles. But it is unavoidable to employ variously different phase of pulse trains for activating appropriate switches which is rather difficult and complicated for realization.

For the analog multiplier utilizing PWAM or PWM principle, it has some interests in similar to that of the switched-capacitor. However, it has also some disadvantages owing to its complicated circuit details. Moreover, circuit calibration is required when carrier frequency is adjusted to completely work as 4-QAM.

The goal of this paper is then to introduce a 4-QAM based on PWM principle to generate the pulse trains from an input signal, the pulse trains are then very readily generated. The pulse trains are applied to activate the appropriate CMOS switches of the switched-capacitor section serving on PWAM principle. The circuit combination is uncomplicated. It can function as analog multiplier against the

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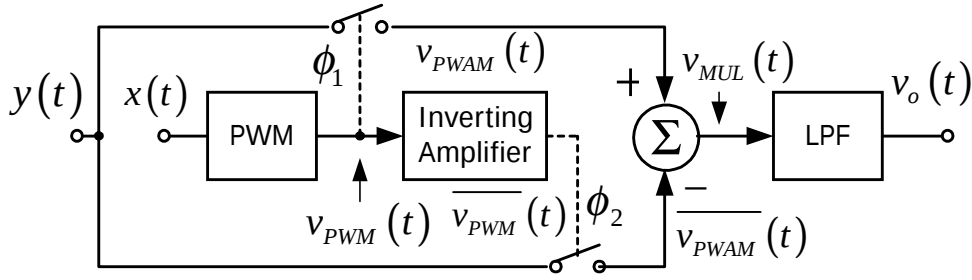


Fig. 1 The principle of proposed 4-QAM

relatively wide dynamic range of input voltage. Another important benefit of the proposed circuit is that no effect from the PWM carrier frequency deviations, it is so unnecessary to calibrate a parameter of the circuit in the case of its variations. It is then easy and suitable for both assembling the circuit and developing it into IC realization. The experimental results are also exhibited to insist on the ability of circuit performance.

2. Principle

2.1 The Proposed 4-QAM Principle

Fig. 1 shows block diagram of the principle of proposed 4-QAM. An input signal being $x(t)$ is modulated by PWM method. The output signal; $v_{PWM}(t)$ is later changed the polarity by inverting amplifier to achieve $\overline{v_{PWM}}(t)$. The pulse trains of $v_{PWM}(t)$ and $\overline{v_{PWM}}(t)$ are used to activate analog switches in the sample and hold section of another input signal; $y(t)$. The results are respectively $v_{PWAM}(t)$ and $\overline{v_{PWAM}}(t)$, the subtracted result of $v_{PWAM}(t)$ and $\overline{v_{PWAM}}(t)$; $v_{MUL}(t)$ is applied to low pass filter (LPF). The multiplied result of the input signal can be achieved at the output of LPF.

To understand the proposed principle, let firstly consider the $v_{PWM}(t)$ which can be easily defined by [7-8, 10]

$$v_{PWM}(t) = \frac{A}{T} [\beta T + \beta T x(t)] + Q_1(t) \quad (1)$$

Whereas A is the carrier amplitude, β is a modulation index of the PWM signal and T is a period of the

PWM signal. While

$$Q_1(t) = \frac{A}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\omega_o t)}{\sin[n\omega_o(t + (1+x(t))\beta T)]} \right\} \quad (2)$$

It is clearly seen that $Q_1(t)$ represents the usual high frequency terms in the PWM signal.

After that the $v_{PWM}(t)$ is utilized to sample the $y(t)$, the $v_{PWAM}(t)$ is obtained as

$$v_{PWAM}(t) = y(t)v_{PWM}(t)$$

$$v_{PWAM}(t) = \frac{A}{T} y(t) [\beta T + \beta T x(t)] + y(t) Q_1(t) \quad (3)$$

Similarly, the $\overline{v_{PWAM}}(t)$ can be derived by

$$\overline{v_{PWAM}}(t) = \frac{A}{T} [(1-\beta)T - \beta T x(t)] + Q_2(t) \quad (4)$$

