A Probabilistic Approach for Cooling Load Calculation

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Abstract

This paper demonstrates how probability can be used as a decision tool for justifying the appropriate results from the cooling load calculation. The ventilation heat gain, as one component in cooling load, is used to demonstrate the method presented in this paper. The probability for each amount of ventilation heat gain under various design conditions is calculated. The results of ventilation heat gain in the studied case vary from 436 to 593 kW and depend on what design condition is used. By using 98 % of cumulative density function, it indicates that the amount of ventilation heat gain is 537 kW. Using this probability information one can now logically decide for an appropriate amount of ventilation heat gain used and hence cooling load amount. This will lead to efficiency in energy management as well as reducing the risk in air conditioning system investment.

Keywords: cooling load, uncertainty, probability, ventilation heat gain

1. Introduction

Cooling load is the rate of heat which must be removed from a space to maintain a specific space air temperature and moisture content. The parameters affecting cooling load calculations are numerous, for example, the outside air temperature, the humidity ratio, the number and activity of people, etc. These parameters are often difficult to precisely define and always intricately interrelated. Many cooling load components vary in magnitude over a wide range during a 24 hr period. These cyclic changes in load components are not often in phase with each other. Each must be analyzed to establish the maximum cooling load for a building or zone. Moreover, effects of thermal accumulation also affect calculating procedure. Therefore various models and assumptions are developed. [The estimated results at the specific time of calculation are normally expected and not the exact ones.]

Referring to or using difference values of parameters at the same specific time of calculation will result in different outcomes of the calculation. Most of the reference data for the parameters used in calculation are from the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) standards which are obtained from data collection and experiments. However, the standards do not present the probability and variance of those data as well as the chances that other data apart from standard data might occur. ASHRAE standards define the design conditions in the form of maximum temperature for dry-bulb and wet-bulb temperature. However, cooling load calculations depend not only on temperature but also on other parameters such as lighting or occupants. These parameters sometimes are uncertain and in some cases it is difficult to find exact information. This leads to uncertainty in the calculation result. Therefore, a safety factor is always involved in the final phase of cooling load calculations.

2. Probabilistic Approach

In order to attack this uncertainty problem in the cooling load calculation, we must first categorize parameters that affect the cooling load. They can be divided into 2 types, i.e. uncontrollable and controllable parameters. Uncontrollable parameters, such as the outside air temperature, affect the change in the probability of cooling load occurrence while controllable parameters, such as the type and shading coefficient of glass, do not affect the change in the probability of cooling load occurrence. Conventionally the standard values of uncontrollable parameters that related to climate such as outside air temperature are obtained from data collection with statistical analysis. For non-climate type uncontrollable parameters, such as lighting, the values are obtained from laboratory experiment for a specific condition. Therefore in each case we are restricted to only one standard value for each parameter considered for the calculation. Under the new concept, all data values must be considered along with a probability density function that indicates the frequencies of occurrence, not just the standard ones. The cooling load received from the calculation will contain its own probability to indicate the chance for that specific cooling load to occur. Therefore, specifying the amount of cooling load used can be logically decided. This will help reduce the risk in air conditioning system investment.

3. Cooling Load Calculation

Heat gains that enter into or are generated in a space are external heat gain, internal heat gain and ventilation heat gain. Only ventilation heat gain is considered in this paper to demonstrate the application of probabilistic in cooling load calculations.

Heat gain from ventilation is important for areas that need a high ventilation rate, for example restaurants, theaters, etc. Ventilation is used to maintain indoor air quality to standard conditions. The ventilation is used to maintain indoor air quality and affects the thermal comfort condition in the buildings.

Ventilation heat gain calculations are normally referred to an ASHRAE standard. The maximum dry-bulb temperature is used to calculate maximum sensible heat gain. At the maximum wet-bulb temperature, if the dry-bulb temperature is high, the latent heat gain may decrease. On the contrary, at the maximum humidity ratio, the latent heat gain is always the maximum. Moreover, the maximum humidity ratio value does not have the same conditions as the maximum wet-bulb temperature. This indicates that the maximum ventilation heat gain can occurr at any point. Therefore, in order to determine the probability of ventilation as well as its maximum heat gain value at any condition, both the probability of dry-bulb temperature and the probability of humidity ratio must be used.