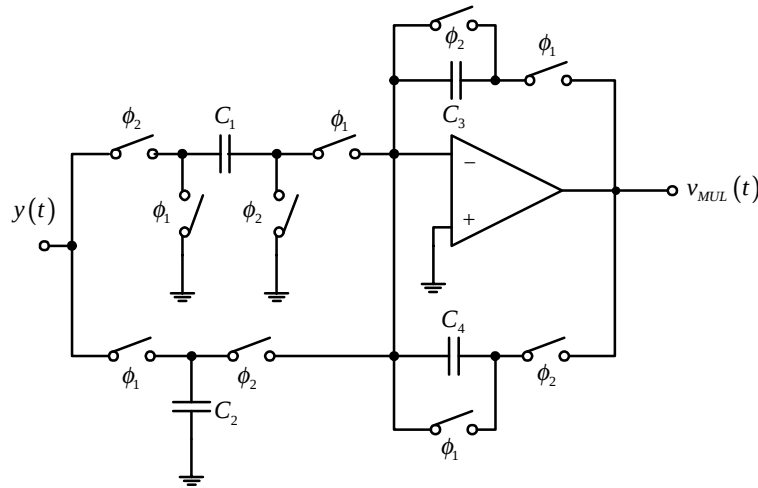
Where

$$Q_2(t) = \frac{A}{\pi} \sum_{n=1}^{\infty} \left\{ \frac{\sin(n\omega_o t)}{\sin[n\omega_o(t + (1-x(t))\beta T)]} \right\} \quad (5)$$

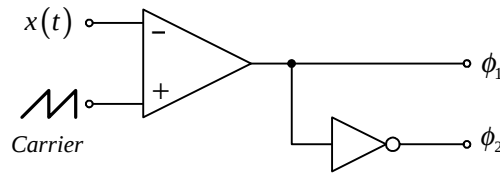
When the $y(t)$ is sampled by the $\overline{v_{PWAM}}(t)$, it yields

$$\overline{v_{PWAM}}(t) = \frac{A}{T} y(t) [(1-\beta)T - \beta T x(t)] + y(t) Q_2(t) \quad (6)$$

The $v_{MUL}(t)$, subtracted result of $v_{PWAM}(t)$ and $\overline{v_{PWAM}}(t)$, can be written as follows



(a) The proposed 4-QAM circuit



(b) The signal generator of PWM pulse trains

Fig. 2 The completely proposed 4-QAM

$$v_{MUL}(t) = Ay(t)(2\beta - 1) + 2A\beta x(t)y(t) + y(t)[Q_1(t) - Q_2(t)] \quad (7)$$

From eqn.(7), if the β is traditionally 0.5, the first term of the right-handed side is disappeared whereas the third term represents the fundamental of high-frequency terms which can be eliminated by using LPF [11]. Thus

$$v_o(t) = Ax(t)y(t) \quad (8)$$

It means that, the multiplier output can be received, whereas A is the multiplying constant value and independent of the carrier frequency.

2.2 The Proposed 4-QAM Circuit

From the principle of the proposed method, the switched-capacitor is applied to reduce the circuit details, it simultaneously functions as sample and hold including subtraction circuit. The completely

proposed 4-QAM is demonstrated in Fig. 2. Circuit operation can be seen by analysis each iteration step.

- ϕ_1 is activated

When ϕ_1 is triggered, the equivalent circuit of proposed 4-QAM is illustrated in Fig. 3(a). In this condition, the is sampled by ϕ_1 and held by capacitor C_2 . The charge at C_2 is given by

$$\Delta q_{C2} = C_2 Y_{S1} \quad (9)$$

Where Y_{S1} represents the instantaneous value of $y(t)$ at sampled time of ϕ_1 .

- ϕ_2 is activated

In this time, shown in Fig. 3(b), the $y(t)$ is sampled by ϕ_2 and held by capacitor C_1 , voltage crossing the C_1 becomes Y_{S2} representing the instantaneous value of $y(t)$ at sampled time of ϕ_2 . Simultaneously, the capacitor C_2 is discharged through

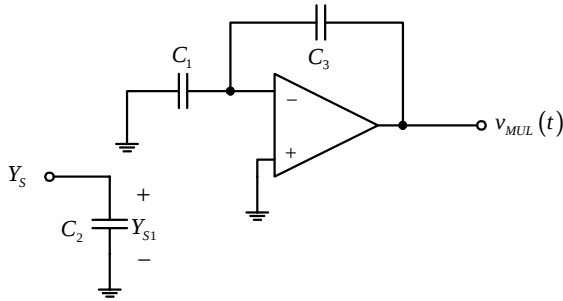
capacitor C_4 . Thus

$$v_{MUL}(t) = -Y_{S1} \quad (10)$$

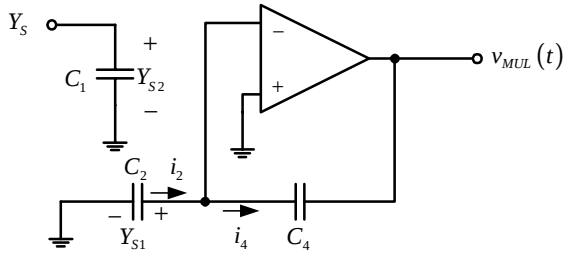
It is seen that, when ϕ_2 is activated, we will get the instantaneously negative value of $y(t)$.

- ϕ_1 is again activated

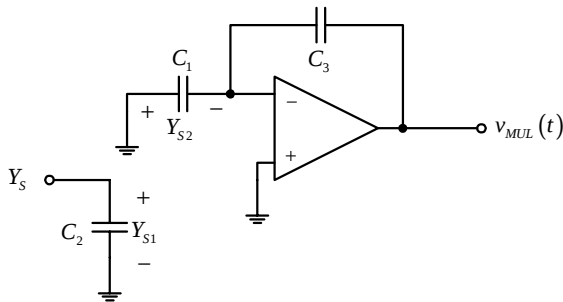
The C_2 charges again followed by Eqn. (9) in this period shown in Fig. 3(c). Simultaneously, the C_1 is discharged through capacitor C_1 with alternating ground. Therefore



(a) When ϕ_1 is activated

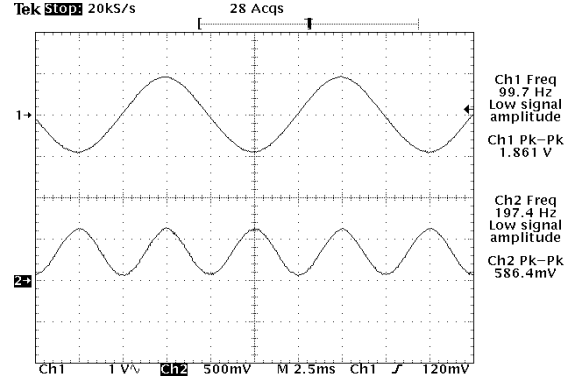


(b) When ϕ_2 is activated

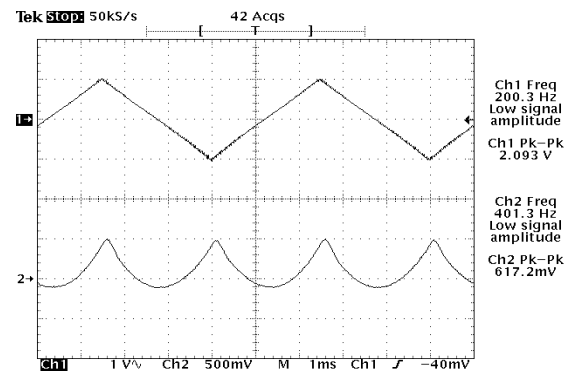


(c) When ϕ_1 is again activated

Fig. 3 The equivalent circuit of proposed 4-QAM



(a) Sinusoidal input



(b) Triangular input

Fig. 4 The multiplier outputs for different input waveforms

$$v_{MUL}(t) = Y_{S2} \quad (11)$$

When ϕ_1 is again activated, it is clear that, the instantaneously positive value of $y(t)$ can be obtained. It is then concluded that the switched-capacitor circuit performs as sample and hold together with subtraction circuit followed by the proposed principle shown in Fig.1. In addition, we can see that the proposed circuit also gives a fast response time.