4. Probabilistic Approach for Ventilation Heat Gain Calculation

For constant $\rho, \dot{Q}, c_p$ *and* T_i, the sensible heat gain is a function of outside dry-bulb temperature and is given by:

$$\dot{q}_{vent,sen,\theta} = f_{vs}(T_{o,\theta}) = \rho\dot{Q}c_p(T_{o,\theta} - T_i) \quad (1)$$

or

$$T_{o,\theta} = f_{vs}^{-1}(\dot{q}_{vent,sen,\theta}) = \frac{\dot{q}_{vent,sen,\theta}}{\rho\dot{Q}c_p} + T_i \quad (2)$$

where

c_p = specific heat of air
ρ = air density
$\dot{Q}$ = air volume flow rate
$T_{o,\theta}$ = outside temperature
T_i = inside temperature

If the probability density function (PDF) of the outside temperature is $f_T(T_{o,\theta})$, then by definition the cumulative density function (CDF) is:

$$F_T(T_{o,\theta}) = \int_{-\infty}^{T_{o,\theta}} f_T(T_{o,\theta})\, dT_{o,\theta} \quad (3)$$

The probability density function of sensible heat gain can be obtained by using the equivalent of cumulative density function on range $\dot{q}_{vent,sen,\theta}$ and domain $T_{o,\theta}$ and is given by:

$$f_{qvs}(\dot{q}_{vent,sen,\theta}) = \frac{1}{\rho\dot{Q}c_p} f_T\left(\frac{\dot{q}_{vent,sen,\theta}}{\rho\dot{Q}c_p} + T_i\right) \quad (4)$$

Latent heat gain is given by:

$$\dot{q}_{vent,lat,\theta} = \rho\dot{Q}h_{fg}(w_{o,\theta} - w_i) \qquad (5)$$

where

h_{fg} = enthalpy of saturated water
$w_{o,\theta}$ = outside humidity ratio
w_i = inside humidity ratio

If $\rho, \dot{Q}, h_{fg}$ *and* w_i are constant then the latent heat gain is a function of outside humidity ratio:

$$\dot{q}_{vent,lat,\theta} = f_{vl}(w_{o,\theta}) \qquad (6)$$

or

$$w_{o,\theta} = f_{vl}^{-1}(\dot{q}_{vent,lat,\theta}) = \frac{\dot{q}_{vent,lat,\theta}}{\rho\dot{Q}h_{fg}} + w_i \qquad (7)$$

If the probability density function (PDF) of humidity ratio is $f_w(w_{o,\theta})$, then by definition the Cumulative density function (CDF) is:

$$F_w(w_{o,\theta}) = \int_{-\infty}^{w_{o,\theta}} f_w(w_{o,\theta})\,dw_{o,\theta} \qquad (8)$$

The probability density function of latent heat gain can be obtained by using the equivalent of cumulative density function on range $\dot{q}_{vent,lat,\theta}$ and domain $w_{o,\theta}$ and is given by:

$$f_{qvl}(\dot{q}_{vent,lat,\theta}) = \frac{1}{\rho\dot{Q}h_{fg}} f_w\left(\frac{\dot{q}_{vent,lat,\theta}}{\rho\dot{Q}h_{fg}} + w_i\right) \qquad (8)$$

The total ventilation heat gain ($\dot{q}_{vent,\theta}$) is the combination of sensible heat gain and latent heat gain.

$$\begin{aligned}\dot{q}_{vent,\theta} &= \rho\dot{Q}c_p(T_{o,\theta} - T_i) \\ &\quad + \rho\dot{Q}h_{fg}(w_{o,\theta} - w_i)\end{aligned} \qquad (9)$$

In order to obtain the probability density function of total ventilation heat gain, we must first find the probability of relative probability density function of sensible heat gain and latent heat gain; $f_{qvsl}(\dot{q}_{vent,sen,\theta}, \dot{q}_{vent,lat,\theta})$ .This relative function can be obtained by using the equivalent of cumulative density function on range $(\dot{q}_{vent,sen,\theta}, \dot{q}_{vent,lat,\theta})$ and domain $(T_{o,\theta}, w_{o,\theta})$ and is given by:

$$\begin{aligned}f_{qvsl}(\dot{q}_{vent,sen,\theta}, \dot{q}_{vent,lat,\theta}) &= \frac{1}{\rho^2\dot{Q}^2c_ph_{fg}} \\ f_{Tw}&\left(\frac{\dot{q}_{vent,sen,\theta}}{\rho\dot{Q}c_p} + T_i, \frac{\dot{q}_{vent,lat,\theta}}{\rho\dot{Q}h_{fg}} + w_i\right)\end{aligned} \qquad (10)$$

The probability density function of total ventilation heat gain can be found from:

$$f_{qv}(\dot{q}_{vent,\theta}) = \int_{-\infty}^{\infty} f_{qvsl}(\dot{q}_{vent,sen,\theta}, \dot{q}_{vent,lat,\theta})\,d\dot{q}_{vent,sen,\theta} \qquad (11)$$

or

$$\begin{aligned}f_{qv}(\dot{q}_{vent,\theta}) &= \int_{-\infty}^{\infty} \frac{1}{\rho^2\dot{Q}^2c_ph_{fg}} \\ f_{Tw}&\left(\frac{\dot{q}_{vent,sen,\theta}}{\rho\dot{Q}c_p} + T_i, \frac{(\dot{q}_{vent,\theta} - \dot{q}_{vent,sen,\theta})}{\rho\dot{Q}h_{fg}} + w_i\right)d\dot{q}_{vent,sen,\theta}\end{aligned} \qquad (12)$$

If we consider that the outside temperature and the humidity ratio are independent, which is normally the assumption for the calculation, then the probability density function of relative probability density function of sensible heat and latent heat gain is given by:

$$\begin{aligned}f_{qvsl}(\dot{q}_{vent,sen,\theta}, \dot{q}_{vent,lat,\theta}) &= \\ f_{qvs}(\dot{q}_{vent,sen,\theta})&f_{qvl}(\dot{q}_{vent,lat,\theta})\end{aligned} \qquad (13)$$

and the probability density function of total ventilation heat gain is:

$$\begin{aligned}f_{qv}(\dot{q}_{vent,\theta}) &= \int_{-\infty}^{\infty} f_{qvsl}(\dot{q}_{vent,sen,\theta}, \dot{q}_{vent,lat,\theta})\,d\dot{q}_{vent,sen,\theta} \\ &= \int_{-\infty}^{\infty} f_{qvs}(\dot{q}_{vent,sen,\theta})f_{qvl}(\dot{q}_{vent,lat,\theta})\,d\dot{q}_{vent,sen,\theta}\end{aligned}$$

$$f_{qv}(\dot{q}_{vent,\theta}) = \frac{1}{\rho^2 \dot{Q}^2 c_p h_{fg}} \int_{-\infty}^{\infty} f_T\left(\frac{\dot{q}_{vent,sen,\theta}}{\rho \dot{Q} c_p} + T_i\right) f_w\left(\frac{(\dot{q}_{vent,\theta} - \dot{q}_{vent,sen,\theta})}{\rho \dot{Q} h_{fg}} + w_i\right) d\dot{q}_{vent,sen,\theta} \tag{14}$$

5. Case Study

The following case studies are used to illustrate the usefulness of the new concept to consider the ventilation heat gain presented in this paper.

Given data: A building in Bangkok, Thailand requires ventilation rate $\dot{Q} = 10\ m^3/s$, inside temperature $T_i = 24\ ^\circ C$ and inside relative humidity ratio $Rh_i = 50\ \%$. Determine the appropriate amount of ventilation heat gain.