3. The Experimental Results

The proposed circuit of Fig. 2 has been tested using experiments. The experiments were set up by utilizing LF351 as op-amp, CD4066s as analog switches and all of capacitors identical to $0.01\mu F$. The circuit was operated by $\pm 5V$ power supply voltage with

carrier frequency of 30kHz and 2V amplitude. The output signal of the proposed circuit is filtered by using second order LPF with 3kHz cutoff-frequency. The first experimental results shown in Fig. 4(a)-(b) are multiplying results of the same signals in which are the sinusoidal and triangular signal of 100Hz frequency, respectively.

Another test for multiplying two sinusoidal signals called as double-sideband modulation of 100Hz with 1kHz frequency is shown in Fig. 5. In addition, the static characteristics of the proposed multiplier are also shown in Fig. 6, it obviously confirms that the proposed multiplier can precisely function as 4-QAM with a wide dynamic range of input signal. Fig. 7 illustrates experimental results of THD against carrier frequency adjustment.

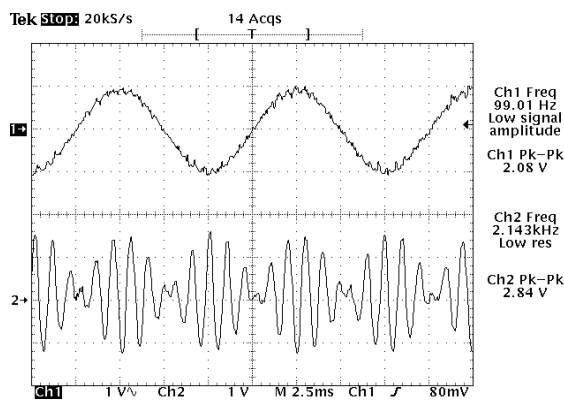


Fig. 5 Product of two sinusoids with different frequencies: 100Hz and 1kHz

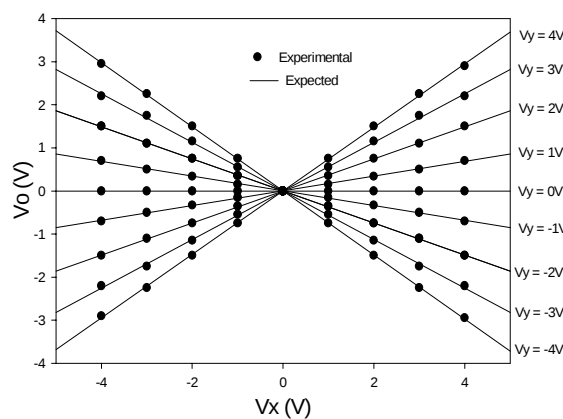


Fig. 6 Static characteristics of the proposed multiplier

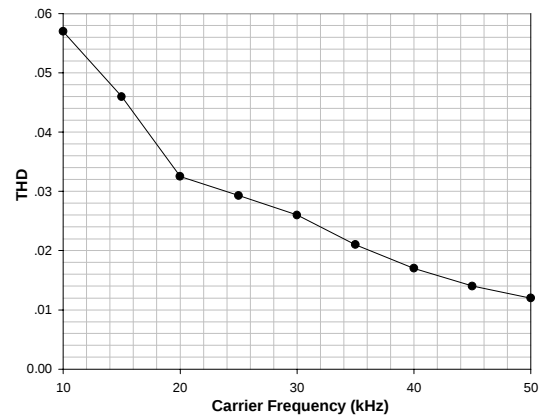


Fig. 7 Experimental results of THD against carrier frequency adjustment

4. Conclusions

The 4-QAM using switched-capacitor and PWAM techniques has been exhibited. It can function as analog multiplier with a wide dynamic range of input signal and no disturbing from deviation of carrier frequency of PWM signal. The circuit simplicity is then easy and applicable for employing it into Integrated Circuit (IC) realization to particularly operate in low-frequency and low-power application. The experimental results, using readily available components, are in correspondence to the prediction of circuit performance.

5. Acknowledgement

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