$c_p = 1000\ J/kg \cdot K$

$h_{fg} = 2500\ kJ/kg$

$\rho = 1.23\ kg/m^3$

Data obtained from the Meteorological department of Thailand at Bangkok station during year 2001-2003 are shown in Table 1:

Table 1 Temperature Data

	Mean	Variance
Dry-bulb temperature (T_{db})	$29.1^\circ C$	$8.12^\circ C^2$
Wet-bulb temperature (T_{wb})	$25.2^\circ C$	$4.13^\circ C^2$
Humidity ratio (w)	0.0187	0.00000681

Note: Humidity ratio is calculated from dry-bulb and wet-bulb temperature data.

Solution

Condition 1: Consider using a dry-bulb temperature at 98.0% design condition and mean coincident wet-bulb temperature that indicate maximum sensible heat gain.

The dry-bulb temperature at 98.0% design condition is $35.0\ ^\circ C$. The mean coincident wet-bulb temperature is $26.8\ ^\circ C$. For $T_{db} = 24\ ^\circ C$ and $Rh = 50\%$, the humidity ratio is $w = 0.0092$

Therefore:

$$\dot{q}_{vent,\theta} = \rho \dot{Q} c_p (T_{o,\theta} - T_i) + \rho \dot{Q} h_{fg} (w_{o,\theta} - w_i)$$

$$\dot{q}_{vent,\theta} = 1.23 \cdot 10 \cdot 1.0 \cdot (35.0 - 24) + 1.23 \cdot 10 \cdot 2500 \cdot (0.0190 - 0.0092)$$

and

$$\dot{q}_{vent,\theta} = 436.65\ kW$$

Condition 2.1: Consider using a wet-bulb temperature at 98.0% design condition and mean coincident dry-bulb temperature that indicate maximum latent heat gain.

The wet-bulb temperature at 98.0% design condition is $29.3\ ^\circ C$. The mean coincident dry-bulb temperature is $33.1\ ^\circ C$. For $T_{wb} = 29.3\ ^\circ C$ and $T_{db} = 33.1^\circ C$, the humidity ratio is $w = 0.0244$.

Therefore:

$$\dot{q}_{vent,\theta} = \rho \dot{Q} c_p (T_{o,\theta} - T_i) + \rho \dot{Q} h_{fg} (w_{o,\theta} - w_i)$$

$$\dot{q}_{vent,\theta} = 1.23 \cdot 10 \cdot 1.0 \cdot (33.1 - 24) + 1.23 \cdot 10 \cdot 2500 \cdot (0.0244 - 0.0092)$$

and

$$\dot{q}_{vent,\theta} = 579.33\ kW$$

Condition 2.2: Consider using a humidity ratio at 98.0% design condition and mean coincident dry-bulb temperature that indicate maximum latent heat gain.

The humidity ratio at 98.0 % design condition is 0.0241. Mean coincident dry-bulb temperature is $29.9\ ^\circ C$.

Therefore:

$$\dot{q}_{vent,\theta} = \rho \dot{Q} c_p (T_{o,\theta} - T_i) + \rho \dot{Q} h_{fg} (w_{o,\theta} - w_i)$$

$$\dot{q}_{vent,\theta} = 1.23 \cdot 10 \cdot 1.0 \cdot (29.9 - 24) + 1.23 \cdot 10 \cdot 2500 \cdot (0.0241 - 0.0092)$$

and

$$\dot{q}_{vent,\theta} = 530.75\ kW$$

Condition 3: Consider using a both dry-bulb temperature at 98.0% design condition and humidity ratio at 98.0% design condition. Therefore:

$$\dot{q}_{vent,\theta} = \rho\dot{Q}c_p(T_{o,\theta} - T_i) + \rho\dot{Q}h_{fg}(w_{o,\theta} - w_i)$$

$$\dot{q}_{vent,\theta} = 1.23\cdot 10\cdot 1.0\cdot(35.0 - 24) + 1.23\cdot 10\cdot 2500\cdot(0.0241 - 0.0092)$$

and

$$\dot{q}_{vent,\theta} = 593.48\ kW$$

As one can see that the amount of ventilation heat gain varies from 436 to 593 kW depending on what design condition is used. The question is what should be the appropriate value for ventilation heat gain to consider in our cooling load calculation. The design condition as we always use in our conventional method is obviously not the logical answer to this question since it is just the condition we select in order to accomplish the calculation.

Forming the least squares method in several distribution models, we found that the suitable PDF of dry-bulb temperature ($f_T(T_{o,\theta})$) and humidity ratio ($f_w(w_{o,\theta})$) are a normal distribution and the dry-bulb temperature and humidity ratio are assumed to be independent. Then using equation (14) the PDF of total ventilation heat gain can be found as follow:

$$f_{qv}(\dot{q}_{vent,\theta}) = \frac{1}{\rho\dot{Q}\sqrt{2\pi((\rho\dot{Q}c_p)^2\sigma_T^2 + (\rho\dot{Q}h_{fg})^2\sigma_w^2)}}\, e\left[-\frac{(\dot{q}_{vent,\theta} - \rho\dot{Q}c_p(E_T - T_i) + \rho\dot{Q}h_{fg}(E_w - w_i))^2}{2((\rho\dot{Q}c_p)^2\sigma_T^2 + (\rho\dot{Q}h_{fg})^2\sigma_w^2)}\right]$$

$$f_{qv}(\dot{q}_{vent,\theta}) = 0.004555\cdot e\left[-\frac{(\dot{q}_{vent,\theta} - 354.86)^2}{15335.51}\right]$$

The above equation is integrated to obtain the cumulative density function of ventilation heat gain. The result is shown in Figure 1.

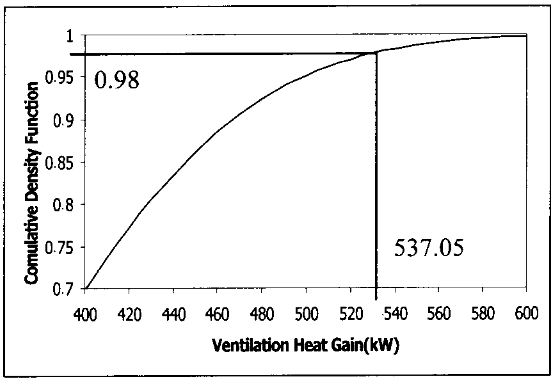

Figure 1: Cumulative density function of ventilation heat gain.

In Figure 1, various ventilation heat gain amounts are now equipped with their own cumulative density function. For example, at 98% of cumulative density function, it indicates that there is only a 2% chance that the ventilation heat gain will be more than 537.05 kW.

The results of ventilation heat gain amount from various design conditions as previously mentioned can be substituted into the graph shown in Figure 1 to find their own cumulative density function. Results are presented in Table 2:

Table 2 CDF for ventilation heat gain at various design conditions

Design Condition	Ventilation	CDF value
98% T_{db} and Mean T_{wb}	436.65 kW	82.49
98% T_{wb} and Mean T_{db}	579.33 kW	99.48
98% w and Mean T_{db}	530.75 kW	97.77
98% T_{db} and 98% w	593.48 kW	99.68
98% Total Heat Gain	537.05 kW	98.00

Using the information in Table 2, one can make a decision to pick up the value for ventilation heat gain with chances that they are willing to take for the 98% design condition.

6. Conclusion

The new concept for determining the cooling load which involves the probability density function is presented using the ventilation heat gain as an example. Under this method, the appropriate amount of cooling load can be logically determined using the known cumulative density function as a tool for decision making. Engineers can now state an amount of cooling load by knowing the probability that the load will occur. This helps in reducing the risk in air conditioning system investment.

7. Acknowledgements

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